Depth conversion and associated uncertainties using consistent velocity model: A probabilistic unified model based on Bayesian approach

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Abstract

Is it reliable to depth convert seismic time interpretation using seismic velocity data? Do these velocity data add value to the depth conversion?
This paper is an attempt to answer the question and to propose a geostatistical model based on Bayesian approach. The proposed methodology is applied to real and test case studies to demonstrate the ability to add value to the depth conversion by properly using seismic velocity models.

Introduction

Geostatistical approach for time-to-depth conversion of seismic horizons is often used in many geo-modelling projects in oil industry and in various contexts. From a geostatistical point of view, the time-to-depth conversion of seismic horizons is a classical estimation problem involving one or more secondary variables $S_l(x) \to \{S_0 = 1, l = 1, \cdots L\}$, ($S_1$= time-migrated horizon in time units) and well markers corresponding to the interpreted horizon $Z(x_i) \to \{i = 1, \ldots N\}$ (in depth units).
Different kriging methods are used for this estimation problem such as Simple Kriging (SK), Kriging with External Drift (KED). As all kriging methods, they provide the depth mapping and an estimation of associated uncertainty.
These kriging methods are based on the dichotomy of the spatial random variable $Z(x)$ (Matheron, 1963) in two parts the mean $m(x)$ and the residual $Z_R(x)$. The mean function is generally done by a linear combination:

$$m(x) = \sum_{l=0}^{L} b_l \cdot S_l(x)$$

In function of available data we can have three different scenarios for defining the trend model:

1. No seismic velocity involved (M0 case):
$$m(x) = b_0 + b1 \cdot T(x)$$
2. Un-scaled seismic velocity model involved (M1 case):
$$m(x) = b_0 + b_1 \cdot Z_{seis}(x)$$
3. Scaled seismic velocity model (M2 case)
$$m(x) = b_0 + b_1 \cdot Y(x) \cdot Z_{seis}(x) + b_2 \cdot Z_{seis}(x)$$

In the case of SK the mean $m(x)$ is assumed to be known and defined as a time-depth regression fit based on well markers. In the case of kriging with external drift (Wackernagel, 1995), the drift coefficients are evaluated in the KED process (called "geo-regression")
All these kriging based estimators are of course conditioned by the data and are model-driven but the only tuning parameter is the variogram model. The drift coefficients are either fixed or estimated. However we can have the a-priori information about the velocity model and for that reason, the Bayesian approach is well adapted to provide a framework for the integration of this a-priori knowledge.

Estimation using Bayesian approach

In the linear Bayesian approach we replace the set of regression linear coefficients b_l by random variables B_l characterised by a known a-priori join distribution. The a-priori knowledge of time-depth relationship or the prior distribution of drift coefficients in Gaussian case can be fully determined by two first moments: the prior expectation vector $\mu_l = E[B_l]$ and associated covariance matrix $Cov(B_l, B_k) = \sigma_{lk}$. The Bayesian extension of KED provides an estimator which is more general and introduces the notion of uncertainty of the mean (or trend) prediction. The kriging formalism is generalised thanks to a Bayesian inference and provide a local estimation with an associated variance of estimation and posterior distribution of drift coefficients (Omre 1987, Omre and al. 1989). The Bayesian Kriging system is then defined as:

$$\sum_{i=1}^{N} \lambda_i \left\{ C_{Z|B}(x_i - x_j) + \sum_{l=0}^{L}\sum_{k=0}^{L} \sigma_{lk} S_l(x_i) S_k(x_j) \right\} =$$
$$C_{Z|B}(x_0 - x_j) + \sum_{l=0}^{L}\sum_{k=0}^{L} \sigma_{lk} S_l(x_0) S_k(x_j)$$

Where $C_{Z|B}(x_i - x_j)$ is the conditional covariance which can be modelled from experimental variogram of residuals (the trend is computed using prior). Considering the Gaussian distribution of drift coefficients, the first two moments of posterior distribution can be obtained by the following equations:

$$E[B|Z] = \mu + (S\Sigma)^T (C + S\Sigma S^T)^{-1}(Z - S\mu)$$
$$Cov[B|Z] = \Sigma - (S\Sigma)^T (C + S\Sigma S^T)^{-1} S\Sigma$$

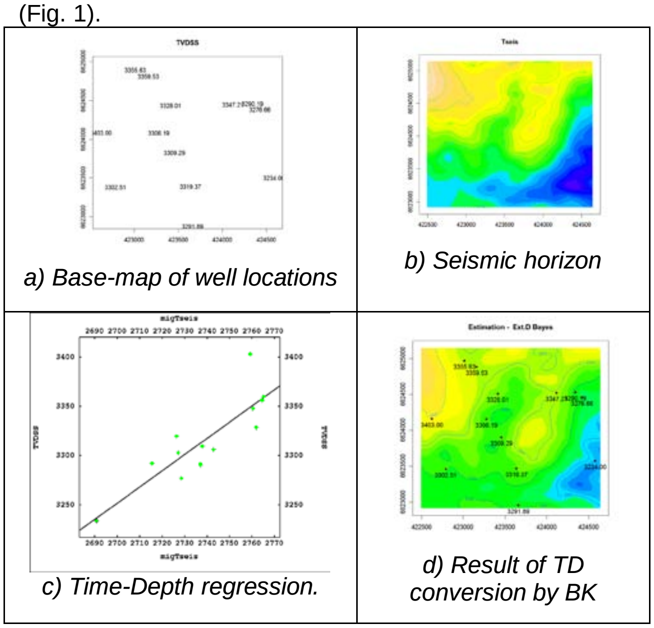
Where: μ is the vector of drift coefficients, S is the drift matrix $S = [S_i(x_j)]$ (seismic time or seismic depth at well markers), $\Sigma = [\sigma_{ij}]$ is the matrix of the covariance of drift coefficients, C is the covariance matrix between data points and Z is the vector of sample depth measurements.

The Bayesian Kriging (BK) stands between Simple Kriging (SK) and Kriging with external Drift (KED). In the case that our knowledge of drift coefficients is very poor (big variance or large prior distribution of drift coefficients) the BK converges to the KED and in the case of good knowledge of these coefficients (low variance or narrow distribution of drift coefficients) the BK converges to the SK. This property of BK provide an general unified probabilistic model for time to depth conversion that can be applied in three mentioned M0, M1 and M2 scenarios and as all kriging estimators Bayesian Kriging provide also a quantification of the depth uncertainty by the kriging variance of estimation.

Another important advantage of the use of Bayesian approach is the fact that the mathematical model accounts for prior knowledge of velocity model. This prior knowledge serves as a geophysical supervision of estimator. That is very important in the case of limited number of wells.

Example: M0 case

The application of BK for time-to-depth conversion was tested in several case studies and provided very promising results. In the extreme cases of our prior knowledge of drift coefficients we found the same results as KS or KED. The result of a test case study is shown (Fig. 1).

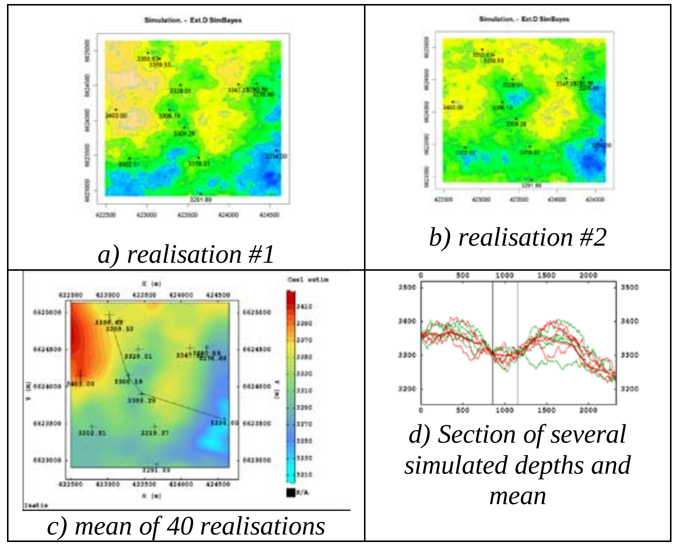


a) Base-map of well locations *b) Seismic horizon* *c) Time-Depth regression.* *d) Result of TD conversion by BK*

Figure 1: Application of Bayesian Kriging for time-to-depth conversion of a seismic horizon

Simulation based on Bayesian approach

Bayesian Kriging is used not only for estimation purposes but it also can be implemented in simulation algorithms. The Bayesian simulation approach provides a quantification of variability of depth integrating the uncertainty of our prior knowledge of the trend.



a) realisation #1 *b) realisation #2* *c) mean of 40 realisations* *d) Section of several simulated depths and mean*

Figure 2: Application of simulation based on Turning Bands and Bayesian Kriging (test case)

Among of different possible algorithms, our choice was the combination of turning bands with BK which provides an accurate and fast simulation method adapted not only for regular grids but for any kind of mesh. The spatial statistics of simulated depths are fully verified in several real and test case studies (Fig. 2).

M1 and M2 case (seismic velocity involved)

The first step before embarking into depth conversion using seismic velocity fields consists in identifying remaining processing artefacts to be filtered out and filtering these denser and denser velocity data sets: a simple procedure based on factorial kriging is proposed and illustrated on a real data case; a specific output of factorial kriging is the Spatial Quality Index (SQI®) cube which provides a handy tool to pinpoint anomalies in the available velocity data cubes. The filtering parameters are easily controlled and fine-tuned. Artefacts and filtered data can be visualized and interpreted. Ghosts of enhancement strategy grids currently used in high density automatic picking algorithm are easily identified and removed in this first step.

Unfortunately, seismic velocities, even properly filtered, diverge from well velocity. The approach proposed in this paper consists in introducing a scaling factor (the R factor) into a classical geostatistical tool applied for years in time to depth conversion. Indeed Bayesian kriging is perfectly adapted to this type of non stationary bivariate problem: in its classical application, a primary variable (marker depth in well) is kriged using a secondary variable (seismic time or seismic depth) as a shape factor, namely the external drift.

This robust geostatistical tool is revisited here to introduce a ratio between well and seismic velocity as a second external drift. In many cases as shown in this real case example, the strong anti-correlation observed between this ratio (the R factor) and the seismic velocities damages the correlation between well velocities and seismic velocities and leads to the wrong decision to ignore seismic velocities in the preferred time to depth conversion approach. The R factor used as a second external drift can be related to the introduction of a laterally varying correction for anisotropy, and\or compaction. It can be intuitively linked to the depth of burial of a classical V0, K approach.

The introduction of a second external drift captures well identified pitfalls of time to depth conversion such as ignoring anisotropy or surface reference for burial.

A flowchart has been developed and is proposed to evaluate the effect of seismic velocity on depth models resulting from different approaches covering the usual range of practical cases: seismic time as external drift, seismic depth as external drift, R factor with a second external drift. Each approach is illustrated on a real case study with blind tests performed to highlight the benefits and limitation of the R factor approach.

Simple case study

Different scenarios of prior velocity model have been tested in a simple study case in order to quantify the quality of estimated depth. For that a simple case study is presented with a limited number of wells (Only 10 wells in our case) (Fig 3).

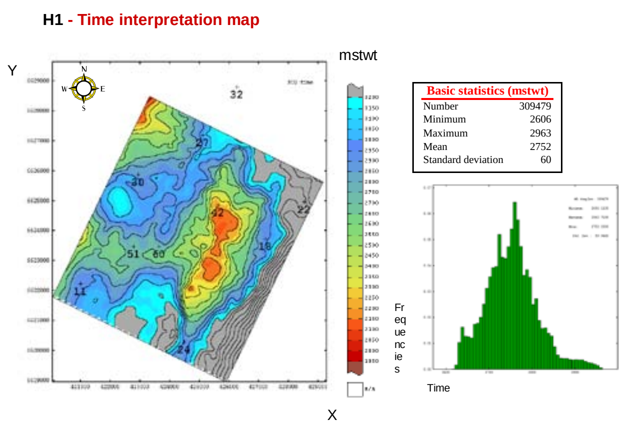


Figure 3: Seismic interpreted horizon in time

A first depth map (un-conditioned by the well markers) was obtained by a simple product of interpreted time and filtered seismic velocity (Fig, 4, 5). The relationship between true depth and seismic measurements (time and depth) was obtained by a simple regression analysis on well markers (Fig 6). In order to define the secondary drift (M2 case) the relationship between Scaling factor and seismic velocities was also defined (Fig 7). The Figures 8, 9 and 1à shows the resulting depths for M0 M1 and M2 case.

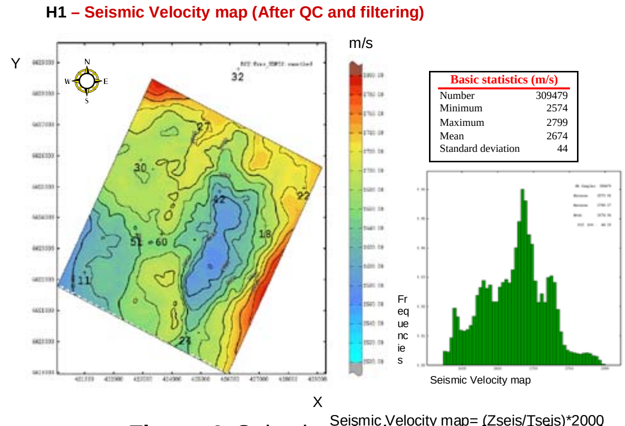


Figure 4: Seismic velocity map after QC end filtering

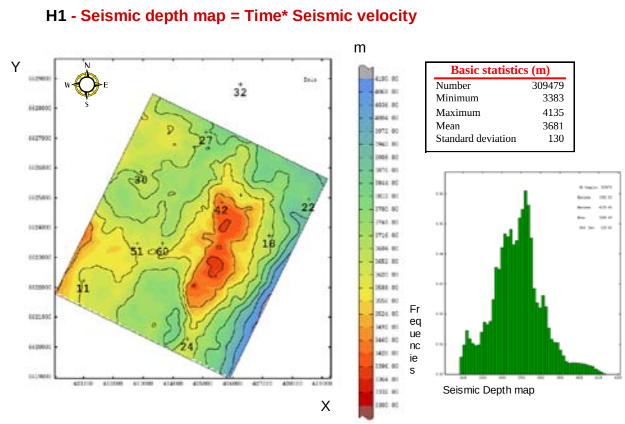


Figure 5: Seismic horizon depth map

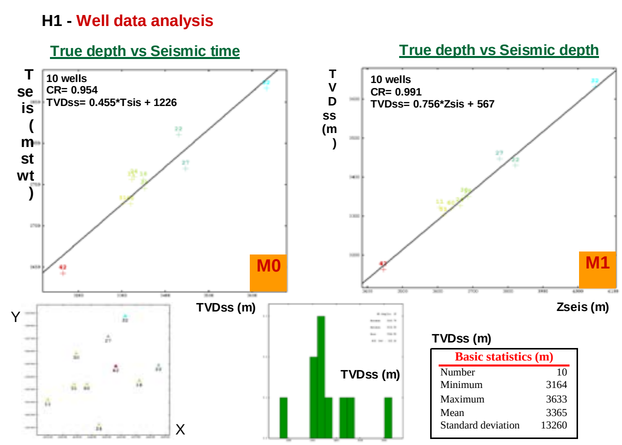


Figure 6: Well data analysis SeismicTime -True Depth and Seismic Depth-True Depth regression

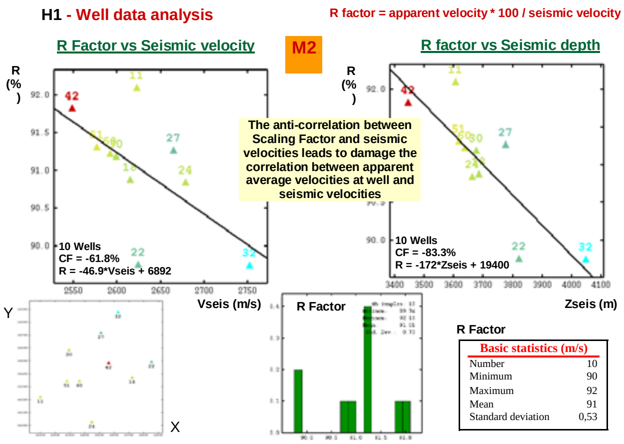


Figure 7: Well data analysis of R factor

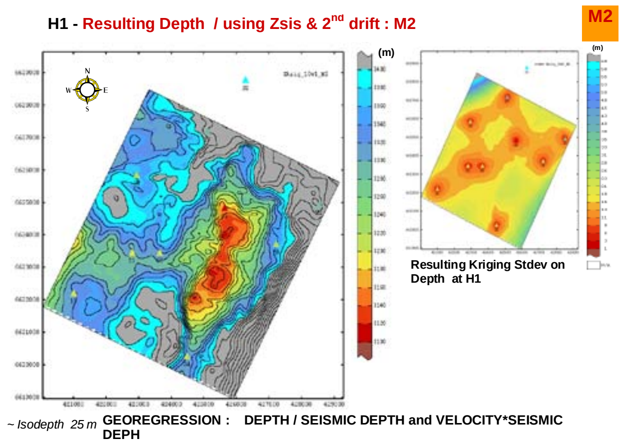


Figure 10: Resulting depth using seismic depth and a second external drift (M2 case)

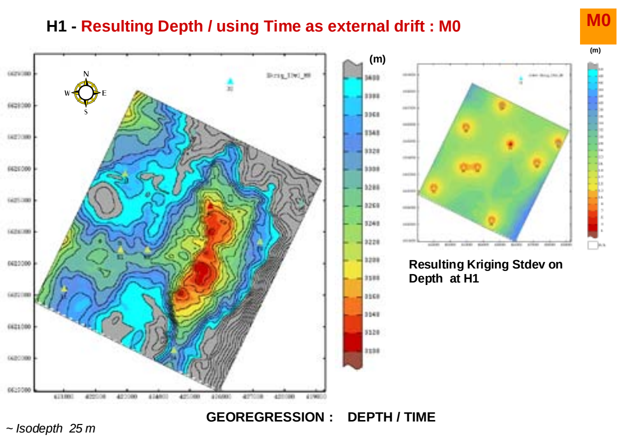


Figure 8: Resulting depth using seismic time as external drift (M0 case)

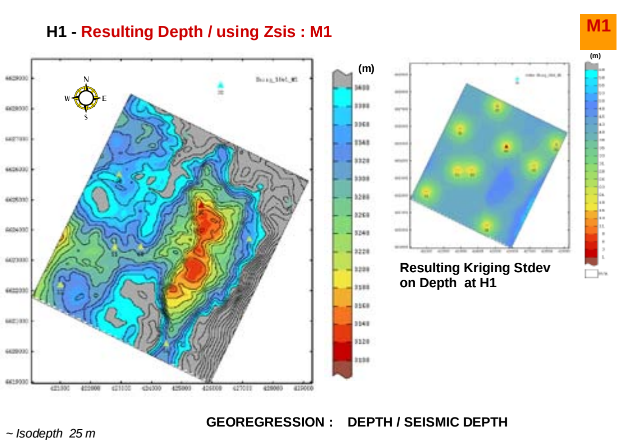


Figure 9: Resulting depth using only seismic depth as external drift (M1 case)

Uncertainty management

Well depth conditional simulations with external drift provide a set of realisations of the depth surfaces. All simulations are conditioned to the depth well markers (eg here 10 wells, Fig. 11) and are consistent with a given scenario approach of depth conversion method (here M2 scenario Fig 11).

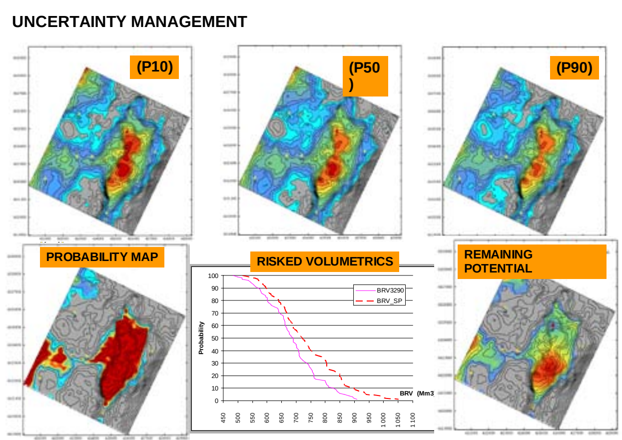


Figure 11: Application of simulation.

In the case of Bayesian approach the base case model can be supervised by the prior knowledge of velocity model. The choice of different prior drift coefficients and their associated variance will provide different base case models (Fig. 12). Bayesian approach integrates in a consistent way not only the uncertainty due to the fluctuations around the mean but also the uncertainty on the base case model.

Kriging in bayesian framework 3 sets of constraints

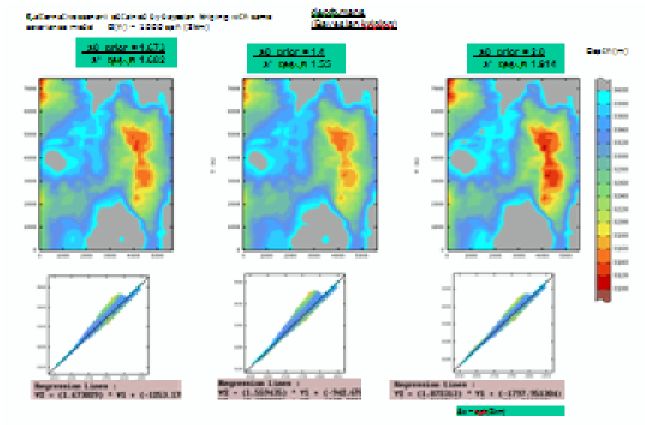


Figure 12: Application of simulation based on Turning Bands and Bayesian Kriging with different priors

Conclusions

Bayesian approach provides an excellent estimator which is more general than the traditional Kriging with external drift(s) and fits very well to the needs for time-to-depth conversion of seismic horizons. The underlying mathematical model infers in an accurate way the prior knowledge of trend model and associated uncertainty. The combination of BK with turning bands simulation algorithm provides an accurate and flexible method for generating conditional simulations with related uncertainty on the drift coefficients, very useful for structural uncertainty analysis. The method has been tested with success in several oil-field case studies.

References

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