Determination of Traveltime Parameters in VTI Media

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Abstract

For modern long-offset acquisition geometries, a hyperbolic traveltime approximation is no longer sufficient to flatten the CMP gather because of medium inhomogeneity or anisotropy. For transversely isotropic media with a vertical symmetry axis (VTI media), just two traveltime parameters are sufficient for performing all time-related processing. Using an estimate of the NMO velocity from a hyperbolic velocity analysis, one can estimate the anisotropic parameter from a more general traveltime approximation. We extend this two-step procedure using a more accurate nonhyperbolicity term in the traveltime approximation. The used traveltime approximations allow to predict the bias in the NMO velocity estimate, thus providing a means of correcting both the estimated NMO velocity and the resulting anisotropy parameter value. By means of a numerical example, we demonstrate that the estimation of both traveltime parameters is improved considerably.

Introduction

The velocity model plays a key role in the processing and migration of seismic reflection data. Conventional velocity analysis (Dix, 1955; Yilmaz, 1987) by the common-midpoint (CMP) method fits a hyperbolic traveltime approximation to a seismic reflection event in a CMP section. In this procedure, a single traveltime parameter, usually expressed as the normal-moveout (NMO) velocity, is estimated using a measure of the quality of the fit. However, for modern long-offset acquisition geometries, a hyperbolic traveltime approximation is no longer sufficient to flatten the CMP gather because of medium inhomogeneity or anisotropy (Alkhalifah et al., 1996; Toldi et al., 1999). Many authors proposed alternative ideas of how to extract seismic velocities from the data. One idea is to use seismic diffractions (Harlan et al., 1984; Landa and Keydar, 1998; Fomel et al., 2007) to extract information about velocity. Schleicher et al. (2008) used image-wave propagation to determine the subsurface velocity model. Even for anisotropic media, there are several methods to obtain information about the velocity model (Tsvankin and Thomsen, 1994; Al-Dajani and Tsvankin, 1998; Sarkar and Tsvankin, 2004; Behera and Tsvankin, 2007).

A particularly important work is that of Alkhalifah and Tsvankin (1995). They demonstrated that, for transversely isotropic media with a vertical symmetry axis (VTI media), just two traveltime parameters are sufficient for performing all time-related processing such as NMO and dip-moveout (DMO) corrections. The two traveltime parameters are usually expressed as the NMO velocity v_{nmo} and the nonhyperbolicity parameter η, a combination of the well-known weak anisotropy parameters ϵ and δ of Thomsen (1986). Using on these parameters, Alkhalifah and Tsvankin (1995) derived a new traveltime approximation based on continued fractions that describes nonhyperbolic traveltimes for larger offsets.

Alkhalifah (1997) showed that using an estimate of v_{nmo} after a hyperbolic velocity analysis, one can estimate the anisotropic parameter η from the more general traveltime approximation of Alkhalifah and Tsvankin (1995). He proposes a two-step procedure. The first step uses conventional velocity analysis in the CMP gather up to a short offset to estimate v_{nmo}. In the next step, assuming that the estimative of v_{nmo} is sufficiently accurate, he proposes to use farther offsets to estimate the anisotropy parameter η. As a drawback of his method, he noted the strong sensitivity of the η estimates on the quality of the estimated NMO velocity.

In this paper, we apply the two-step procedure of Alkhalifah (1997) using the new nonhyperbolic traveltime approximations of Schleicher and Aleixo (2008), based on anelliptic approximations (Fomel, 2004). These traveltime approximations allow to predict the bias in the NMO velocity estimate, thus providing a means of correcting both the estimated NMO velocity and the resulting η value. In this way, the extraction procedure leads to more reliable estimates of v_{nmo} and η.

Method

For a homogeneous VTI medium the hyperbolic traveltime approximation is only valid for small offsets, and the velocity coefficient is an NMO velocity that differs from the vertical velocity (Thomsen, 1986). Extending the Taylor series of the traveltime approximation up to fourth order does not extend the validity range significantly (Tsvankin and Thomsen, 1994). However, other types of traveltime approximation can be found that are valid for longer offsets the most famous one being (Tsvankin and Thomsen, 1994; Alkhalifah and Tsvankin, 1995)

$$t^2(x) = 1 + x^2 - \frac{2\eta x^4}{1 + (1 + 2\eta)x^2}. \qquad (1)$$

Here, we use the normalized half-offset, $x = h/\tau_0 v_{nmo}$, and the normalized traveltime $t(x) = \tau(x)/\tau_0$, where h is half-offset and τ_0 is the zero-offset traveltime. Moreover, the anisotropy parameter is

$$\eta = \frac{\epsilon - \delta}{1 + 2\delta} \qquad (2)$$

and the normal-moveout (NMO) velocity

$$V_{nmo} = V_{p0}\sqrt{1 + 2\delta}, \qquad (3)$$

where ϵ and δ are Thomsen's (1986) parameters, and V_{p0} is the vertical P-wave velocity.

Alkhalifah (1997) proposed to use a hyperbolic approximation

$$t^2(x) = 1 + x^2 \qquad (4)$$

to estimate v_{nmo} by a short-offset conventional velocity analysis. Thereafter, assuming that the estimate of v_{nmo} is sufficiently accurate, the traveltime correction of equation (1) can be used to estimate the anisotropic parameter η. Introducing the notation

$$\Delta t^2 = (1 + x^2) - t^2(x) = \frac{2\eta x^4}{1 + (1 + 2\eta)x^2} \qquad (5)$$

for the traveltime correction of equation (1), η can be obtained at a given normalized half-offset x from

$$\eta = \frac{\Delta t^2(1 + x^2)}{2x^2(x^2 - \Delta t^2)}. \qquad (6)$$

To measure Δt^2 in the data, Alkhalifah (1997) suggests to apply an NMO correction using v_{nmo} from the first step and then compute $\Delta t^2 = 1 - (t^2(x) - x^2) = 1 - t^2_{cor}$, where t_{cor} corresponds to the moveout traveltime after NMO correction. The second quantity needed in equation (6) is the normalized half-offset x. Alkhalifah (1997) showed that the reliability of the estimate increases with increasing offset. Thus, equation (6) should be applied at the farthest offsets available.

Recently, Schleicher and Aleixo (2008) derived a set of new more accurate traveltime approximations in VTI media. These approximations have the form

$$t^2(x) = 1 + \frac{x^2}{Q} + B_i \frac{x^2}{1 + x^2/Q}\ , \qquad (7)$$

where $Q = 1 + 2\eta$. For the factor $B_i(\eta)$, they derived five different forms, being

$$\begin{aligned} B_1(\eta) &= 2\eta/Q, & (8)\\ B_2(\eta) &= 2\eta/(1+\eta)Q, & (9)\\ B_3(\eta) &= 2\eta/(1+\eta)^2, & (10)\\ B_4(\eta) &= 2\eta/Q^2, & (11)\\ B_5(\eta) &= 8\eta(1+\eta)/5Q. & (12) \end{aligned}$$

The aim of this work is to use traveltime approximations (7) in the two-step procedure of Alkhalifah (1997) to obtain a more accurate estimative for parameter η. The first step of estimating v_{nmo} remains the same as before. The second step needs to be slightly altered due to the different traveltime approximation. To simplify the expressions, we introduce a new traveltime parameter y defined as

$$y = \frac{x^2 - \Delta t^2}{x^2}\ . \qquad (13)$$

Manipulating equation (7), we can write

$$y = \frac{Q + x^2 + B_i Q^2}{Q^2 + Qx^2}. \qquad (14)$$

Substituting the different expressions for B_i in equation (14) and linearizing numerator and denominator separately, we find for B_1 to B_4

$$y = \frac{1 + 2\eta + x^2 + 2\eta}{1 + 4\eta + (1 + 2\eta)x^2}\ , \qquad (15)$$

which leads to the following extraction formula for η:

$$\eta = \frac{(1 + x^2)(1 - y)}{4y + 2x^2 y - 4} \qquad (16)$$

Correspondingly, we obtain for B_5

$$y = \frac{5 + 10\eta + 5x^2 + 8\eta}{5 + 20\eta + (5 + 10\eta)x^2}\ , \qquad (17)$$

which results in the following expression for η:

$$\eta = \frac{5(1 + x^2)(1 - y)}{20y + 10x^2 y - 18}\ . \qquad (18)$$

Using formulas (16) and (18), we can estimate η from the picked traveltime at any chosen offset x using the estimated value of y according to expression (13). Here, Δt^2 is determined from the data as described in connection with equation (6).

If the estimate of v_{nmo} is precise, these estimates for η are generally of higher accuracy than the ones obtained with equation (6). However, they suffer from the same sensitivity problems already reported by Alkhalifah (1997). This is a severe drawback, since the estimate of v_{nmo} in the first step is already influenced by anisotropy. However, while traveltime approximation (1) does not predict such a behavior, approximations (7) do. Rewriting equation (7) as

$$t^2(x) = 1 + \frac{x^2}{Q} + B_i \frac{x^2 + x^4/Q - x^4/Q}{1 + x^2/Q} = 1 + \left(B_i + \frac{1}{Q}\right) x^2 - \frac{B_i}{Q} \frac{x^4}{1 + x^2/Q} \quad (19)$$

we see that in this description, the short-offset term is already influenced by the presence of η, resulting in an apparent NMO velocity

$$v_{nmo}^{ap} = v_{nmo}/\sqrt{B_i + 1/Q} \, . \quad (20)$$

For B_1, we have $B_1 + 1/Q = 1$. Thus, in this approximation, the NMO velocity does not depend on η. However, for all other choices of B_i, the true NMO velocity can be calculated from the apparent one by

$$v_{nmo} = C_i v_{nmo}^{ap} = \sqrt{B_i + 1/Q}\, v_{nmo}^{ap} \, . \quad (21)$$

where the correction factor satisfies

$$C_i = \frac{v_{nmo}}{v_{nmo}^{ap}} = \sqrt{B_i + 1/Q} < 1 \, . \quad (22)$$

We thus expect apparent NMO velocities with $v_{nmo}^{ap} > v_{nmo}$. Equation (21) has an important consequence. Once η has been estimated, this equation allows to iteratively correct the estimate of v_{nmo} with the estimate of η until both values are consistent.

The iteratively procedure is summarized in the following steps:

(1) Use hyperbolic velocity analysis for the shortest offset to determine a first estimate for the apparent NMO velocity v_{nmo}^{ap}.

(2) Use this NMO velocity estimate together with equation (16) or equation (18) to obtain a first estimate for η from the farthest offsets.

(3) Use the η estimate to correct the apparent NMO velocity for an improved estimate v_{nmo} accordint to equation (21).

(4) While the correction factor $C_i = B_i + 1/Q$ to v_{nmo} still significantly differs from 1, for instance $1 - C_i > \epsilon$, go to step (2) using the new v_{nmo} estimate.

After convergence of this iterative procedure, the estimated parameters are the final estimates for η and v_{nmo}.

Numerical Examples

In this section we present some numerical examples for the new expressions for the η extraction. We consider a single-layer homogeneous VTI medium with $v_{nmo} = 2.5\ km/s$. The value of η varies from 0.05 to 0.5. Figure 1 shows a synthetic seismogram in such a medium for $\eta = 0.3409$, that represents the Greenhorn shale (Jones and Wang, 1981), with random noise with a signal-to-noise ratio of 10.

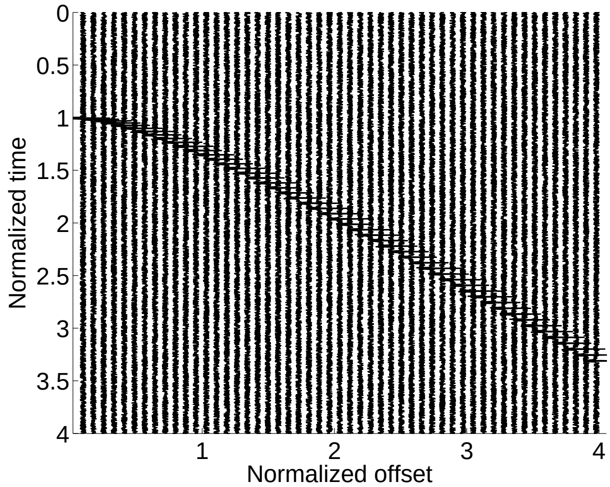

Figure 1: Synthetic seismogram for horizontal reflector at 1 km depth below a homogeneous VTI layer with $\eta = 0.3409$ and $v_{nmo} = 2.5\ km/s$. Signal-to-noise ratio is 3.

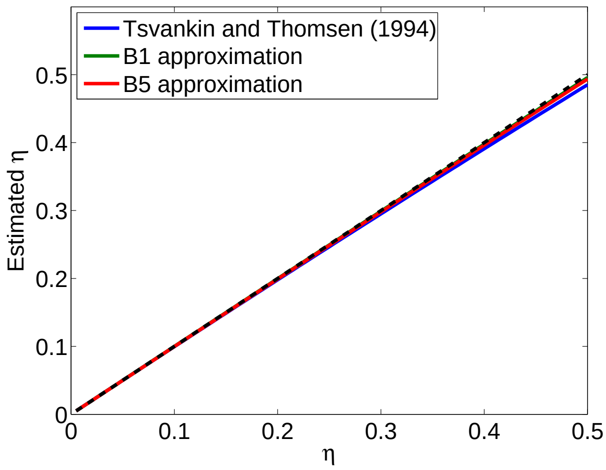

Figure 2: η extraction with formulas (6) (Tsvankin and Thomsen, 1994), (16) (B_1), and (18) (B_5) using the exact NMO velocity.

Our first experiment was to extract η using equations (16) and (18) under the assumption that v_{nmo} is known exactly. Of course, for this constant-velocity layer, $v_{nmo} = v = 2.5$ km/s. We have tested the extraction for 50 different values of η between $\eta = 0.01$ and $\eta = 0.5$. Figure 3 compares the relative error of the estimates for η with formulas (6) (Tsvankin and Thomsen, 1994), (16) (B_1), and (18) (B_5). We see that the extraction using formula (16) is the most accurate one, with a relative error below 1% for the complete range of η. Formula (18) is slightly less accurate, with a error of about 0.25% even for the

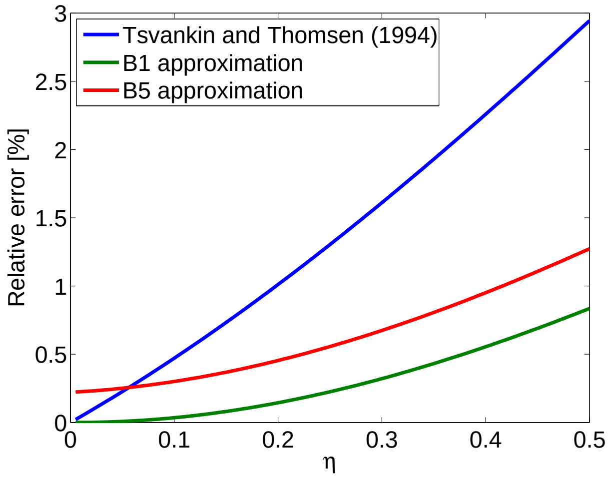

Figure 3: Relative error of η extraction with formulas (6) (Tsvankin and Thomsen, 1994), (16) (B_1), and (18) (B_5) using the exact NMO velocity.

smallest η and with a maximum error of about 1.25%. The error of Tsvankin and Thomsen's formula has an error close to zero at the smallest η, but increases much faster with η than the two B_i estimates, reaching a maximum of 3% at $\eta = 0.5$.

However, this kind of comparison suffers from an important lack of practicality. In practice, the exact value of the NMO velocity is not known a priori. It is necessary to estimate η using the value for v_{nmo} as obtained from a short-offset conventional velocity analysis. Thus, we repeated the above experiment using the estimated apparent NMO velocities. We carried out a conventional hyperbolic velocity analysis in an offset range of $0.1 \leq x \leq 0.5$. Figure 4 shows the estimated apparent NMO velocity as a function of η. We observe a rather strong dependence of the estimated NMO velocity on η.

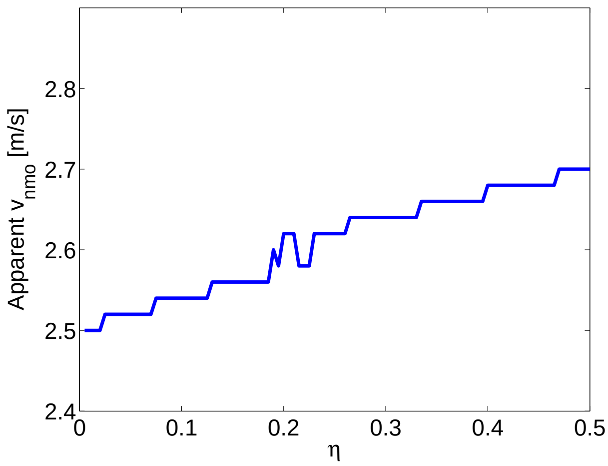

Figure 4: Apparent NMO velocity, estimated from a short-offset conventional hyperbolic velocity analysis, as a function of η.

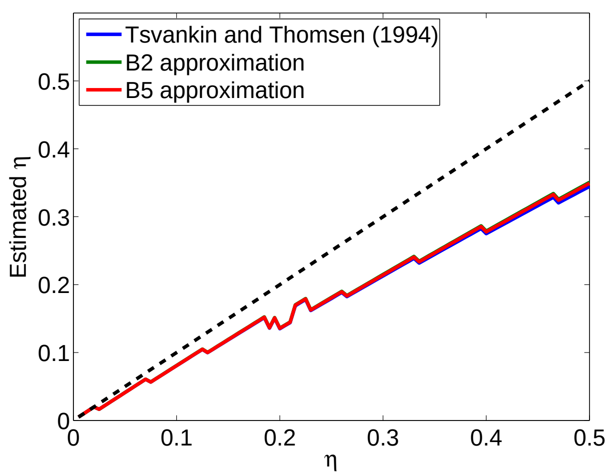

Figure 5: Estimated values of η when using the estimated v_{nmo} from a short-offset hyperbolic velocity analysis.

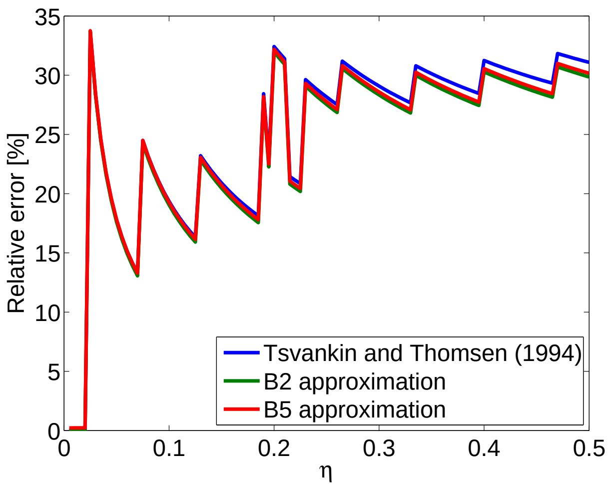

Figure 6: Error of the estimated values of η when using the estimated v_{nmo} from a short-offset hyperbolic velocity analysis.

We then used these estimated values of v_{nmo} in the estimation of η. The incorrect estimation of v_{nmo} strongly deteriorates the quality of the η estimates. Figure 6 shows the relative error of the η estimates with formulas (6) (Tsvankin and Thomsen), (16) (B_1), and (18) (B_5), using the estimated NMO velocities. We see that the error of the three estimates is of comparable size, reaching about 30%. We conclude that the error in the estimate of v_{nmo} affects the η estimates much more than the choice of the traveltime approximation.

Thus, it turns out that the most important feature of the new traveltime approximations is their ability to allow for a correction of the apparent NMO velocity. Figure 7 shows the predicted values of v_{nmo}^{ap} as a function of η for the four values of B_i that predict an

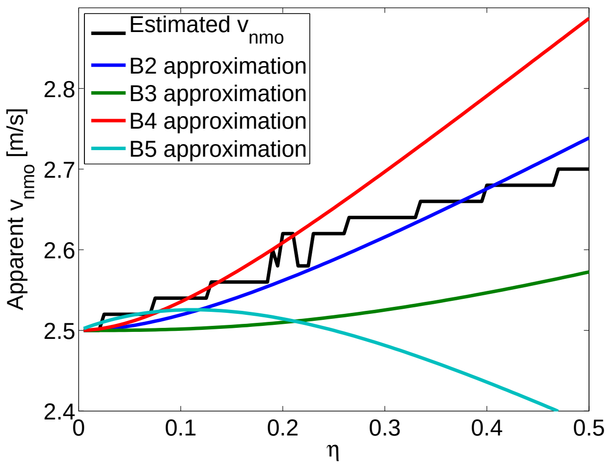

Figure 7: Prediction of apparent v_{nmo} according to equation (19) using four different choices of B_i. Also shown is the observed trend of Figure 4 (black curve).

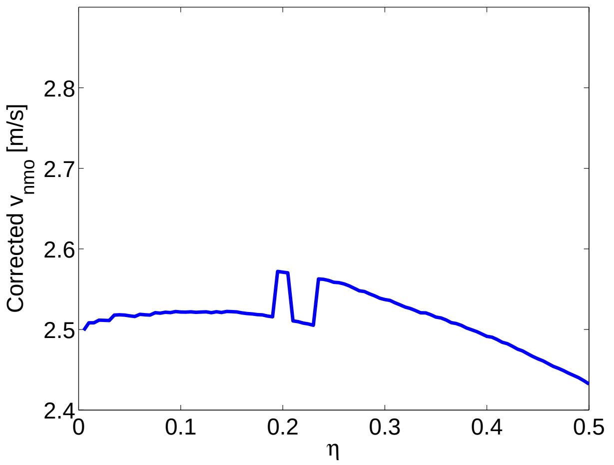

Figure 8: Estimates of v_{nmo} after correction according to equation (19) with B_2 of equation (9).

η-dependent NMO velocity. Also shown is the observed trend of Figure 4 (black curve). We observe that the different approximations predict the apparent NMO velocity with different quality. The best approximation for η below 0.1 is the one using B_5. Up to about 0.2, the B_4 approximation remains closest. The approximation that most closely resembles the observed curve over the full range of tested values of η is the one using B_2.

Using the estimate of η, we can correct the estimate for v_{nmo} according to equation (21). This in turn gives a new estimate for η. We continue this process iteratively until both values are consistent, i.e., until the correction factor C_i differs from one by less than $\epsilon = 10^{-4}$. In principle, and depending on the actual value of η, this should be possible with all four possible choices of B_i for which the correction factor is different from one. Because of the fact that B_2 best predicts the bias in the estimation of v_{nmo} for the full range of tested η (see Figure 4), we use only this approximation for the correction procedure.

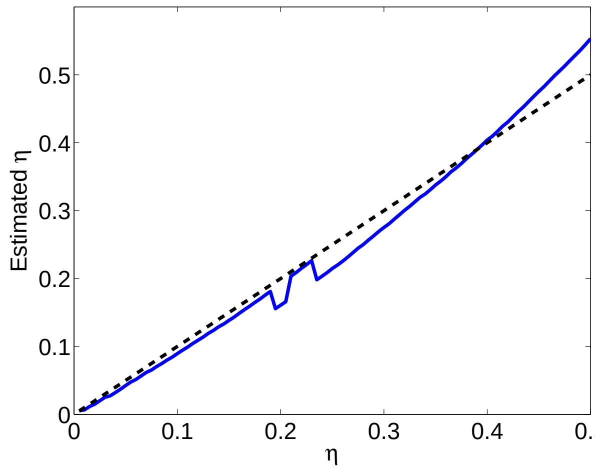

Figure 9: Final estimated values of η after iterative correction of v_{nmo} and η.

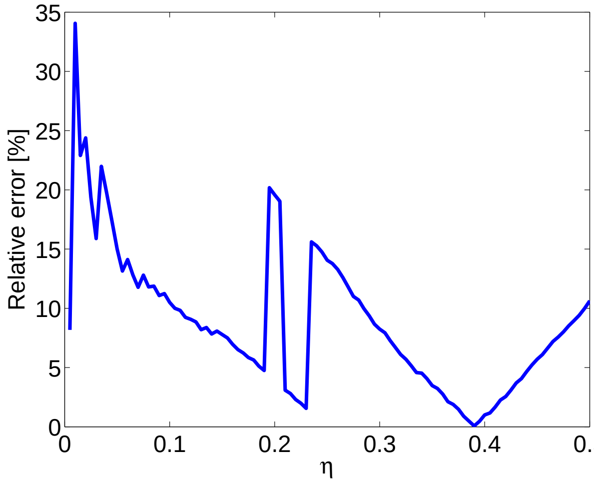

Figure 10: Relative error of the final estimated values of η after iterative correction of v_{nmo} and η.

Figure 8 shows the final corrected values of v_{nmo} after this iterative procedure. We see that the final estimates for v_{nmo} are improved quite considerably. Of course, because of the strongly nonlinear behavior of the apparent NMO velocity, complete correction is impossible. The deviation from the true value of 2.5 km/s is largest at an η of about 0.2. Correspondingly, Figure 10 shows the relative error of the resulting final η estimate after the iterative procedure. As we can see in Figure 10, the resulting error of the η estimates has been reduced significantly in comparison to Figure 6, except in the range of very small values of η. The errors are below 20% for all η values above 0.03 and below 10% almost everywhere except in the range around $\eta = 0.2$ where the appar-

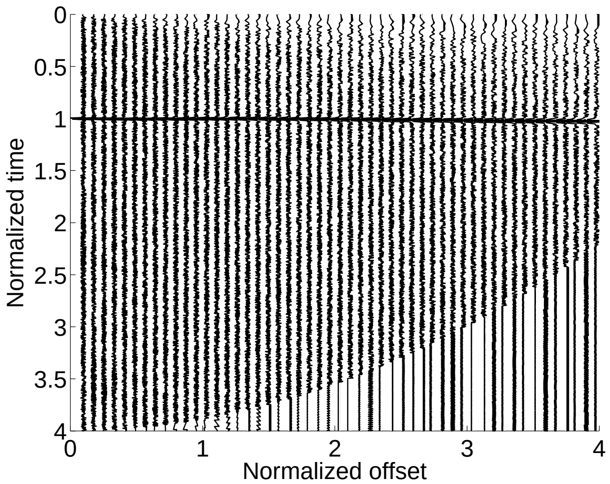

Figure 11: Section of Figure 1 after nonhyperbolic NMO-correction with final estimated values for v_{nmo} and η.

ent NMO velocity has the most nonlinear behaviour. Around $\eta = 0.4$, the error becomes close to zero.

Finally, Figure 11 depicts the synthetic data section of Figure 1 after nonhyperbolic NMO-correction using the described iterative procedure for the extraction of v_{nmo} and η. The extracted values for these data where $v_{nmo} = 2.5162$ km/s and $\eta = 0.3285$.

Conclusions

We extended the technique of Alkhalifah (1997) to compute the anisotropic parameter η. The technique consists of a conventional velocity analysis for short offsets plus a calculation of η based on the nonhyperbolicity term, assuming that an accurate value for the NMO velocity has been obtained. In our analysis, we replaced the nonhyperbolicity term from the traveltime approximation derived by Tsvankin and Thomsen (1994) by those of the more recent ones of Schleicher and Aleixo (2008).

We have seen that the η extraction is more precise if the NMO velocity is known exactly. The general problem of the technique, however, is its sensitivity to errors in the estimate of the NMO velocity. The traveltime approximations of Schleicher and Aleixo (2008) allow to predict the bias in the NMO velocity estimate, thus providing a means of correcting both the estimated NMO velocity and the resulting η value. By means of a numerical example, we have demonstrated the improvement in the estimation of v_{nmo} and η that can be achieved in this way.

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