Power-gradient velocity model

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Abstract

I propose a power-gradient velocity model which incorporates several well-known velocity models as special cases. The proposed velocity model covers wide range of possible velocity distributions and has four parameters, which gives more flexibility in velocity-model manipulation. For the proposed non-linear velocity model, I compute the kinematical characteristics: offset-traveltime parameteric equations, traveltime parameters, relative geometrical spreading and the phase of the propagator. The kinematical characteristics are investigated with respect to a parameter responsible for non-linearity of velocity distribution. The inversion of traveltime parameters is discussed in three- and four-parameter framework.

Introduction

The velocity model is very important for both seismic modeling and inversion. In order to invert the kinematic parameters obtained in velocity analysis into model parameters, the model has to be properly defined. The uncertainty in the velocity model is very important for inversion, interpretation (Hajnal and Sereda, 1981; Bickel, 1990; Lines, 1993; Al-Chalabi, 1997; Kosloff and Sudman, 2001) and imaging (Fomel and Landa, 2005).

There are many different velocity models which are used in seismic processing and interpretation in order to fulfill the inversion from reflection traveltimes, to test different seismic processing techniques and so on. It is very common to imply the models with velocity varying with depth only. The simplest velocity model is the one with constant velocity. The inversion of traveltime-parameters within the framework of this model is given by very simple Dix equations. If the velocity is varying with depth, the inversion requires more complicated techniques.

The most frequently applied vertically heterogeneous velocity models are: the linear velocity model and the linear sloth model.

In this paper I present the power-gradient velocity model which is a four-parameter model and incorporates several well-known velocity models as the special cases. When reduced to three-parameter model, the power-gradient velocity model results in the series of the kinematically equivalent velocity models (Stovas, 2007). To analyze the kinematic characteristics of the power-gradient velocity model, I choose the parameter that controls the curvature of the velocity function. For given values of this parameter, the power-gradient velocity model is reduced to the well-known velocity models.

I show that the non-hyperbolicity of the reflection curve is offset-dependent and illustrate the accuracy of all known traveltime approximations. The geometrical spreading and the phase factor from the propagator can be given in terms of traveltime parameters.

The four-parameter inversion from the power-gradient model shows the uncertainties from the inversion of traveltime parameters into the model parameters.

Definition

I introduce a four-parameter non-linear velocity model by

$$v(z) = v_0\left(1+\left(\gamma^n-1\right)\frac{z}{H}\right)^{1/n}, \quad 0 \le z \le H, \qquad (1)$$

where H is layer thickness, v_0 is velocity to the top of the layer, the velocity ratio parameter $\gamma = v(H)/v_0$, $n \in \mathbf{R}$ is the parameter controlling the curvature of velocity distribution. It is convenient to introduce the function

$$\Phi_n(\gamma) = \frac{\gamma^n - 1}{n}.$$

For $n = 0$ $\Phi_0(\gamma) = \ln\gamma$.

To illustrate the family of velocity models I choose layer thickness $H = 1km$, velocity to the top $v_0 = 2\,km/s$ and velocity ratio parameter $\gamma = 1.5$. The parameter n is taking the values: $0, \pm1, \pm2, \pm4, \pm8$. With respect to the function $v(z)$ it means that I fix the depth interval, the points of $v(z)$ at $z = 0$ and $z = H$, and manipulate with parameter n only. I shall name these models M_n. The velocity distributions for these models are illustrated in Figure 1. It is clear that parameter n controls the curvature of the velocity distribution. With $n > 1$, the velocity function is concave, and if $n < 1$, the velocity function is convex.

This model reduces to well-known velocity models for some specific values of the parameter n. With $n \to \pm\infty$, it is constant velocity model. The velocity models M_{-2}, M_{-1}, M_0, M_1 and M_2 are known as the linear sloth model, the linear slowness model, the exponential velocity model, the linear velocity model and the square root velocity model, respectively.

Offset-traveltime parametric equations

I derive the offset-traveltime equations

$$x_n(p)=\frac{2pv_0H}{\Phi_n(\gamma)(n+1)}DF(n+1)$$
$$t_n(p)=\frac{2H}{\Phi_n(\gamma)(n-1)v_0}DF(n-1)$$
$$DF(k)=\gamma^k\,{}_2F_1\left(\frac{k}{2},\frac{1}{2};\frac{k}{2}+1;p^2v_0^2\gamma^2\right)-{}_2F_1\left(\frac{k}{2},\frac{1}{2};\frac{k}{2}+1;p^2v_0^2\right), \qquad (2)$$

where ${}_2F_1(a,b;c;u)$ is the hypergeometric function. The equations (2) can be used for modelling and ray tracing. The traveltime curves are shown in Figure 2.

Traveltime parameters

The traveltime parameters for this model are:

$$t_0=\frac{2H}{v_0}\frac{\Phi_{n-1}(\gamma)}{\Phi_n(\gamma)}$$
$$v_{nmo}^2=v_0^2\frac{\Phi_{n+1}(\gamma)}{\Phi_{n-1}(\gamma)}$$
$$S_2=\frac{\Phi_{n+3}(\gamma)\Phi_{n-1}(\gamma)}{\Phi_{n+1}^2(\gamma)}, \qquad (3)$$
$$S_3=\frac{\Phi_{n+5}(\gamma)\Phi_{n-1}^2(\gamma)}{\Phi_{n+1}^3(\gamma)}$$
$$\ldots$$
$$S_k=\frac{\Phi_{n-1+2k}(\gamma)\Phi_{n-1}^{k-1}(\gamma)}{\Phi_{n+1}^k(\gamma)}$$

where t_0 is two-way vertical traveltime, v_{nmo} is normal moveout velocity and S_2, S_3 are heterogeneity coefficients of second and third order, respectively. S_k is the heterogeneity coefficient of order k. The coefficients S_2, S_3 are plotted versus n in Figure 3.

Fomel and Stovas (2007) proposed the generalized non-hyperbolic approximation based on an additional ray with the slowness $p_{\max}$. The total number of parameters in this approximation is five. The first three parameters are defined at zero offset and correspond to the traveltime parameters t_0, v_{nmo} and S_2. The last two parameters are defined from the additional ray. The approximation has the following form

$$t^2(\tilde{x})=t_0^2\left[1+\tilde{x}^2+\frac{1-S_2}{4y(\tilde{x})}\tilde{x}^4\right]$$
$$y(\tilde{x})=\frac{1}{2}\left(1+B_{FS}\tilde{x}^2+\sqrt{1+2B_{FS}\tilde{x}^2+C_{FS}\tilde{x}^4}\right) \qquad (4)$$

where $\tilde{x}=x/v_{nmo}t_0$ is the normalized offset and parameters B_{FS} and C_{FS} are

$$B_{FS}=\frac{1+3n}{10}(S_2-1)+\ldots$$
$$C_{FS}=\frac{1}{35}\left(1+6n-6n^2\right)(S_2-1)^2+\ldots \qquad (5)$$

Examples of velocity spectra by using equations (4) and (5) are shown in Figure 4. I also derive equations for the relative geometrical spreading (Figure 5) and one–way propagator (Figure 6) for the power-gradient velocity model.

Inversion of the traveltime parameters

The standard way to estimate the traveltime parameters is to perform the velocity analysis. Since models M_n are four-parameter models (layer thickness H, velocity to the top of the layer v_0, velocity ratio γ and parameter n), the accurate inversion requires four traveltime parameters to be estimated. In velocity analysis we compute the two-way traveltime, the normal moveout velocity and heterogeneity coefficients S_2 and S_3. The first problem is that there is no traveltime approximation which can be used to estimate four traveltime parameters. The maximum applicable number of traveltime parameters is three. In practice, estimation of S_3 from seismic data is hardly possible. The accuracy of the parameter estimation is decreasing with increase of the order of traveltime parameter. Nevertheless, let us first assume that we succeeded to estimate both S_2 and S_3. Then we have to solve equations for S_2 and S_3 (i.e., equations 3) for γ and n. There is no analytical solution for this problem. The computation of γ and n is a non-trivial task. The resolution of this problem is very low, especially for small values of γ. Not all combinations of S_2 and S_3 are physically possible. It is confirmed by the series for heterogeneity coefficients S_2 and S_3, where the parameter n comes into the series coefficient from the fourth order. It means that for low values of γ, we can estimate γ, not n. For large values of γ, the problem is non-unique if we consider uncertainties in heterogeneity coefficients. We also have to constrain heterogeneity coefficients such that $S_3\geq S_2\geq 1$. For a given values of S_2 and S_3 there is a minimum physically possible value of γ. From these considerations we can see that in practice the inversion of four traveltime parameters into the velocity model parameters within the framework of the power-gradient velocity model is hardly possible. If we consider the uncertainties in estimated traveltime parameters: $v_{nmo} = 2.8 \pm 0.025\, km/s$ and $S_2 = 1.03 \pm 0.01$, the depth dependent uncertainties in the velocity distributions depend on parameter n (as it shown in Figure 7).

Conclusions

I defined four-parameter power-gradient velocity model which has several well-known velocity models as special cases. The offset-traveltime equations are defined by the hypergeometric functions. The traveltime parameters and the series coefficients for traveltime squared are defined by the velocity model parameters. All known traveltime approximations are tested for this model, and the generalized non-hyperbolic approximation performs the best. The geometrical-spreading factor and the phase factor in one-way propagator are defined. The parameter n controls the curvature of the velocity model. This parameter is entering the higher-order series coefficients for all traveltime parameters, and if velocity ratio is closed to one, this parameter can be neglected.
The three- and four-parameter inversion is discussed for the power-gradient velocity model. I show that it is practically impossible to resolve the heterogeneity coefficients for this model against velocity ratio and parameter n. The uncertainty in the estimated normal-moveout velocity and the heterogeneity coefficient S_2 results in uncertainty in the model parameters and all kinematic characteristics.

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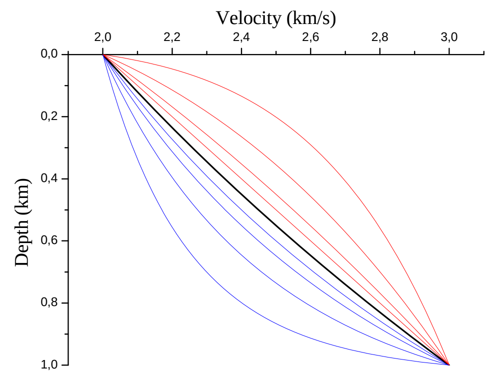

Figure 1. Velocity profiles for the models $M_n, n = 0, \pm1, \pm2, \pm4, \pm8$. The sign of n indicated by the line colour: blue lines for negative n and red lines for positive n. The black line corresponds to $n = 0$.

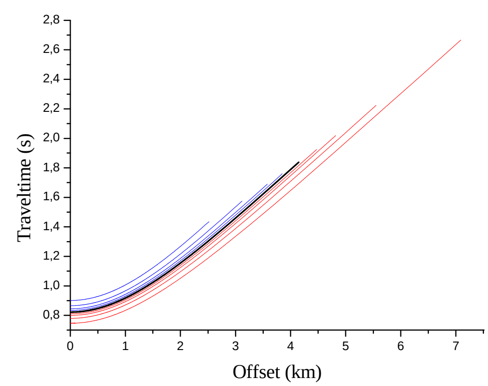

Figure 2. The traveltime curves for models shown in Figure 1.

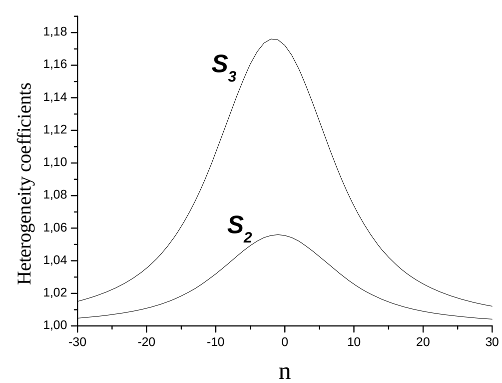

Figure 3. The heterogeneity coefficients S_2 and S_3.

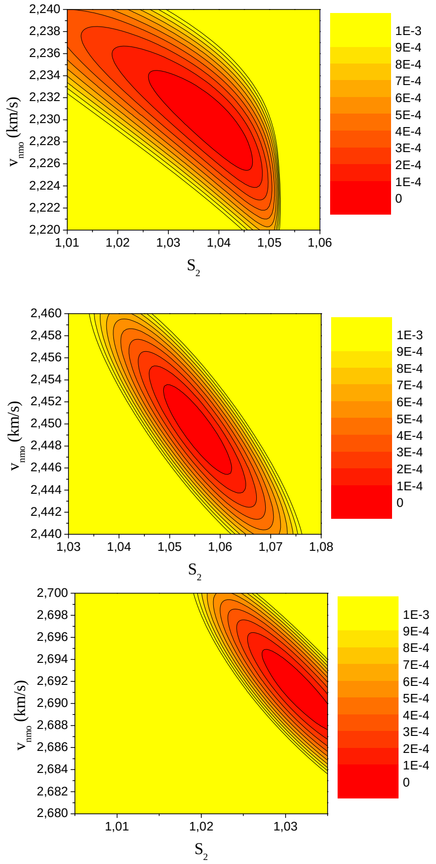

Figure 4. The velocity spectra for models $M_n, n = -8, 0, 8$ (from top to bottom) using the generalized traveltime approximation.

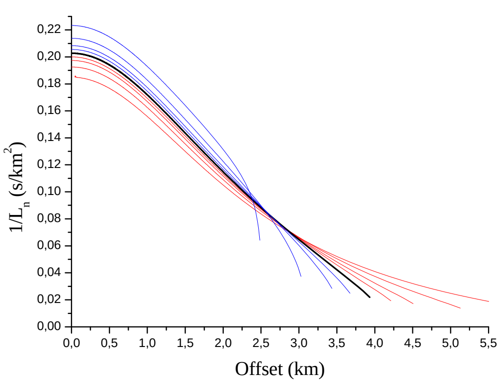

Figure 5. The relative geometrical spreading inverse for models shown in Figure 1.

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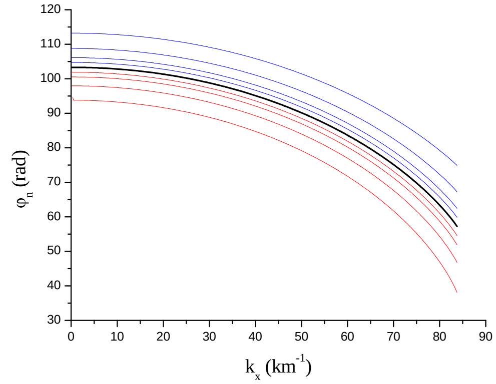

Figure 6. The phase factor for one-way propagator for models shown in Figure 1.

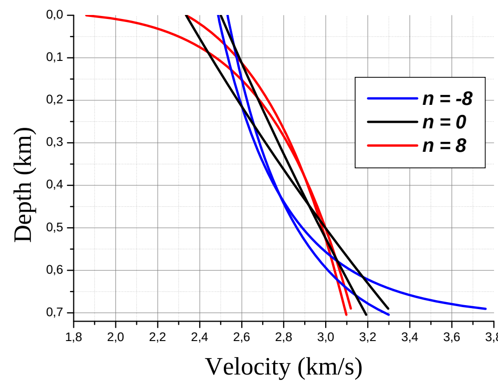

Figure 7. The depth-dependent uncertainties in the velocity distribution for models $M_n, n = -8, 0, 8$ due to uncertainties in estimated traveltime parameters.