# Shear-wave coupling in inhomogeneous weakly anisotropic media

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## Abstract

We present an approximate procedure for computing coupled S waves in inhomogeneous weakly anisotropic media. The procedure can be used to compute S waves propagating in smooth inhomogeneous isotropic or anisotropic media. In anisotropic media, it can describe behaviour of coupled as well as decoupled S waves. Basic part of the procedure is an approximate computation of the common S-wave ray, a trajectory, along which properties of both S waves propagating in anisotropic media are computed. For computation of the common ray and of the approximate geometrical spreading along it, we use the first-order ray tracing and dynamic ray tracing concept, developed for computations of P waves in inhomogeneous weakly anisotropic media. The amplitude coefficients of the coupled S waves are computed by solving a coupled system of ordinary differential equations - the coupling equations - along the common S-wave ray. The performance of the procedure is illustrated on several models of varying strength of anisotropy. Its accuracy is tested by comparing its results with results of the standard ray theory.

## Introduction

Shear waves propagating in inhomogeneous, weakly anisotropic media or in a vicinity of singularities do not propagate independently, they are coupled. Coupling can be described in various ways, the most familiar being the coupling ray theory (Coates and Chapman, 1990; Bulant and Klimeš, 2002). There are various versions of the coupling ray theory, which differ by different levels of approximations used in their applications, see details in Klimeš and Bulant (2004).

One of the approximations of the coupling ray theory is the quasi-isotropic approach (Kravtsov, 1968). For applications to elastic media see Pšenčík (1997, 1998), Pšenčík and Dellinger (2001). In the quasi-isotropic approach, the coupled shear waves are computed along the common S-wave ray - a trajectory common to both S waves - traced in a reference isotropic medium approximating closely the studied anisotropic medium. This reduces accuracy of the approach. In order to increase it, Bakker (2002) proposed the use of a common S-wave ray, which is traced in the studied anisotropic medium, see also Klimeš (2006). Farra and Pšenčík (2008) used Bakker's (2002) approach and combined it with the first-order ray tracing (FORT) and first-order dynamic ray tracing (FODRT) concept, which they used before for the computation of P waves in inhomogeneous weakly anisotropic media (Pšenčík and Farra, 2005; 2007). The use of the term "first-order" in FORT and FODRT reflects the fact that the ray tracing and dynamic ray tracing are based on the first-order perturbation theory, in which the role of a small quantity is played by deviations of anisotropy from isotropy.

Coupling equations of Farra and Pšenčík (2008) are derived under the assumption that the the deviations of anisotropy from isotropy are of the order $O(\omega^{-1})$, where $\omega$ is the circular frequency. The equations are specified in the vectorial frame situated in the plane tangent to the wavefront, which represents the zero-order polarization plane of a common S wave. Substitution of the vectors defining the zero-order polarization plane by their first-order counterparts leads to a considerable increase of accuracy of coupling equations. We illustrate performance of such a procedure by comparing its results with results of the standard ray theory. For illustration, we use several models of varying strength of anisotropy.

## Description of Algorithm

First step of the procedure is the computation of the common S-wave ray. The trajectory of the common ray corresponds to the Hamiltonian obtained from the average of the first-order approximations of two smaller eigenvalues of the Christoffel matrix. In this way, we obtain first-order common ray, and along it the first-order common traveltime. Since the common ray is computed in the studied medium, it is not necessary to specify a reference medium as in the quasi-isotropic approach. Use of common ray in the anisotropic medium leads to more accurate results than its use in a reference isotropic medium even in cases when both trajectories are close to each other. The fact that parameters of the studied medium are used in the former case (and are not approximated by parameters of a nearby isotropic medium) increases accuracy of computations significantly. Moreover, the common S-wave ray computed in the studied anisotropic medium behaves like a P-wave ray. It is regular everywhere, even in S wave singular regions.

Once the common S-wave ray is traced, we can perform FODRT along it. As in the case of first-order common ray, we use again the Hamiltonian obtained from the average of the first-order approximations of the two smaller eigenvalues of the Christoffel matrix. FODRT is solved along the common S-wave ray and provides, among other useful quantities, the first-order common S-wave geometrical spreading.

The amplitude coefficients computed along the common ray are related to a vectorial frame specifying S-wave polarization. Basic frame is identical with the so-called wavefront orthonormal coordinate system (Červený, 2001), which can be easily computed along a ray. It consists of the unit vector perpendicular to the wavefront and two mutually perpendicular unit vectors situated in the plane tangent to the wavefront. This plane represents the zero-order polarization plane of S waves. Instead of the two vectors situated in the plane tangent to the wavefront, we use their first-order counterparts, which define the first-order polarization plane.

Once the common ray, common geometrical spreading and first-order polarization plane are determined, we can solve coupling equations along the common ray. The coupling equations consist of two coupled frequency-dependent, linear ordinary differential equations for two S-wave amplitude coefficients. Use of the first-order polarization plane instead of the zero-order one requires minimum additional computational effort, but it leads to a considerable increase of accuracy of traveltimes.

## Numerical Examples

In order to illustrate properties of the procedure based on the solution of the coupling equations we consider the VSP configuration shown in Fig.1. The source and the borehole are situated in a vertical plane $(x, z)$. The borehole is parallel to the $z$ axis, the vertical single-force source is located on the surface at $z = 0$ km, at a distance of 1 km from the borehole. The source-time function is a windowed symmetric Gabor wavelet, $\exp[-(2\pi f/\gamma)^2 2t^2] \cos(2\pi ft)$, with the dominant frequency $f = 50$ Hz and $\gamma = 4$. There are 29 receivers in the borehole, distributed with a uniform step of 0.02 km, with receiver depths ranging from 0.01 to 0.57 km. In the following, we show only transverse components of the wavefield (components perpendicular to the plane $(x, z)$) since they show most interesting effects. If the above configuration were used in an isotropic medium, the transverse components would be identically zero (vertical force cannot generate transverse motion). Due to anisotropy of the medium, specified below, the transverse components are non-zero. All calculated seismograms are shown with no differential scaling between traces, so true relative amplitudes can be seen. The red seismograms corresponding to the proposed procedure are plotted over the black ones corresponding to standard ray seismograms in presented comparisons.

We consider three models, QI, QI2 and QI4, used by Klimeš & Bulant (2004) and Bulant & Klimeš (2008). The models are vertically inhomogeneous HTI media with constant vertical gradient of elastic moduli. At any depth, the axis of symmetry is rotated in the horizontal plane by 45$^o$ out of the vertical plane $(x, z)$. The S-wave anisotropy defined as $(c_{S1} - c_{S2})/c_{average} \times$ 100% ranges from horizontal to vertical direction from 1% to 4%, from 4% to 7% and from 11% to 13% for QI, QI2 and QI4 models, respectively. Variations of the S-wave phase velocities in the $(x, z)$ plane for all three models are shown in Fig. 2. Left plots correspond to $z = 0$ km, right plots to $z = 1$ km. The sections of

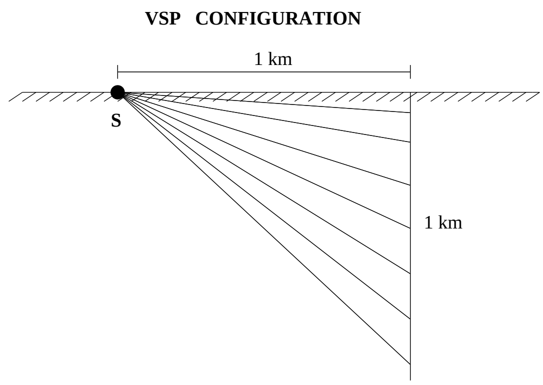


Figure 1: Schematic configuration of experiments.

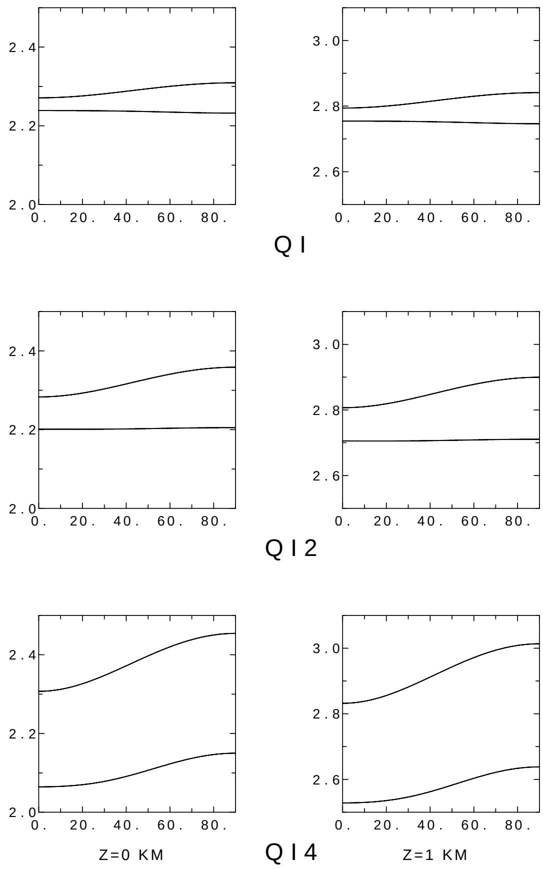


Figure 2: S-wave phase-velocity sections by the $(x, z)$ plane for the models QI (top), QI2 (middle) and QI4 (bottom). Velocities vary from horizontal ($0^0$) to vertical ($90^0$) direction of the wave normal. Left plots correspond to $z = 0$ km, right plots to $z = 1$ km.

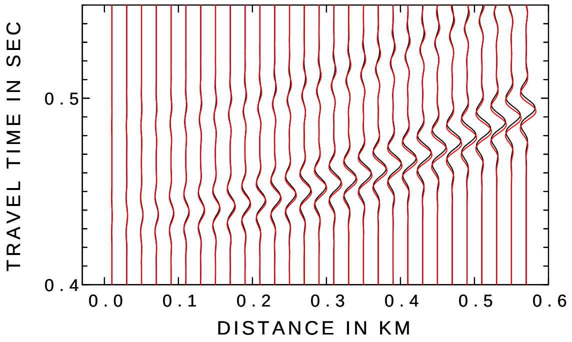


Figure 3: Comparison of the transverse components of seismograms computed using coupling equations (red) and the standard ray theory (black) for the vertical single-force source in the homogeneous QI4 model.

the model QI are shown in the top, of QI2 in the middle and of QI4 in the bottom plot. Velocities are shown as functions of the angle of incidence. They vary from $0^0$ (horizontal propagation) to $90^0$ (vertical propagation).

Fig.3 shows a comparison of seismograms obtained by the proposed procedure with ray seismograms for a homogeneous model resulting from the QI4 model at $z = 0$km, see the bottom plot of the left column of Fig.2. The seismograms shown in Fig.3 are not affected by coupling because there is no coupling in homogeneous media. We can thus estimate accuracy of the traveltime and spreading approximations. We can see that the proposed procedure (red) yields a very good fit with standard ray seismograms (black) even for anisotropy of 11%-13%.

In Fig.4, we show comparison of transverse components of seismograms computed using coupling equations (red) and the standard ray theory (black) for QI, QI2 and QI4 models. We can see misfit of seismograms for the QI and slightly also for QI2 model, especially at shallow receivers. This misfit is caused by the failure of the standard ray theory to describe coupling. Standard ray theory works well when the waves are decoupled as it is the case in the models QI2 (for deeper receivers) and QI4. We can see that in such regions, the coupling procedure yields seismograms, which fit very well those obtained by standard ray theory. The coupling procedure works well even in situations when the S waves are well separated.

## Conclusions

The proposed procedure has several interesting and useful properties. First of all, it considers automatically coupling of shear waves, which standard ray theory ignores. The procedure can be used to compute S waves propagating in smooth inhomogeneous isotropic or anisotropic media of arbitrary symmetry. In anisotropic media, it can describe behaviour of coupled as well as decoupled S waves. The procedure is based on computation of a single ray - common ray, along which properties of both waves are evaluated. Note that separate shear waves shown in red in the top plot in Fig.4 are computed along single rays to each receiver. The common S-wave ray tracing has important property of being regular everywhere including singular regions. It thus avoids problems known from ray tracing of separate S waves in anisotropic media. The FORT, FODRT and coupling equations have a simpler structure than the exact ones. This leads to a reduction in the number of algebraic operations involved, and to greater transparency of equations. As a byproduct of the coupling procedure, we can get formulae for approximate evaluation of traveltimes of separate S waves. These formulae can find applications in migration and traveltime tomography. The coupling

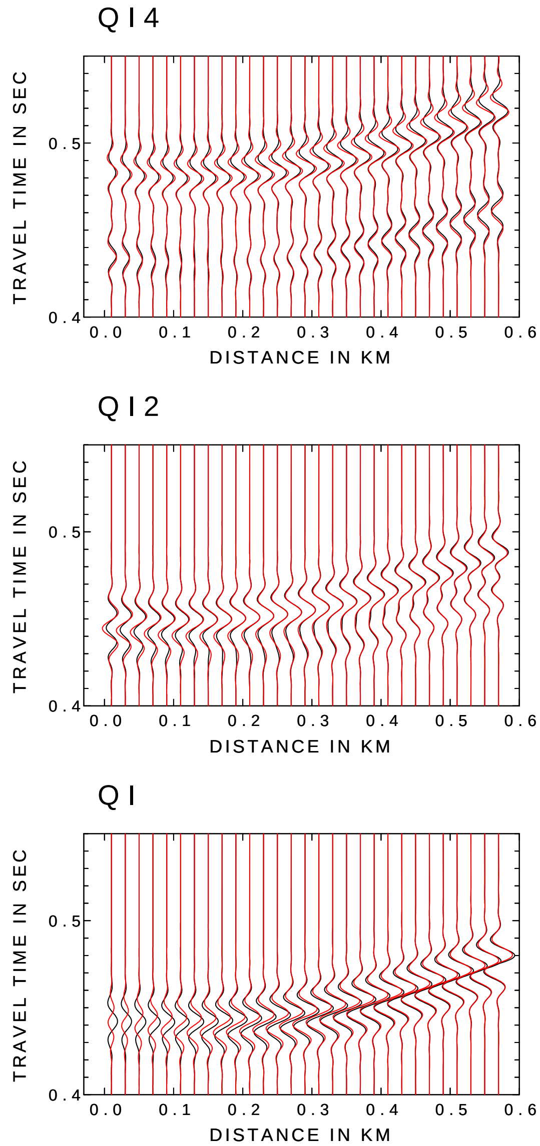


Figure 4: Comparison of the seismograms computed using coupling equations (red) and the standard ray theory seismograms (black) for the vertical single-force source in the QI (bottom), QI2 (middle) and QI4 (top) models.

procedure can be simply generalized for laterally varying, layered, weakly anisotropic structures. The main application of the proposed procedure will be in modelling S-wave fields in inhomogeneous weakly anisotropic media. Further applications are expected in multicomponent seismic imaging, in Kirchhoff migrations including S or PS waves, in tomography. In all these applications, the proposed procedure can simply substitute currently used isotropic ray tracers.

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