Bandwidth extension using harmonics

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Abstract

We show a new method of seismic resolution improvement using the Continuous Wavelet Transform (CWT). Using the CWT and the available bandwidth in the seismic, the phase and amplitude spectra of harmonics and sub-harmonics can be computed. These harmonic and sub-harmonic frequencies are then convolved onto the input data. Only frequencies for reflectivity that is above the ambient noise level in the CWT domain is added to the seismic wavelet. This process broadens the bandwidth of the signal which increases the resolution of the seismic data.

Introduction

The subsurface is neither fully elastic nor homogenous and as a result we have dissipation of high frequency energy (conversion to heat) and velocity dispersion. All of these effects give us a distorted and stretched wavelet (Wang, 2006). The end result is poorer resolution of our seismic data.

The algorithm we introduce attempts to recover the lost wavelet characteristics by using the available bandwidth in the seismic data. The available bandwidth acts as the fundamental frequencies, for which harmonics will be computed from, and added back into the wavelet by a convolutional like process in the CWT domain. This effectively reshapes the wavelet and broadens the spectrum. Any harmonic frequencies that do not match reflectivity above the ambient noise level in the CWT domain will not remain in the final result.

This process is not limited to the high frequency end only. The same algorithm can be applied to the low end of the spectrum by computing sub-harmonics from the fundamental frequencies (available bandwidth). This can be important in areas where much of the lower frequency data has been suppressed due to ground roll and other low frequency noise trains.

Theory

The CWT is defined as the convolution of a time series f(t) with a scaled (s) and translated (τ) wavelet $\Psi(t)$.

$$W(\tau,s)=\int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{|s|}}\psi*\left(\frac{t-\tau}{s}\right)dt$$

where (*) indicates the complex conjugate

The wavelet Ψ must meet the admissibility condition if the analyzing wavelet is going to be used to reconstruct the original time series (Qian, 2002).

The admissibility condition is given by:

$$C_\Psi=\int_{-\infty}^{\infty}\frac{|\Psi(\omega)|^2}{|\omega|}d\omega<\infty$$

where $\Psi(\omega)$ is the Fourier transform of $\psi(t)$.

The Admissibility Condition implies that $\Psi(\omega)=0$ for $\omega = 0$ and $\int_{-\infty}^{\infty}\psi(t)dt=0$. This tells us that $\psi(t)$ has a zero mean and is a wavelet.

The scaled wavelets are called daughter wavelets as they are scaled from the mother wavelet ψ**.** Because the implementation of the CWT is a discrete operator and not a truly continuous operator, a choice needs to be made as to how many daughter wavelets will be used, thus how much redundancy. A minimum of 10 voices per octave is sufficient to recreate the input time series from the transform by computing its reconstruction.

The CWT, although highly redundant, provides a very detailed description of the signal in terms of time and frequency (Walker, 1999). These properties are utilized to predict the harmonics and sub-harmonics used for bandwidth extension.

We chose the Morlet wavelet as our mother wavelet which is a complex function representing a plane wave modulated by a Gaussian. The complex nature of the wavelet permits the calculation of amplitude and phase for each scale at distinct times. The Morlet wavelet is given by:

$$\psi_0(t)=\pi^{-1/4}e^{i\omega_0\eta}e^{-t^2/2}$$

where the real and imaginary parts are:

$$\psi(t)_R=\frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}\cos(2\pi\omega_0 t)$$

$$\psi(t)_I=\frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}\sin(2\pi\omega_0 t)$$

The Morlet wavelet is also optimal in that it closely follows the uncertainty principle as it is defined in time-series analysis. This helps to give us an optimal distribution of time vs. frequency balance (Teolis, 1998). The uncertainty principle places limits on our time-frequency analysis. These limits prevent us from knowing both the exact frequency and time simultaneously.

To reconstruct the time series, a double integral is required since we went from a function in time f(t) to a function of time and scale W(τ,s). The reconstruction formula is given by:

$$f(t)=\frac{1}{C_{\Psi}}\int_{0}^{\infty}\int_{-\infty}^{\infty}\frac{1}{\sqrt{s}}W(\tau,s)\psi\left(\frac{t-\tau}{s}\right)\frac{ds\,d\tau}{s^{2}}$$

where C_{Ψ} is given by the Admissibility Condition.

One other limitation should be noted in this process of bandwidth extension. Any such endeavor will be limited by the Sampling Theorem and can not recover reflectivity information higher than the Nyquist frequency. Furthermore, because of leakage from filters (side lobes), anti-aliasing filters often limit the maximum recoverable reflectivity to less than Nyquist.

Examples

The first example is a classic wedge model with equal positive reflectivity for both the top and bottom reflectors. The top reflector is flat, and the bottom reflector has a dip of .3ms per trace. The frequency content of the low frequency wedge is 7-55 Hz. The bandwidth extended wedge and the high frequency wedge both have frequency content of 7-85 Hz. As can be seen in this example the resolution of the low frequency wedge by has been significantly improved by bandwidth extension.

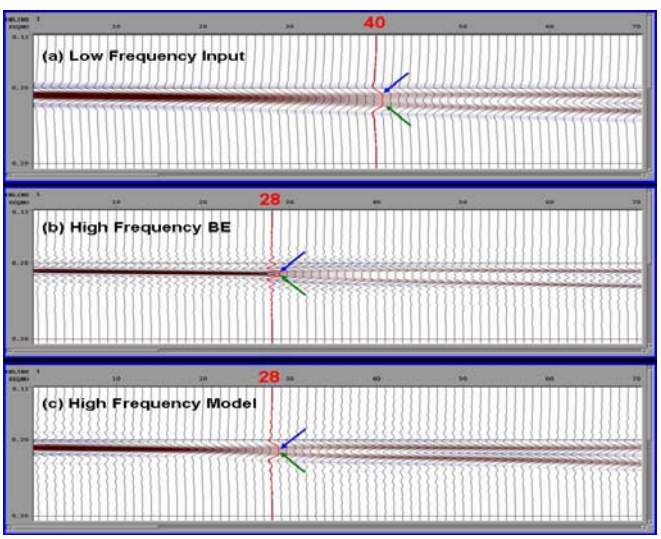

Figure 1 Bandwidth extension applied to a wedge model. (a) Low-frequency synthetic input to bandwidth extension. (b) High-frequency extension of input (top). (c) High frequency synthetic to compare to bandwidth extension results. Blue and green arrows indicate the limit of resolution of the top and bottom reflectors of the wedge respectively.

Real Data Examples

The next example is from an onshore 3D survey. The wells ties are shown along with the cross-correlations for the well ties. The input data (figure 2) has large side lobes indicated on the cross-correlation for the tie. This suggests that although the tie is good, the data probably has a "ringy" (repetitive) appearance (figure 4). After bandwidth extension we see a much better cross-correlation with the peak/trough ratio much smaller for the side lobes. Figure 5, shows the bandwidth extended data and it can be seen that the data has a much better character than the input data (figure 4).

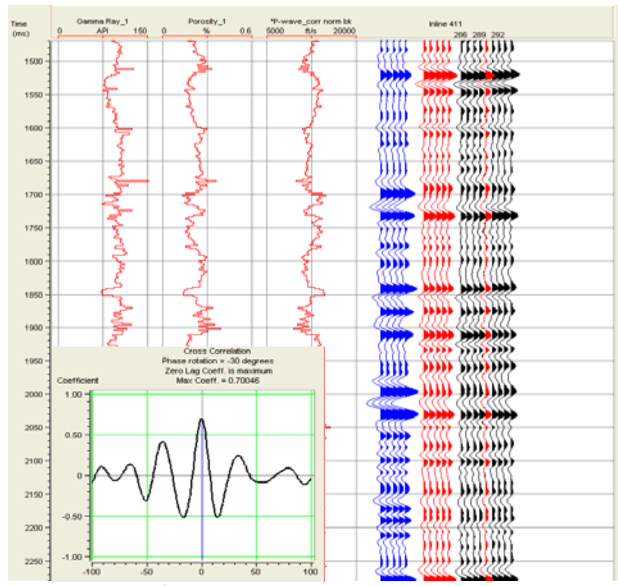

Figure 3 Low frequency input data well tie. Note the large side lobes on the cross-correlation.

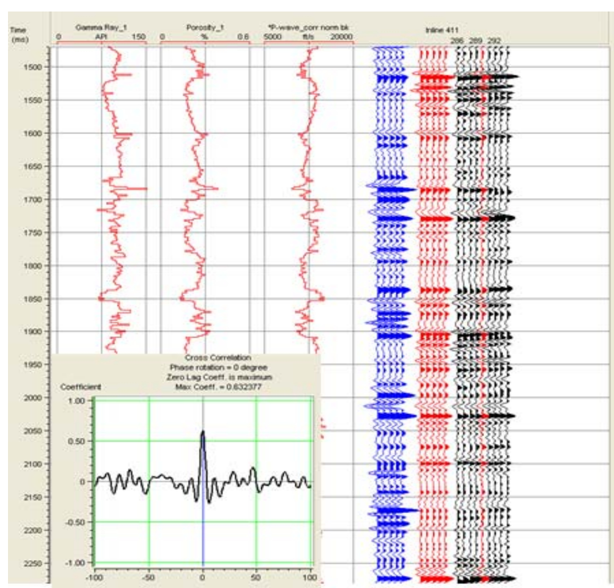

Figure 4 Bandwidth extended data well tie. Note the small side lobes on the cross-correlation,

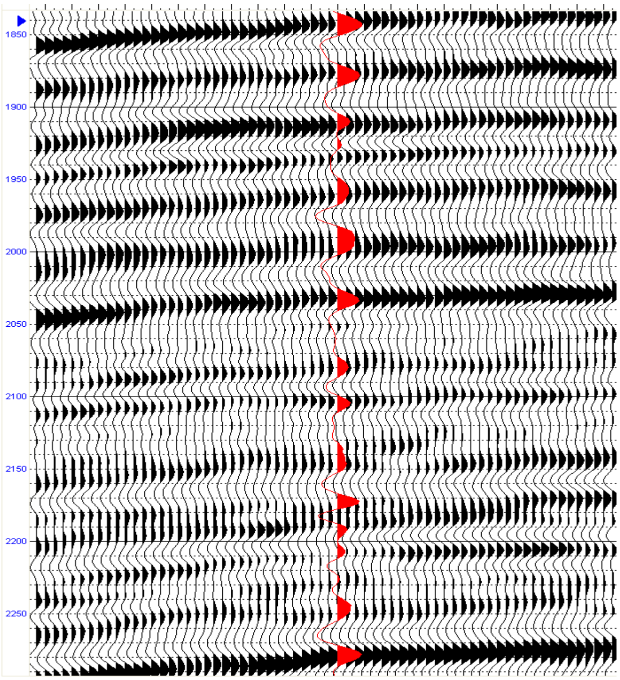

Figure 5 Low frequency input data 17-55Hz showing "ringy" or repetitive character. Red curve is an overlaid zero offset synthetic with an extracted wavelet.

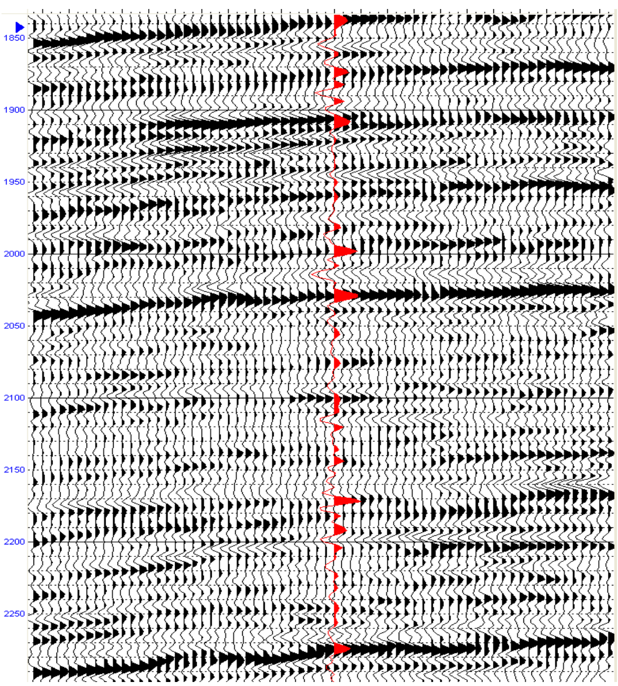

Figure 6 Bandwidth extended data 12-120Hz showing more character than input data. Red curve is an overlaid zero offset synthetic with an extracted wavelet.

Our final example is from another 3D onshore survey. The large amplitude reflector is a shale-limestone interface (figure 7). The limestone when porosity is present and is on a high structurally is usually a good gas reservoir. On the low frequency data it is difficult to ascertain as to whether porosity is present in the limestone. The bandwidth extended data shows interesting character that is related to the presence of porosity in the limestone (figure 8).

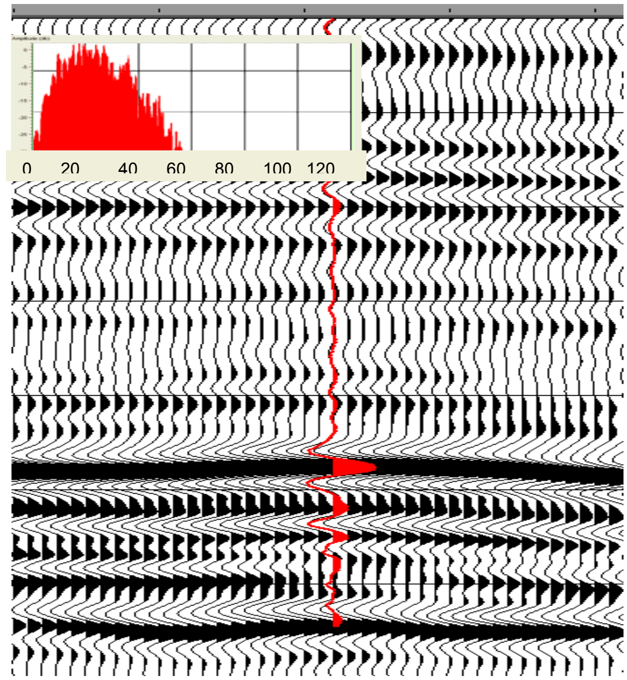

Figure 7 Low frequency data, large amplitude peak is shale-limestone interface. Red curve is zero-offset synthetic overlaid with extracted wavelet.

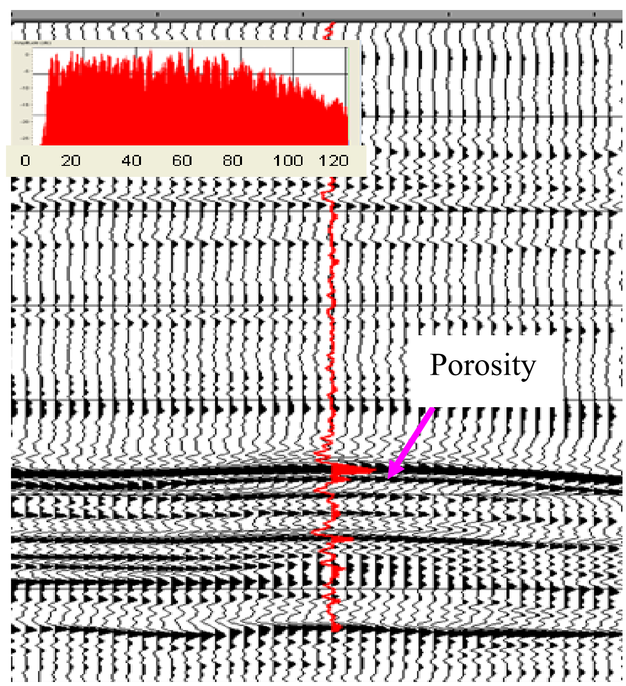

Figure 8 Bandwidth extended data porosity zone is now evident. Red curve is zero-offset synthetic overlaid with extracted wavelet.

Conclusions

Bandwidth extension has been demonstrated to be possible and fruitful for seismic enhancement and interpretation. There are limitations: the Sampling Theorem limits the maximum recoverable reflectivity to Nyquist and anti-aliasing filters will often set this limit below Nyquist.

The 2-layer wedge model demonstrated that bandwidth extension can help resolve pinch outs and other similar stratigraphic features. The real data examples show that besides just enhancement of bandwidth we also get the benefit of reduced “ringy” character and can help reveal such stratigraphic or facies changes related to porosity changes.

The Widess Model (Widess 1975) suggests that there is seismic reflectivity available below the dominant frequency wavelength. This information can be extracted, resulting in an increase in resolution by adding harmonic frequencies back to the data. Once this is done many features such as minor faults, on-laps, pinch outs and other stratigraphic features come to light. All of these features can have a significant impact on interpretation of seismic data.

References

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