Generalized nonhyperbolic moveout approximation

Sergey Fomel* and Alexey Stovas†

* Jackson School of Geosciences, The University of Texas at Austin, University Station, Box X, Austin, TX 78713-8924, USA

† Dept. of Petroleum Eng. and Appl. Geophysics, NTNU, S.P. Andersensvei 15A, NO-7491 Trondheim, Norway

This paper was prepared for presentation at the 11th International Congress of The Brazilian Geophysical Society held in Salvador, Brazil, August 24-28, 2009.

Abstract

We introduce a novel functional form for approximating the moveout of reflection traveltimes at large offsets. While the classic hyperbolic approximation uses only two parameters (the zero-offset time and the moveout velocity), our form involves five parameters, which can be determined, in a known medium, from zero-offset computations and from tracing one non-zero-offset ray. We call it a generalized approximation because it reduces to some known three-parameter forms (the shifted hyperbola of Malovichko, de Baziliere, and Castle; the Padé approximation of Alkhalifah and Tsvankin; and others) with a particular choice of parameters. By testing the accuracy of the proposed approximation with analytical and numerical examples, we show that it can bring several-orders-of-magnitude improvement in accuracy compared to known approximations, which makes it as good as exact for all practical purposes.

INTRODUCTION

The reflection traveltime as a function of the source-receiver offset has a well-known hyperbolic form in the case of plane reflectors in homogeneous isotropic (or elliptically anisotropic) overburden. A hyperbolic behavior of the PP moveout is always valid around the zero offset thanks to the source-receiver reciprocity and the first-order Taylor series expansion. However, any deviations from this simple model may cause nonhyperbolic behavior at large offsets (Fomel and Grechka, 2001).

Considerable research effort has been devoted to developing nonhyperbolic moveout approximations in both isotropic and anisotropic media. The work on isotropic approximations goes back to Bolshykh (1956), Taner and Koehler (1969), Malovichko (1978), de Bazelaire (1988), Castle (1994), and others. Fowler (2003) provides a comprehensive review of many different approximations developed for non-hyperbolic moveout in anisotropic (VTI) media. A particularly simple “velocity acceleration” model for nonhyperbolic moveout is suggested by Taner et al. (2005, 2007). Causse (2004) approximates nonhyperbolic moveout by expanding it into a sum of basis functions. Douma and Calvert (2006) build an accurate moveout approximation by using rational interpolation between several rays.

In this paper, we propose a general functional form for nonhyperbolic approximations that can be applied to any kind of seismic media. The proposed form includes five coefficients as opposed to two coefficients in the classic hyperbolic approximation. In certain cases, the number of coefficients can be reduced. In the case of a homogeneous VTI medium and the “acoustic approximation” of Alkhalifah (1998), our approximation becomes identical to the one proposed previously by Fomel (2004). In the general case, determining the optimal coefficients requires tracing of only one non-zero-offset ray.

NONHYPERBOLIC MOVEOUT APPROXIMATION

Let $t(x)$ represent the reflection PP (or SS) traveltime as a function of the source-receiver offset x. We propose the following general form of the moveout approximation:

$$t^2(x) \approx (1-\xi)\,(t_0^2 + a\,x^2) + \xi\,\sqrt{t_0^4 + 2\,b\,t_0^2\,x^2 + c\,x^4}\ . \quad (1)$$

The five parameters a, b, c, ξ, and t_0 describe the moveout behavior. By simple algebraic manipulations, one can also rewrite equation (1) as

$$t^2(x) \approx t_0^2 + \frac{x^2}{v^2} + \frac{A\,x^4}{v^4\left(t_0^2 + B\,\frac{x^2}{v^2} + \sqrt{t_0^4 + 2\,B\,t_0^2\,\frac{x^2}{v^2} + C\,\frac{x^4}{v^4}}\right)}\ , \quad (2)$$

where the new set of parameters A, B, C, v, and t_0 is related to the previous set by the equalities

$$a = \frac{A\,B + B^2 - C}{v^2\,(A + B^2 - C)}\ ; \quad (3)$$

$$b = \frac{B}{v^2}\ ; \quad (4)$$

$$c = \frac{C}{v^4}\ ; \quad (5)$$

$$\xi = \frac{A}{C - B^2}\ . \quad (6)$$

The inverse transform is given by

$$v^2 = \frac{1}{a\,(1-\xi) + b\,\xi}\ ; \quad (7)$$

$$A = \frac{\xi\,\left(c - b^2\right)}{\left[a\,(1-\xi) + b\,\xi\right]^2}\ ; \quad (8)$$

$$B = \frac{b}{a\,(1-\xi) + b\,\xi}\ ; \quad (9)$$

$$C = \frac{c}{\left[a\,(1-\xi) + b\,\xi\right]^2}\ . \quad (10)$$

The nonhyperbolic part of the traveltime approximation (1) and (2) is controlled by parameters ξ or A, correspondingly. When $\xi = 0$ or $c = b^2$, approximation (1) is hyperbolic. When both B and C are very large, approximation (2) also reduces to the hyperbolic form.

Equation (2) can be also rewritten in the following form

$$t^2(x) \approx t_0^2 \left[1 + \tilde{x}^2 + \frac{A\,\tilde{x}^4}{y(\tilde{x})}\right] , \qquad (11)$$

where $\tilde{x}$ is the normalized offset $\tilde{x} = x/(v\,t_0)$ and

$$y(\tilde{x}) = 1 + B\,\tilde{x}^2 + \sqrt{1 + 2\,B\,\tilde{x}^2 + C\,\tilde{x}^4} \qquad (12)$$

is the positive solution of the quadratic equation

$$y^2 - 2\,(1 + B\,\tilde{x}^2)\,y + (B^2 - C)\,\tilde{x}^4 = 0 . \qquad (13)$$

The Taylor series for traveltime squared from equations (11) around $\tilde{x} = 0$ is

$$t^2(\tilde{x}) = t_0^2 \left[1 + \tilde{x}^2 + \frac{A}{2}\,\tilde{x}^4 - \frac{A\,B}{2}\,\tilde{x}^6 + \ldots\right] . \qquad (14)$$

Connection with other approximations

Equations (1-2) reduce to some well-known approximations with special choices of parameters.

- If $A = 0$, the proposed approximation reduces to the classic hyperbolic form

$$t^2(x) \approx t_0^2 + \frac{x^2}{v^2} , \qquad (15)$$

  which is a two-parameter approximation.

- The choice of parameters $A = (1-s)/2$; $B = s/2$; $C = 0$ reduces the proposed approximation to the shifted hyperbola (Malovichko, 1978; de Bazelaire, 1988; Castle, 1994), which is the following three-parameter approximation:

$$t(x) \approx t_0 \left(1 - \frac{1}{s}\right) + \frac{1}{s}\sqrt{t_0^2 + s\,\frac{x^2}{v^2}} . \qquad (16)$$

- The choice of parameters $A = -4\,\eta$; $B = 1 + 2\,\eta$; $C = (1 + 2\,\eta)^2$ reduces approximation (2) to the form proposed by Alkhalifah and Tsvankin (1995), which is the following three-parameter approximation:

$$t^2(x) \approx t_0^2 + \frac{x^2}{v^2} - \frac{2\,\eta\,x^4}{v^4\left[t_0^2 + (1 + 2\,\eta)\,\frac{x^2}{v^2}\right]} . \qquad (17)$$

- The choice of parameters $A = -2\,\gamma\,t_0^2\,v^2$; $B = -A/2$; $C = A^2/4$ reduces approximation (2) to the following three-parameter approximation suggested by Blias (2007) and reminiscent of the "velocity acceleration" equation proposed by Taner et al. (2005):

$$t^2(x) \approx t_0^2 + \frac{x^2}{v^2\,(1 + \gamma\,x^2)} . \qquad (18)$$

- The choice of parameters $A = 2\,\tan^2\theta$, $B = 1 - \tan^2\theta$, $C = 1/\cos^4\theta$ reduces the proposed approximation to the double-square-root expression

$$t(x) \approx \frac{\sqrt{z^2 + (y + x/2)^2}}{V} + \frac{\sqrt{z^2 + (y - x/2)^2}}{V} , \qquad (19)$$

  where $V = v\,\cos\theta$, $z = (t_0\,V/2)\,\cos\theta$, and $y = (t_0\,V/2)\,\sin\theta$. Equation (19) describes moveout precisely for the case of a diffraction point in a constant velocity medium.

Thus, the proposed approximation encompasses some other known forms but introduces more degrees of freedom for the optimal selection of parameters.

General method for parameter selection

Zero-offset ray

The Taylor expansion of approximation (2) around the zero offset

$$t^2(x) = t_0^2 + \frac{x^2}{v^2} + \frac{A}{2}\,\frac{x^4}{v^4\,t_0^2} + O(x^6) \qquad (20)$$

provides a convenient method for evaluating coefficients t_0, v, and A by matching expansion (20) to the corresponding expansion of the exact traveltime. This is the method used previously for deriving approximations (15) and (16). In an isotropic $v(z)$ medium, the coefficients are readily available and reduce to statistical averages of the velocity distribution (Bolshykh, 1956)

$$t_0 = 2\,m_{-1} , \qquad (21)$$

$$v^2 = \frac{m_1}{m_{-1}} , \qquad (22)$$

$$A = \frac{1}{2}\left(1 - \frac{m_3\,m_{-1}}{m_1^2}\right) , \qquad (23)$$

where

$$m_k = \int\limits_0^z V^k(\zeta)\,d\zeta$$

Equations (21-23) are easily extensible to the vertical transverse isotropy (VTI) case (Ursin and Stovas, 2006).

Nonzero-offset ray

To determine uniquely the remaining coefficients B and C, we propose to use just one additional ray reflected at a nonzero offset. Suppose that a reflection ray with the ray parameter P arrives at the offset X and traveltime T. Substituting approximation (2) into equations $t(X) = T$ and $t'(X) = P$ and solving for B and C produces the explicit analytical solution

$$B = \frac{t_0^2\,(X - P\,T\,v^2)}{X\,(t_0^2 - T^2 + P\,T\,X)} - \frac{A\,X^2}{X^2 + v^2\,(t_0^2 - T^2)} , \qquad (24)$$

$$C = \frac{t_0^4\,(X - P\,T\,v^2)^2}{X^2\,(t_0^2 - T^2 + P\,T\,X)^2} + \frac{2\,A\,v^2\,t_0^2}{X^2 + v^2\,(t_0^2 - T^2)} . \qquad (25)$$

Horizontal ray

If the reference ray happens to be horizontal, both X and T are infinite, and equations (24-25) are not directly applicable. However, one can use the same principle and match two terms for the behavior of the traveltime at infinitely large offsets. If the traveltime behaves as

$$t^2(x) \approx T_\infty^2 + P_\infty^2\, x^2 \qquad (26)$$

for x approaching infinity, then, matching the corresponding behavior of approximation (2), we find that

$$B = \frac{t_0^2\,(1 - v^2\,P_\infty^2)}{t_0^2 - T_\infty^2} - \frac{A}{1 - v^2\,P_\infty^2}\;, \qquad (27)$$

$$C = \frac{t_0^4\,(1 - v^2\,P_\infty^2)^2}{(t_0^2 - T_\infty^2)^2}\;. \qquad (28)$$

ACCURACY TESTS

To illustrate the applicability of the proposed approximation, we try several analytical and numerical models. Using this set of models, we test the proposed approximation against the hyperbolic approximation, the shifted hyperbola approximation, and the Alkhalifah-Tsvankin approximation.

Analytical examples

Linear velocity

We start with an analytical isotropic three-parameter model: the linear velocity model. In this models, it is possible to compute the exact moveout analytically and thus to compare directly the accuracy of different approximations. We show this comparison in Figure 1, where the relative approximation error is plotted for different approximations against a large range of the offset-to-depth ratio and the maximum-to-minimum velocity ratio. As evident from the figures, three-parameter approximations (shifted-hyperbola and Alkhalifah-Tsvankin) improve the accuracy of the two-parameter hyperbolic approximation. However, the proposed five-parameter generalized approximation brings a more significant improvement and reduces the error by several orders of magnitude.

Homogeneous VTI layer

Our next analytical example is a horizontal reflector in a homogeneous VTI (vertically transverse isotropic) medium. According to the "acoustic" approximation of Alkhalifah (1998), one can use the following parametric equations to define the traveltime-offset relationship in this model:

$$x(p) = \frac{2\,H}{v_z}\,\frac{p\,v^2}{(1 - 2\,\eta\,p^2\,v^2)^2\,\sqrt{1 - \frac{p^2\,v^2}{1-2\,\eta\,p^2\,v^2}}}\;, \qquad (29)$$

$$t(p) = \frac{2\,H}{v_z}\,\frac{(1 - 2\,\eta\,p^2\,v^2)^2 + 2\,\eta\,p^4\,v^4}{(1 - 2\,\eta\,p^2\,v^2)^2\,\sqrt{1 - \frac{p^2\,v^2}{1-2\,\eta\,p^2\,v^2}}}\;, \qquad (30)$$

where p is the ray parameter, H is the depth of the reflector, v_z is the vertical velocity, v is the NMO velocity, and η is

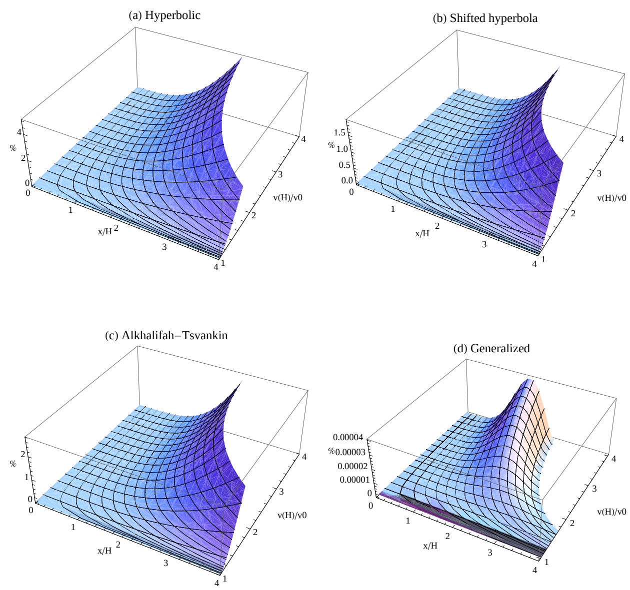

Figure 1: Relative error of different traveltime approximations as a function of velocity contrast and offset/depth ratio for the case of a linear velocity model. (a) Hyperbolic approximation, (b) Shifted hyperbola approximation, (c) Alkhalifah-Tsvankin approximation, (d) Generalized nonhyperbolic approximation. The proposed generalized approximation reduces the maximum approximation error by several orders of magnitude.

the dimensionless parameter introduced by Alkhalifah and Tsvankin (1995).

At small offsets, the homogeneous VTI traveltime behaves as

$$t^2(x) \approx t_0^2 + \frac{x^2}{v^2} - \frac{2\,\eta\,x^4}{t_0^2\,v^4}\;, \qquad (31)$$

which allows us to define $A = -4\,\eta$ according to equation (20).

At large offsets, the homogeneous VTI traveltime behaves as

$$t^2(x) \approx t_0^2\,(1 + 2\,\eta) + \frac{x^2}{v^2\,(1 + 2\,\eta)}\;. \qquad (32)$$

Comparing with equation (26), we note that $T_\infty = t_0\,\sqrt{1 + 2\,\eta}$ and $P_\infty = 1/(v\,\sqrt{1 + 2\,\eta})$. Substituting into equations (27-28), we define the coefficients B and C to be

$$B = \frac{1 + 8\,\eta + 8\,\eta^2}{1 + 2\,\eta}\;, \qquad (33)$$

$$C = \frac{1}{(1 + 2\,\eta)^2}\;. \qquad (34)$$

Equation (2) with coefficients given by equations (33-34) is precisely equivalent to the traveltime approximation suggested previously by Fomel (2004). Fomel (2004) shows comparisons with alternative non-hyperbolic approximations, which demonstrate superior accuracy of equation (2) in a case of strongly anisotropic material.

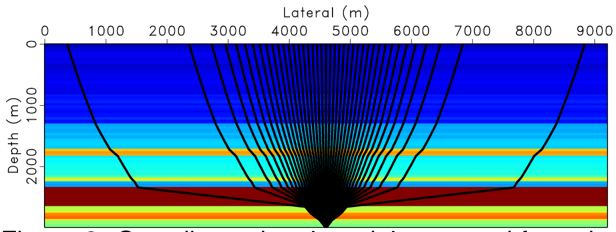

Figure 2: One-dimensional model extracted from the left column of the anisotropic Marmousi model and corresponding reflection rays.

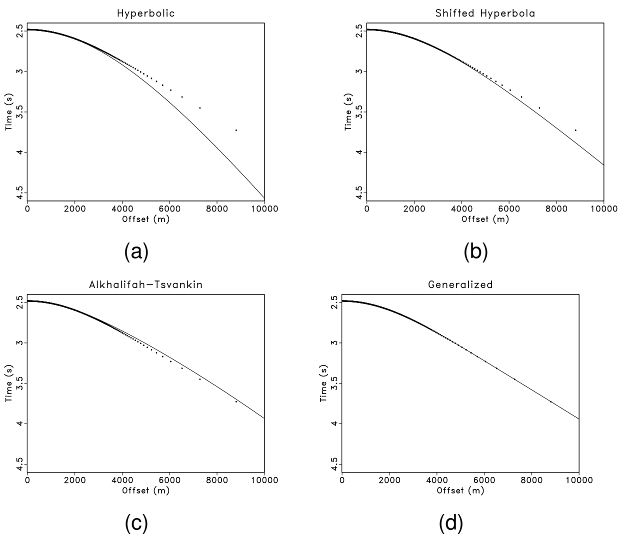

Figure 3: Exact moveout from ray tracing in the one-dimensional anisotropic Marmousi model (dots) and different approximations (solid lines). (a) Hyperbolic approximation, (b) Shifted hyperbola approximation, (c) Alkhalifah-Tsvankin approximation, (d) Generalized nonhyperbolic approximation.

Numerical example

For a numerical test, we create a one-dimensional velocity model by extracting a depth column out of the anisotropic Marmousi model, created by Alkhalifah (1997). We evaluate exact reflection traveltimes by ray tracing (Figure 2). Next, we compare the exact time for different reflection rays with values predicted by different traveltime approximations. As shown in Figure 3, only the proposed generalized approximation is able to predict the true traveltime accurately over the full range of offsets.

CONCLUSIONS

We propose a five-parameter nonhyperbolic moveout approximation that generalizes the classic two-parameter hyperbolic approximation as well as some known three-parameter approximations. We propose a method for selecting the approximation parameters, which involves only two rays: the normal-incident ray and one additional ray, preferably at a large offset. The special case of the additional ray being horizontal can be handled as well.

A comparison with the classic hyperbolic approximation, the shifted hyperbola approximation and the Alkhalifah-Tsvankin approximation for analytical and numerical isotropic and transversely isotropic models shows that the proposed generalized nonhyperbolic approximation can bring an improvement of several orders of magnitude in approximation accuracy. Based on these experiments, we claim that, for all practical purposes, the proposed approximation is as good as the exact moveout.

References

Alkhalifah, T., 1997, An anisotropic Marmousi model, *in* SEP-95: Stanford Exploration Project, 265–282.

———, 1998, Acoustic approximations for processing in transversely isotropic media: Geophysics, **63**, 623–631.

Alkhalifah, T., and I. Tsvankin, 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, 1550–1566.

Blias, E., 2007, Long-spreadlength approximations to NMO function for a multi-layered subsurface: Recorder, **3**, 36–42.

Bolshykh, S. F., 1956, About an approximate representation of the reflected wave traveltime curve in the case of a multi-layered medium: Applied Geophysics (in Russian), **15**, 3–15.

Castle, R. J., 1994, Theory of normal moveout: Geophysics, **59**, 983–999.

Causse, E., 2004, Approximations of reflection travel times with high accuracy at all offsets: Journal of Geophysics and Engineering, **1**, 28–45.

de Bazelaire, E., 1988, Normal moveout revisited - Inhomogeneous media and curved interfaces: Geophysics, **53**, 143–157.

Douma, H., and A. Calvert, 2006, Nonhyperbolic moveout analysis in VTI media using rational interpolation: Geophysics, **71**, D59–D71.

Fomel, S., 2004, On anelliptic approximations for qP velocities in VTI media: Geophys. Prosp., **52**, 247–259.

Fomel, S., and V. Grechka, 2001, Nonhyperbolic reflection moveout of p waves. an overview and comparison of reasons.: Technical Report CWP-372, Colorado School of Mines.

Fowler, P. J., 2003, Practical VTI approximations: a systematic anatomy: Journal of Applied Geophysics, **54**, 347–367.

Malovichko, A. A., 1978, A new representation of the traveltime curve of reflected waves in horizontally layered media: Applied Geophysics (in Russian), **91**, 47–53.

Taner, M. T., and F. Koehler, 1969, Velocity spectra - Digital computer derivation and applications of velocity functions: Geophysics, **34**, 859–881. (Errata in GEO-36-4-0787).

Taner, M. T., S. Treitel, and M. Al-Chalabi, 2005, A new travel time estimation method for horizontal strata: 75th Ann. Internat. Mtg, Soc. of Expl. Geophys., 2273–2276.

Taner, M. T., S. Treitel, M. Al-Chalabi, and S. Fomel, 2007, An offset dependent NMO velocity model: EAGE 69th Conference and Exhibition, EAGE, P036.

Ursin, B., and A. Stovas, 2006, Traveltime approximations for a layered transversely isotropic medium: Geophysics, **71**, D23–D33.