A comparison of two true-amplitude Gaussian beam migration/inversion operators

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Abstract

The use of the concept of Gaussian beams in seismic migration has been highly considered in recent times and it is not a new issue. With the advent of non-conventional and mandatory new exploration targets in Brazil (such as the presalt), the imaging task has been highly demanded in order to bypass common (known ones) and new features of the wavefield propagation, such as anisotropy and some other factors. Since migration algorithms today are not anymore imaging-only process, comes into play the task of inversion, alongside with interpretation, prospecting and drilling. This paper then compares the theoretical results of two true-amplitude migration algorithms and their abilities as attributes estimators.

Introduction

In 1954 J. G. Hagedoorn published an article entitled "*A process of seismic reflection interpretation*" in the *Geophysical Prospecting* journal where he introduced an heuristic seismic imaging technique. This graphical imaging construction technique became known as the "swinging arm technique", "string construction" or "ruler and compass method" and, according to some authors, constitutes what it is today known as Kirchhoff migration/inversion. In summary the method uses isochrones and diffraction surfaces to construct coherent events either in the time and depth domains.

Although Hagedoorn's ideas considered only zero-offset data, it was pioneering in its sense of the duality among the (acquired) seismic data and its respective diffractors within the Earth. The images of reflection surfaces, in this manner, are obtained by a superposition of arcs of circles in which the envelope of a set of these circles forms a curve where every point of it satisfies the condition of especular reflection. With the development of the oil industry and the growing use of digital computers in the seismic data processing, the graphical technique was gradually substituted by robust and efficient algorithms that perform the same task, wherein among them the Kirchhoff is the most known and widely used, especially in 3D processing. All Hagedoorn's original ideas are present in the Kirchhoff migration/inversion algorithm, including velocity estimation and frequency filtering.

Apparently the first works to reproduce Hagedoorn's graphical ideas date from the beginning of the 70's, including French (1974). But the best reference related to its application to seismic data is referred to Schneider (1978). In this work only the process of migration of zero-offset data is considered as the main objective in 2D and 3D media. Inversion works – i.e., process of determination of seismic attributes at the same time as imaging – are first referred in works of Cohen and Bleistein (1979) and Clayton and Stolt (1981). In these works inversion is obtained through multifold integrals along several domains and through the Born-WKBJ scattering theory, by the use of Fourier transform techniques. Only after Beylkin (1985) migration/inversion theory took the shape it has today: Cohen et al. (1986), Sullivan and Cohen (1987), Bleistein et al (1987) and Bleistein (1987) are examples of works where inversion according to Beylkin's idea is analyzed to 2D, 2.5D and 3D media. The papers by Bleistein (1999), in the *The leading edge*, and the one by Bleistein and Gray (2000), in the *Geophysical Prospecting*, discuss how the graphical imaging of Hagedoorn was substituted by the Kirchhoff migration/inversion.

The Beylkin (1985) approach of an inversion operator for the problem of a wavefield scattering is based on the imaging of discontinuities present in the propagation media considered, where pseudo-differential operators associated to Radon transform properties are used. Interpreting this inversion for the seismic case, Cohen et al (1986), Bleistein (1987) and Bleistein et al. (1999) considered the mapping of these "physical properties" as band-limited Dirac delta functions that are present, for example, in the modeling operators or when the seismic data is acquired. Thus, when defining an operator for the modeling (forward) problem, it must be inserted in its context some function that represents the discontinuities present in the media, such as the reflection coefficient or a perturbation in the velocity field. When these functions are inserted in the inversion equation, a mapping is associated to a discontinuity function in itself, by the use of the properties of the Dirac delta function, which permits to determine an approximate form of the weight-function of the inversion operator. This procedure introduces in the inversion scheme the famous Jacobian known as the Beylkin determinant, which represents the transformation of variables ξ_1, ξ_2, ω to k_1, k_2, k_3.

In this work we theoretically investigate the use of the Bleistein-Cohen formalism for an inversion operator in a set of seismic data formed by the superposition of Gaussian beams (GB's). Disregarding terms related to the Beylkin determinant and the products of amplitude terms – the cases analysed so far by Bleistein (1987) – the resultant weight-function then derived here contains terms (i.e., functions) that are referred to the considered media. In the case of the GB's, these terms are the function that transforms one surface integral into a volume integral and the weight-function of GB's superposition integral. The latter weight-function agrees with the one derived in Ferreira (2006). After that, we then compare the present migration/inversion operator to the operator derived by Albertin et al. (2004).

Our derivation here shows that the two operators are equivalent as a local slant-stacking process, except for some terms regarding the weight-function of the GB's, but they differ in the sorting of the data. Even so, we successively show that, after some small manipulations of the resulting operator and some a priori consideration, the GB operator yields Albertin's true-amplitude operator.

Gaussian beams and the inversion operator

Following Bleistein (1987), who applied the Bleistein-Cohen methodology to Kirchhoff-modeled seismic data, in what follows we advocate the same considerations using a superposition of GB's (Červený, 2001), which also represents a high-frequency approximation of the seismic wavefield.

The GB operator is an integral that represents in 3D a summation of (paraxial) rays that depart from the (point or line) source and, after traversing several layers of isotropic and smooth set of reflectors (i.e., one seismic system, see Bortfeld, 1989), are reflected over a specific surface, proceeding upwards, being detected by a dense coverage of geophones. These rays are summed along their emergence points and their contributions are registered in their vicinities, in a given reference geophone. This integral operator, in the frequency domain, is given by:

$$\psi(\vec{\xi},\omega)=\frac{i\omega}{2\pi}\int_{A_P(\vec{\xi})} d^2\vec{\xi}^P\,\sqrt{det\,\mathbf{H}_P(\vec{\xi}^P)}\;D_L(\vec{\xi}^P,\vec{\xi})\,\psi(\vec{\xi}^P,\omega)\,\mathrm{e}^{-i\omega\phi_B(\vec{\xi}^P,\vec{\xi})} \quad (1)$$

where $\phi_B(\vec{\xi}^P,\vec{\xi})=\vec{\mathbf{p}}^T(\vec{\xi}^P-\vec{\xi})+\frac{1}{2}(\vec{\xi}^P-\vec{\xi})^T\mathbf{H}_P(\vec{\xi}^P-\vec{\xi})$ is the complex paraxial traveltime at $\vec{\xi}$ (observation point or reference trace) due to the traveltime of a ray that emerged at $\vec{\xi}^P$. Here, A_P is the projected Fresnel zone aperture for trace $\vec{\xi}$; $\vec{\mathbf{p}}$ is the central ray slowness-vector; $\mathbf{H}_P(\vec{\xi}^P)$ is the projected Fresnel zone matrix; $D_L(\vec{\xi}^P)$ is the Gaussian taper function and $\psi(\vec{\xi}^P,\omega)$ represents a window of the seismic data $\psi(\vec{\xi},\omega)$ located inside the Fresnel zone aperture A_P. So, the seismic data on the left hand side of Eq. (1) is a "copy" of itself, wherein each trace then carries the local stacking of paraxial events (amplitudes) reflected inside every Fresnel zone, for each seismic experiment between a source and a receiver, contained in the integration domain A_P. The Gaussian taper function, centered at the reference-trace $\vec{\xi}$, weighs the relative contributions smeared along $\phi_B(\vec{\xi}^P,\vec{\xi})$, inside each A_P.

In the original notation of Červený (2001), the integrating parameters are usually the take-off angles of the rays or, generally speaking, any ray parameters representing individual seismic rays. Here our interpretation of the integrating parameters is that they are projections of the area of the Fresnel zone in depth, around the reflection point, towards the acquisition surface. This mapping (Schleicher et al., 2002) is linear and centered around the beam that is supported by a central ray, containing its Fresnel volume.

Ferreira (2006) used the operator described in Eq. (1) into the kernel of a true-amplitude, diffraction stack, 3D migration inversion operator. His interpretation of the modeling operator, based on the asymptotic ray theory, led to the definition of the weight-function of the GB operator as a function of the Fresnel volume elements of the seismic wavefield (Kravtsov and Orlov, 1980). This is considered as a physical interpretation of the GB weight-function, since the former well-known interpretation given by Klimeš (1984) is only numeric.

Here the inversion operator (Cohen et al., 1986; Bleistein, 1987) to be considered is defined as

$$\beta(\vec{\mathbf{x}})=\int_A d^2\vec{\xi}\;b(\vec{\mathbf{x}},\vec{\xi})\int i\omega d\omega F(\omega)\mathrm{e}^{-i\omega\phi(\vec{\mathbf{x}},\vec{\xi})}\,\psi(\vec{\xi},\omega). \quad (2)$$

The quantities appearing in Eq. (2) are: $F(\omega)$, the source spectrum; *A*, migration aperture; $\vec{\mathbf{x}}$, vector-coordinates of a imaging point in depth; $\vec{\xi}$, source-receiver parameter coordinate vector; ω, the angular frequency; $\phi(\vec{\mathbf{x}},\vec{\xi})$, diffraction traveltime (Huygens surface); and $\psi(\vec{\xi},\omega)$, the seismic data to be inverted. The philosophy of inversion is that inserting the representation of the seismic wavefield given by Eq. (1) into Eq. (2), this mapping should result in:

$$\beta(\vec{\mathbf{x}})=\int_V d^3\mathrm{x}'\,\beta(\vec{\mathbf{x}}')\,\delta(\vec{\mathbf{x}}'-\vec{\mathbf{x}})\;. \quad (3)$$

Eq. (3) is a mapping of the reflectivities existing in the Earth where it is, i.e., over the reflector surfaces. Using a Born modeling operator, Cohen et al. (1986) derived the following weight-function for the inversion operator

$$b(\vec{\mathbf{x}},\vec{\xi})=\frac{1}{8\pi^3}\frac{c^2(\vec{\mathbf{x}})\left|h(\vec{\mathbf{x}},\vec{\xi})\right|}{a(\vec{\mathbf{x}},\vec{\xi})}\;. \quad (4)$$

in which $\left|h(\vec{\mathbf{x}},\vec{\xi})\right|$ is the absolute value of the Beylkin determinant, $c^2(\vec{\mathbf{x}})$ is the square of the medium reference velocity and $a(\vec{\mathbf{x}},\vec{\xi})$ is the product of amplitudes among the triplets of the coordinates $\vec{\mathbf{x}},\vec{\xi}$. If, by chance, the data to be inverted were of the Kirchhoff-type, one additional transformation should be included in the formalism, which transforms a surface integral into a volume integral, and the mapping represented by Eq. (3) should proceed normally.

In the next section it is shown the formalism used to get a similar result as in Eq. (4), considering the use of Eq. (1).

The weight-function for the GB data

An inversion operator, in the Cohen-Bleistein approach, is represented by a volume integral in the variables $\vec{\xi}=(\xi_1,\xi_2)^T$ and ω acting over a filtered version of the seismic data $\psi(\vec{\xi},\omega)$. This is represented in Eq. (2). It is considered that $\psi(\vec{\xi},\omega)$ must contain the reflectivity function $\beta(\vec{\mathbf{x}}')$ to map the wavefield via a Dirac completness relation [see Eq. (3)]. And, last but not least, $\psi(\vec{\xi},\omega)$ must represent one approximation that maps reflection points $\vec{\mathbf{x}}'=(x_1',x_2')^T$ over some reflecting surface and register them over an acquisition surface in positions parameterized by $\vec{\xi}$ [see Eq. (1)]. In the present case we consider that the seismic data is of zeroth order ray theory and represented by $\psi(\vec{\xi}^P,\omega)=A(\vec{\xi}^P)\,e^{i\omega\phi(\vec{\xi}^P)}$.

A priori Eq. (1) does not seem to be subtle to be used in the Cohen-Bleistein inversion method because the integral used to describe the seismic wavefield is written in projected surface coordinates. We then rewrite Eq. (1) using the linear relationship $\vec{\xi}^P=\mathbf{\Lambda}^{-1}\mathbf{H}_P\,\vec{\mathbf{x}}'+\vec{\xi}$ (Schleicher et al., 1997) among projected and in depth coordinates $\vec{\mathbf{x}}'=(x_1',x_2')^T$, and following Ferreira (2006) approach. Then the following mapping is obtained: $A(\vec{\xi}^P)=A(\vec{\mathbf{x}}',\vec{\xi})$, $\phi(\vec{\xi}^P)=\phi(\vec{\mathbf{x}}',\vec{\xi})$, $\phi_R(\vec{\xi}^P,\vec{\xi})=\phi_R(\vec{\mathbf{x}}')$ and $D_L(\vec{\xi}^P,\vec{\xi})=D_L(\vec{\mathbf{x}}')$. The Jacobian of the transformation of variables is written as

$$d^2\xi^P=\left|\frac{\partial(\xi_1^P,\xi_2^P)}{\partial(x_1',x_2')}\right|d^2x'=\frac{det\,\mathbf{H}_F(\vec{\mathbf{x}}')}{det\,\mathbf{\Lambda}}\,d^2x' \qquad (5)$$

That follows from the decomposition of the transformation Jacobian into products between the projection of A_F (Fresnel zone in depth) into a tangent plane centered at the reflection point and the projection of A_F towards A_P over the acquisition surface (Ferreira, 2006). Eq. (1) in the new coordinates is now given by

$$\psi(\vec{\xi},\omega)=-\frac{i\omega}{2\pi}\int_{A_F}d^2x'\sqrt{det\,\mathbf{H}_F(\vec{\mathbf{x}}')}\,D_L(\vec{\mathbf{x}}')\,e^{i\omega\phi_B(\vec{\mathbf{x}}')}\,A(\vec{\mathbf{x}}',\vec{\xi})e^{i\omega\phi(\vec{\mathbf{x}}',\vec{\xi})}\,. \qquad (6)$$

Eq. (6) was "deprojected" from the projected Fresnel zone A_P, on the acquisition surface, towards the reflecting element A_F, i.e., the Fresnel zone area in depth. Considering that a ray can be decomposed into two branches (Hubral et al., 1993), the amplitude function $A(\vec{\mathbf{x}}',\vec{\xi})=\dfrac{a(\vec{\mathbf{x}}',\vec{\xi})}{p_{zS}\,p_{zR}}R(\vec{\mathbf{x}}',\vec{\xi})$ and the phase function $\phi(\vec{\mathbf{x}}',\vec{\xi})$ are to be viewed as products and sums of individual branch terms, respectively. We have then a complete picture of ray tracing and suitable components for the inversion scheme we are claiming here. In the decomposition of the amplitude term above, $R(\vec{\mathbf{x}}',\vec{\xi})$ is the reflection coefficient.

To transform Eq. (6) into a volume integral, we must introduce a function $\gamma(\vec{\mathbf{x}}')=\delta(x_3-\Sigma(x_1,x_2))$ with support only over the Fresnel zone A_F portion of the reflector represented by $\Sigma(x_1,x_2)$. The multiplication of this singular function with the reflection coefficient, e.g. $\beta(\vec{\mathbf{x}}')=R(\vec{\mathbf{x}}',\vec{\xi})\gamma(\vec{\mathbf{x}}')$, gives rise to the reflectivity function we were searching for (Cohen et al., 1986) and that is the discontinuity mapped by the present inversion scheme. With these considerations, Eq. (6) becomes

$$\psi(\vec{\xi},\omega)=-\frac{i\omega}{2\pi}\int_{V_F}d^3x'\sqrt{det\,\mathbf{H}_F(\vec{\mathbf{x}}')}\,D_L(\vec{\mathbf{x}}')\,e^{i\omega\phi_B(\vec{\mathbf{x}}')}\,a(\vec{\mathbf{x}}',\vec{\xi})\beta(\vec{\mathbf{x}}')e^{i\omega\phi(\vec{\mathbf{x}}',\vec{\xi})} \qquad (7)$$

The physical meaning of Eq. (7) is clear now. It simply states that given the Fresnel volume for a (central) seismic ray, the cross-sections of this volume that happen to be located over reflector surfaces and that are intercepted by the volume when the central ray reflects over some point on these surfaces determine the Fresnel zone A_F for the reflection point at R. The volume integral evaluates the support of its third coordinate x_3 only when surface-function $\Sigma(x_1,x_2)$ is equal to A_F, and this collapses the volume integral into a surface integral. Thus, only the region belonging to A_F contributes then to the observation in $\vec{\xi}$. The mapped reflectivity $\beta(\vec{\mathbf{x}}')$ determines then the physical property of only the area A_F and not solely that of the reflection point in R.

Since all the terms in Eq. (7), including the reflectivity-function $\beta(\vec{\mathbf{x}}')$, are functions only of $\vec{\mathbf{x}}'=(x_1',x_2')^T$, the kernel of the inversion in Eq. (2) follows the paths described in Cohen et al. (1986). But there is one difference that must be taken into account: the oscillatory factor $e^{i\omega\phi_B(\vec{\mathbf{x}}')}$, along with the factors including the Gaussian taper function and the square root of the determinant of the Fresnel zone matrix, cannot be simply eliminated by the weight-function $b(\vec{\mathbf{x}},\vec{\xi})$. In this case the only possible result for the weight-function on the inversion is

$$b(\vec{\mathbf{x}},\vec{\xi})=\frac{1}{8\pi^3}\frac{\left|h(\vec{\mathbf{x}},\vec{\xi})\right|}{a(\vec{\mathbf{x}},\vec{\xi})}\,. \qquad (8)$$

This equation again grants that the mathematical manipulation described so far transforms Eq. (2) into a 3D Dirac delta function. But, this time, Eq. (3) will be of the form

$$\beta(\vec{\mathbf{x}})=\int_{V_F}d^3x'\,w_B(\vec{\mathbf{x}}')\,\beta(\vec{\mathbf{x}}')\,\delta(\vec{\mathbf{x}}'-\vec{\mathbf{x}})=w_B(\vec{\mathbf{x}})\,\beta(\vec{\mathbf{x}}), \qquad (9)$$

where $w_B(\vec{\mathbf{x}}')$ is a function that carries all the terms that are only function of $\vec{\mathbf{x}}'=(x_1',x_2')^T$. Eq. (9) means that we have obtained a *weighted inversion* and that there must be one controlling factor for this weigh. The answer for this dilemma is in the oscillatory factor $e^{i\omega\phi_B(\vec{\mathbf{x}}')}$, which is function of the knowledge of the slowness vector $\vec{\mathbf{p}}$ and of the Fresnel zone matrix $\mathbf{H}_F(\vec{\mathbf{x}}')$. The choice of one of these two controlling factors precludes the use of the other. While vector $\vec{\mathbf{p}}$ controls the illumination, the matrix $\mathbf{H}_F(\vec{\mathbf{x}}')$ is related to the size of the Fresnel zone in depth, a frequency-dependent factor. Although similar in physical meaning, the approach differs for the inversion scheme. But since our aim is to compare it to the inversion operator studied by Albertin et al. (2004), the choice remains with vector $\vec{\mathbf{p}}$ as key controlling factor. Thus, Eq. (9) must be rewritten as

$$\beta(\vec{\mathbf{x}})=\int_{-\infty}^{+\infty}d^2p\int_{V_F}d^3x'\,w_B(\vec{\mathbf{x}}',\vec{\mathbf{p}})\,\beta(\vec{\mathbf{x}}')\,\delta(\vec{\mathbf{x}}'-\vec{\mathbf{x}})=\int_{-\infty}^{+\infty}d^2p\,w_B(\vec{\mathbf{x}},\vec{\mathbf{p}})\,\beta(\vec{\mathbf{x}}). \qquad (10)$$

The final inversion operator then becomes

$$\beta(\vec{\mathbf{x}}) = \frac{1}{8\pi^3}\int_A d^2\xi \left|h(\vec{\mathbf{x}},\vec{\xi})\right| \int_{-\infty}^{+\infty} d\,\omega i\,\omega \int_{-\infty}^{+\infty} d^2 p \frac{p_{zS}p_{zG}}{a(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})} \mathrm{e}^{-i\omega\phi(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})}\, \psi(\vec{\xi},\vec{\mathbf{p}},\omega), \quad (11)$$

which means that the transformation in Eq. (1) must now be viewed as a *local slant stacking* of the seismic data in the A_P domain of integration with respect to the controlling factor $\vec{\mathbf{p}}$. The consequences for the controlling factor considering the frequency-dependent Fresnel zone matrix $\mathbf{H}_F(\vec{\mathbf{x}}')$ will not be discussed here.

Another consequence of this whole process of an inversion is the fact that the weigh-function chosen for the GB operator in Eq. (1), introduced in Ferreira (2006), is corrected when related to the elements of the Fresnel volume raytracing.

Comparison with the inversion operator of Albertin et al. (2004)

Albertin et al. (2004) describe a ray-based beam inversion theory for common-offset data using a true-amplitude imaging GB operator. Their approach starts with a Kirchhoff scattering formula for the forward-modeling operator using the Green's function theorem (Morse and Feshbach, 1954). In their case, the Green's functions are expanded into Gaussian beams as follows

$$G_S(\vec{\xi},\vec{\mathbf{x}},\omega) = \frac{i\,\omega}{2\pi}\int_{-\infty}^{+\infty} \frac{d^2 p}{p_{zS}}\, \tilde{a}(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})\, \mathrm{e}^{i\omega\phi_S(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})}\,, \quad (12)$$

where $\tilde{a}$ and ϕ_S are, respectively, the complex amplitude and traveltime of the beam for the source *S* location and $\vec{\mathbf{p}} = (p_1, p_2)^T$ is the take-off ray parameter vector. Inserting Eq. (12) for the source – and, similarly, its contribution G_R for the receiver – into the forward-modeling equation, results in a general scattering equation that considers the contributions of plane-waves from sources and receivers. The data is then sorted in common-offset parameters and local slant stacked, and the slowness integrals evaluated as steepest descents, giving rise to a forward-scattering equation for a single beam center *p*-component of the wavefield (in their notation) given by

$$\psi(\vec{\xi}_0,\vec{\mathbf{p}},\omega) = i\,\omega \int_{-\infty}^{+\infty} d^2 x \frac{\left|\nabla\phi_{Re}(\vec{\xi}_0,\vec{\mathbf{p}},\omega)\right|}{p_{zS}\, p_{zR}}\, a(\vec{\xi}_0,\vec{\mathbf{x}},\vec{\mathbf{p}})\, \mathrm{e}^{i\omega\phi(\vec{\xi}_0,\vec{\mathbf{x}},\vec{\mathbf{p}})}\, R(\vec{\mathbf{x}})\,, \quad (13)$$

in which ϕ_{Re} refers to the real part of the total traveltime ϕ for a fixed *h*, and the amplitude term is the product of common-offset amplitudes. The authors assert that Eq. (13) should be compared to equations (20)-(22) of Hill (2001), except for amplitude factors. The adjoint (or pseudo-inverse, see Beylkin, 1985) of Eq. (13), following the lines of Cohen et al. (1986) and Bleistein (1987), is the asymptotic true-amplitude beam migration/inversion operator.

Analysis of the kernel for the Cohen-Bleistein inversion for Eq. (13) shows that a exponential factor related to the imaginary part of the traveltime ϕ_{Im} smears the 3D Dirac delta function, and in being so, it must be integrated over the *p*-components of the common-offset ray in order to grant the inversion result. This fact constrains data to be locally slant stacked prior to diffraction stacking. Thus, in Albertin et al. (2004) notation, the migration/inversion equation is

$$\beta(\vec{\mathbf{x}}) = -\frac{i}{8\pi^3}\int_{-\infty}^{+\infty} d\,\omega\omega \int_A d^2\xi \int d^2 p\, w_A(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})\, \mathrm{e}^{-i\omega\phi(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})}\, \psi(\vec{\xi},\vec{\mathbf{p}},\omega)\,, \quad (14)$$

where

$$w_A(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}}) = \frac{p_{zS}p_{zR}}{a(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})\left|\nabla\phi_{Re}(\vec{\xi},\vec{\mathbf{x}},\vec{\mathbf{p}})\right|}\left|h_{\mathrm{GB}}(\vec{\mathbf{x}},\vec{\xi})\right| \quad (15)$$

is the weight-function and in which $\left|h_{\mathrm{GB}}(\vec{\mathbf{x}},\vec{\xi})\right|$ is a version of the Beylkin determinant for the common-offset case. Comparing Eq. (14) to Eq. (11) it is obesrved that the two inversion operators present the same structure, differing only in their weight-functions, specially that in Eq. (11) the final weight-function does not present the gradient of the real part of the total traveltime.

Conclusions

We have theoretically investigated the Bleistein-Cohen inversion procedure (Bleistein, 1987) of a common-offset operator in which the seismic data is viewed as a local slant stack around a reference trace bounded by the projected Fresnel zone (PFZ) of the simulated seismic experiment. The idea used followed the approach studied in Ferreira (2006), in which the data to be migrated is first beam stacked along a Huygens surface (Schleicher et al., 1993), using relative traveltime surfaces or curves in each point of the diffraction curve, previously to mapping the amplitudes to depth. This approach is entirely carried out in the time domain, in contrast to the present procedure, which is carried out in the frequency-wavenumber domain and partially interpolated through the τ-p (slant stack) domain.

The approach of using the PFZ in Ferreira (2006) considered elements of the Fresnel volume of a seismic wavefield as boundary condition for the use of Gaussian Beams (GB's) in the seismic migration operator. The radius of PFZ acts as constraint for the half-beam width of the GB to be considered for each emergent paraxial wavefield of each geophone. In previous approaches (see, e.g., Hill, 1990 and Hill, 2001), in order to consider such situation, the data must be decomposed in gathers that simulate a superposition of GB's and that locate seismic events according to their dips present in the common-offset section and around every midpoint. The idea here is precisely the same, although seen by a different point of view, using the Bleistein-Cohen methodology.

Another difference of the present approach regarding the one adopted in Ferreira (2006) is that presently there is an explicit sum (i.e., integration) over seismic dips (slowness) instead of a sum over frequency values (Ferreira, 2006). The latter was necessary in order to modify the curvature (i.e., the Hessian) of the traveltime approximation of the relative traveltime curves along the diffraction (Huygens) curve. This step is not necessary when dealing with slant stackings.

When inserted in the procedures considered in Cohen et al. (1986) to find an inversion operator for the seismic case (Beylkin, 1985), the data must be considered to be a superposition of GB's (Ferreira, 2006), so that the Bleistein-Cohen inversion machinery must act over events that were previously projected towards the acquisition surface and were stacked in reference traces – i.e., every trace along a common-offset section. But to consider these paraxial contributions and relate them to points in depth they must be "deprojected" back to depth variables. When this is applied, the operator is redefined as an integration over the cross section of the reflector in depth and restricted by the Fresnel zone around the reflection point. Since this is the only region of the reflector to influence the amplitudes in the reference geofone, its mapping is restricted to its Fresnel volume. In that between the Bleistein-Cohen methodology comes in, and the new inversion may be seen again as a volume mapping (Cohen et al., 1986). Thus the result is a weighted inversion with respect to every plane wave component and that must be summed (or integrated) along all slowness vectors in order to produce the complete image.

When compared to Albertin et al. (2004) inversion operator, it is noticed that Eq. (11) is analogous, differing only in terms in the structure of each weight-function. **Figure 1** summarizes all ideas described along the present paper.

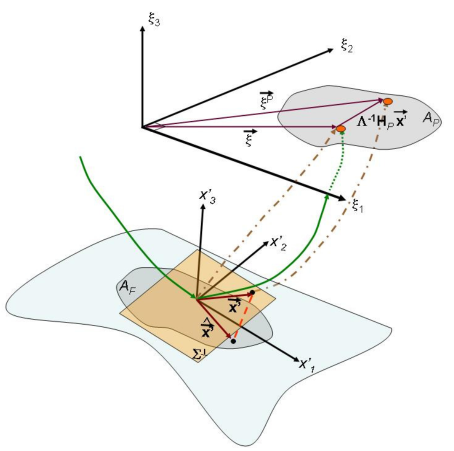

Figure 1 – The seismic experiment including elements of the Fresnel volume, including the Fresnel zone in depth and its projected counterpart along the acquisition surface.

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