Optimization of Absorbing Boundary Methods for Acoustic Wave Modelling

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Abstract

This article presents numerical simulations obtained via a 10-2 (10th order in space and 2nd order in time) finite difference scheme applied to acoustic waves. Nonreflecting boundary conditions – namely damping zone and perfect matched layer (PML) – are implemented and tested for different coefficients and varying absorbing layers. It has been found that the optimized PML and a modified version of Cerjan's method can reduce significantly the artificial reflections at the boundaries when compared to the conventional attenuation coefficients.

Introduction

The appearance of fast processing computers and the continuous advances in numerical analysis have allowed new developments in geophysical wave modelling. For imaging the subsurface, many articles have been published dealing with numerical simulations of wave propagation using finite difference, finite element and boundary integral methods (Virieux 1986, Marfurt 1984, Schuster 1985, Durran 1999).

A typical difficulty that arises when solving numerically such boundary value problems is how to express the radiation condition mathematically at a contour which is only at a finite distance from the energy source (Sommerfeld 1949). The boundary condition should allow travelling disturbances to pass through the contour without generating spurious reflections that propagate back toward the interior, which may eventually override the original emitted seismic signals.

To avoid these side effects, researchers used to enlarge the computational domain, delaying the backward reflections, though increasing the numerical mesh and its computational demand. In the late 70's, nonreflecting boundary condition techniques were introduced aiming to treat such problems. Clayton and Engquist (1977) and Reynolds (1978) proposed an absorbing boundary condition by applying a one-way wave equation in the boundary region, which proved to be efficient for events not at shallow angles on the contour.

In the early 80's, Cerjan et al. (1985) introduced the damping zone concept in which a gradual reduction of the wave amplitudes is imposed along an absorption layer, without any loss of effectiveness due to shallow angles of wave incidence. More recently, Berenger (1994) and Collino and Tsogka (2001) proposed the PML method for solving electromagnetic and elastic wave equations. A new matched medium is designed to absorb without reflection the incident waves at any frequency and at any incidence angle.

This work aims to perform effectiveness tests for optimized nonreflecting boundary conditions in a finite difference time domain (FDTD) scheme applied to acoustic wave modelling. In the following sections, PML and Cerjan's methods are presented and modified aiming to reduce wave reflection at the borders. Results are shown in terms of the time sum of squared energy difference between infinite and nonreflecting models for varying absorbing layers.

Optimized PML Technique

When the disturbances generated by a source reach the limits of the computational domain, reflected waves are spread throughout the medium. To avoid this problem, Berenger (1994) developed the PML technique, in which a new region that surrounds the FDTD domain is defined, where a set of non-physical equations are applied giving a high attenuation of the incident waves.

For acoustics, the 2D linearized continuity and Euler equations take the following form at the PML absorbing layer,

$$p_t + B\alpha p = -B\nabla \cdot \vec{u}\,, \qquad (1)$$

$$\vec{u}_t + B\alpha \vec{u} = -\frac{1}{\rho}\nabla p\,, \qquad (2)$$

where ρ, p and $\vec{u}$ are, respectively, the medium density and the acoustics pressure and vector velocity, while α is the attenuation coefficient and B $(=\rho c^2)$ the medium bulk modulus. c is the medium wave speed.

Differentiating in time and space equations (1) and (2) and subtracting the resulting expressions gives the PML acoustic equation,

$$p_{tt} + \alpha(1+B)p_t + \alpha^2 Bp = c^2\nabla^2 p\,. \qquad (3)$$

The attenuation coefficient α varies accordingly to,

$$\alpha(i) = \frac{1}{B\delta t}\ln\left(\frac{1}{r_{PML}}\right)\left[\frac{x(i)}{x(n_{PML})}\right]^k, \qquad (4)$$

in which originally the maximum applied absorption rate r_{PML} is equal to $1/10$ and the exponent k=2. Therefore α oscillates from 0 (when x is at the border of the absorbing layer, thus satisfying the acoustic wave equation) to $ln(10)/(B\delta t)$, where δt is the time step and n_{PML} the number of PML grid elements. As it will be discussed later on, changing the values of r_{PML} and k for a fixed n_{PML} improves the effectiveness, reducing its side effects by increasing the maximum absorption rate and using smoother polynomials at the absorbing layer.

Conventional Cerjan's Technique

Conventional Cerjan's method introduces a damping zone around the domain consisting of Na points where the wave amplitude is absorbed by the relation,

$$Fac = e^{-(factor*(Na-i))^2} . \tag{5}$$

The coefficient *factor* is 0,015 for Na boundary layer points in the damping layer and i varies from 1 to Na. The factor remains constant for various numbers of points in the boundary layer.

The wave amplitude gradually diminishes, but at the end of the process a small amount of energy is reflected. Though being small, the energy reflection cannot be accepted for more accurate analysis. To minimize the reflected energy, a common procedure is to increase the number of points in the damping layer. At first, the reflected energy diminishes, but from a certain number of points, it tends to remain constant. A procedure to minimize the energy reflected was then developed to try to diminish the error.

Methodology

In this study, the effectiveness of three different algorithms for wave absorption in the boundary layer was compared. For all algorithms, absorption layers with different thickness were tested. At time $t=0$, a Ricker type source is generated at the center of the model and propagated through a finite difference method.

At first, the boundary layers were made large enough that no waves reach the boundaries of the numerical model during the simulation's time and no absorption is applied, representing an infinite model. The wave field is recorded at receiver points located at a fixed distance from the source for all the propagation time, as shown in Figure 1.

After this initial simulation, the test is repeated, but now with the application of the boundary layers with different thickness and the effectiveness of the absorption algorithms is studied. By taking the square of the amplitude difference between the infinite and the absorbing model, it is possible to evaluate the effectiveness of each absorption boundary,

$$E_{Effectiveness} = \sum_{t=0}^{t=T_f} (U_{\inf inite} - U_{absorption})^2 . \tag{6}$$

The model used a 2D constant velocity (3000 m/s) grid. As shown in Figure 1, the region without the absorption layer has 601x601 grid points. Around this region, a boundary layer was created with thickness varying from 20 to 150 grid points. The wave absorption algorithms were applied in these boundary layers. The finite difference scheme uses a 2nd order in time and a 10th order in space operator. To avoid instability and divergence problems with the numerical method, the grid spacing has 5 meters and the time step 0,0002 s. The distance between source and receiver is of 294 grid points or 1470 m.

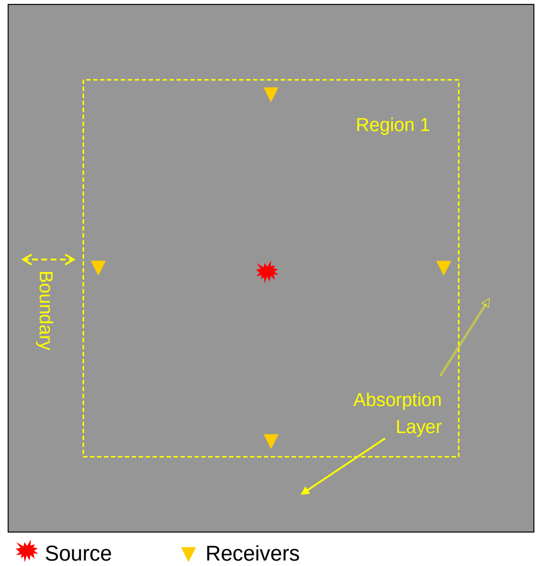

Figure 1: Wave propagation domain with absorbing boundary conditions.

PML Results

Figure 2 compares the effectiveness of wave absorption for the original (PML2-10: k=2, r_{PML}=1/10) and optimized PML methods (PML5-10: k=5, r_{PML}=1/10; PML2-1.1: k=2, r_{PML}=9/10; PML5-1.1: k=5, r_{PML}=9/10; PML7-10: k=7, r_{PML}=1/10) with varying absorbing layers. Results show that, for a small number of PML grid elements (n_{PML}=*25*, *50*), the application of larger maximum absorption rates (r_{PML}=9/10) proves to be more efficient to absorb incident waves, reducing significantly wave reflections at the border.

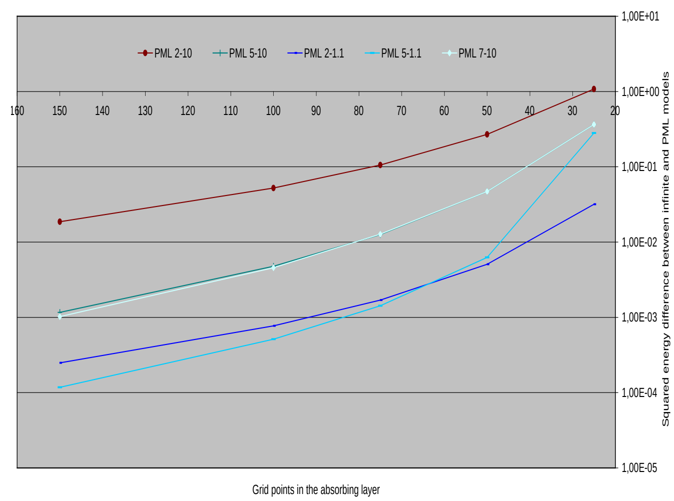

Figure 2: Time sum of squared energy difference between infinite and nonreflecting boundary methods for varying absorbing layers.

On the other hand, for larger PML layers ($n_{PML}=75$, *100*, *150*), a conjugated use of maximum absorption rates (r_{PML}=9/10) and higher order polynomials (k=5, 7) improves the effectiveness of the absorbing layer. In fact, figure 2 illustrates that the proposed optimized PML models are more effective than the original PML method.

Optimized Cerjan's Method

In order to try to improve the conventional Cerjan's method, the coefficient factors are calculated by minimizing the effectiveness equation (6) for each number of points on the boundary layer (20 to 100 points). Minimization was achieved by choosing starting values for Cerjan´s factor, followed by a standard numerical method to find the minimum. The factors computed are shown in figure 3.

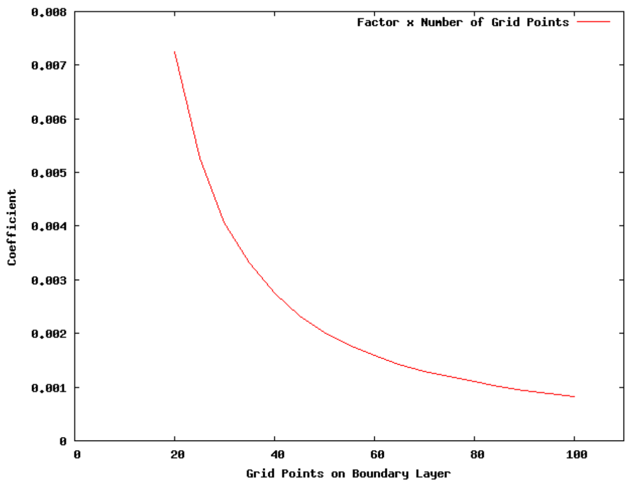

Figure 3: The coefficient factor of the optimized Cerjan's method and the number of boundary layer points.

Figure 4 shows the amount of energy reflection and the number of boundary points. It shows a comparison between the original Cerjan's method and its optimization. It can be seen that the original Cerjan's curve is constant after 25 grid points in the boundary layer, while on the optimized Cerjan's curve the error decreases with the number of grid points in the boundary layer. The main goal is to develop a method that minimizes the error and does not increase the computational effort.

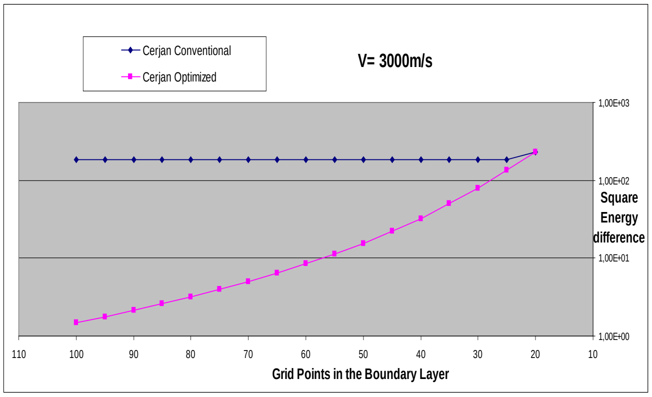

Figure 4 Comparison between conventional and optimized Cerjan's method.

Conclusions

Two classical nonreflecting boundary methods – PML and Cerjan – were optimized aiming to reduce wave reflections at the borders of the 2D FDTD computational domain. Our main motivation was to reduce the number of grid points at the absorbing boundary layer for the least reflected waves inside the medium. It has been found that both optimizations increase the effectiveness of the absorbing layer, with better absorption efficiencies for the optimized Cerjan and PML methods. Results also show that side effects are very sensitive to the number of grid points used in the absorbing layer, with better results found for larger discretization points.

Acknowledgments

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