Embedding of the Theoretical Solution to an Inverse Elastic Problem into Observed Data within Spectral Domain

Georgy Mitrofanov, IGG /SB RAS, Russia
Viatcheslav Priimenko, LENEP/UENF, Brazil

Abstract

We present studies of some issues which relate to embedding of theoretical solutions into observed real data. The study has been conducted on the basis of solutions of inverse dynamic problems or a waveform inversion problem for thin layer elastic models within the frequency domain. Possibilities for the embedding of the theoretical solution into real data are considered from two basic positions: (1) a transformation of the observed data for the purpose of their improved correspondence to the modeling assumptions, and (2) the analysis of influence of the characteristic properties of real data on the theoretical solution. Such approach permits us to improve the solution of complex problems of multi-component and multi-wave seismic prospecting.

Introduction

The solution to the inverse dynamic problems or the seismic waveform inversion by existing methods is well established provided that theoretical schemes and the solutions obtained on their basis correspond to the observed data. These solutions can be achieved in time or frequency domains, using nonlinear methods and concepts of the generalized derivatives, see Tarantola (1987), Pratt (1999). In relation to this the clear internal structure of the methods allows us to provide effective realization on the program level. However, the practical application of this approach to the processing of the real seismograms does not cause that effect, which could be assumed, based on the preliminary theoretical prerequisites.

The outlined situation is connected to that in the theoretical schemes the basic attention is given to the internal structure and efficiency of the methods. Thus, it is presupposed that the real data can correspond to the initial theoretical assumptions. For the most part in the theoretical constructions difference between the real and modeling data is represented with an additive component similar to noise. It is the erroneous representation because the real data can differ from the modeling assumptions much more substantially. As a consequence it requires an essential simplification of the model assumptions in the practice, and also to spend "hard" processing adjusting the real data to the preliminary theoretical assumptions. An example of such an approach is the seismic inversion for acoustic models of media.

In this paper we have tried to analyze the questions, outlined above, concerning the solution to the inverse problems for the thin-layers elastic models of media. We used the decomposition of the wave field developed in Mitrofanov, Helle at al. (1993), and also the solution of the inverse problem in the frequency domain, obtained using the Laplace transform, see Mitrofanov, Priimenko and Missagia (2007(a)).

Method

Figure 1 illustrates the principal moments of the embedding of the theoretical solution into the observed data. Here IP (inverse problem) is the basic object of the theoretical investigations. In addition it is often presupposed that the difference between the real data $\bar{u}^{\mathrm{r}}$ and the theoretical solution $\bar{u}^{\mathrm{f}}$ is not so large. If this proximity is not provided with the initial experiment it can be reached by some transformation for both synthetic and real data. Basically such a general scheme is quite true. However it is necessary to take into account that the required transformations can be influenced by both features of the real data on the structure of the theoretical solution, and the structure of the theoretical solution on the transformation of the initial observations. For example, the area Σ, where the theoretical solution is under construction, can essentially differ from the area Σ_0, where the initial observations have been received.

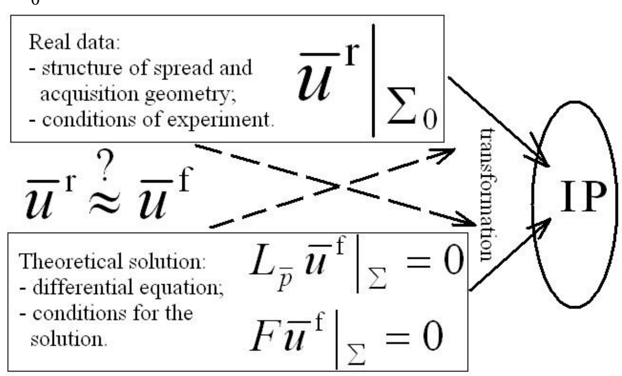


Figure 1

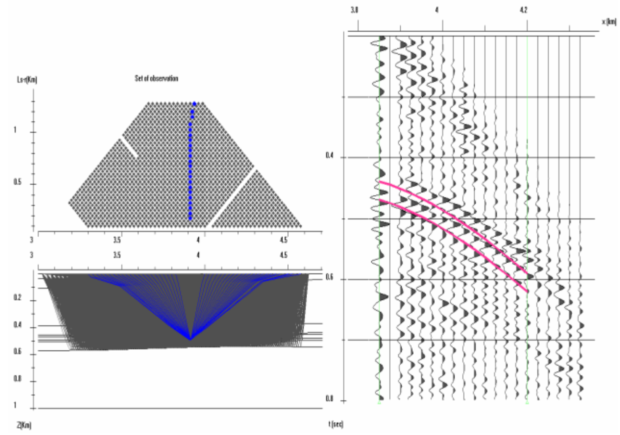
If the inverse problem is solved in the frequency domain, then the questions of the embedding of spectra, constructed on the basis of the theoretical solution of the direct problem, into spectra, calculated using the real data, become essential, i.e., the problem is complex. We need to control the correspondence between the transforms of the real and synthetic data.

In the case of the inverse problem related to target thin-layers objects there is an additional serious problem, causing a considerable difference between the transformations of the real and synthetic data within the frequency domain. Solving the direct problem we suppose the homogeneity of overlying formation and an invariance of the stimulation and registration conditions of signals. All the specified characteristics are not realized in the real experiment. Besides, the real media is not horizontally layered as it is supposed to be in the theoretical models. Therefore, a preliminary processing of the seismograms is required to eliminate the specified differences. Using simplified schemes, developed for the acoustic models and basically directed on the selection of monotype reflected waves, it is impossible in such processing in the case of multi-component observations and elastic models. An optimal strategy in this case is the scheme of decomposition of the wave field, based on a "frame" thick layer model proposed in Mitrofanov, Helle at al. (1993). The scheme is based on the ray tracing procedures for waves of the certain type (for example, PP and PS) and on the identification of corresponding waves on multi-component seismograms. It allows us to select the reflected waves of required types from the general structure of the registered wave field, and further to consider changes in the form of the selected reflected signals, connected with heterogeneity of the stimulation and registration areas, in additional to the area situated upper of the reflecting thin layer object as well. It also guarantees the solution of the inverse dynamic problems for target thin-layers objects in the spectral domain for the horizontally layered elastic models. Besides, the specified procedures of the decomposition give an opportunity to approximate the model of local target object to the one-dimensional theoretical model.

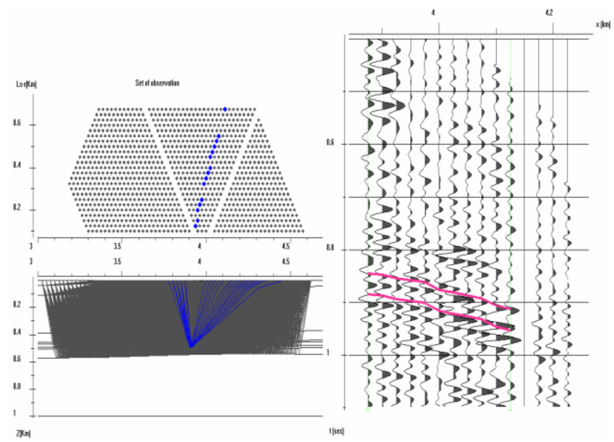
Results

Using the decomposition process for selection of the PP (a) and PS (b) waves on real seismograms is shown on Figure 2. An essential distinction in the selection of the specified waves, as on the structure of the initial observations and on the initial wave field, is distinctly shown. When the selection of signals is made, we can transform the selected observations to the multicomponent seismograms, corresponded to the local one-dimensional model. Thus we will have the characteristics of the modified selected observations. They will allow us to define the type of necessary "windows" for the embedding of the spectra, constructed on the basis of the theoretical solution of the direct problem, into the spectra, calculated using the real seismograms.

Figure 3 presents the spectral characteristics of special two-dimensional "windows", calculated using two types of the seismograms constructed for the model showing at Figure 1. The aperture of observations for these seismograms was differed (by the spatial variable). Besides, the order of the Bessel function, used in the solution of the corresponding direct problem, has a certain influence on the type of the "window", see Mitrofanov, Priimenko and Missagia (2007(b)). Also the "window" structure is defined by α, presented in the Laplace transform parameter $p=-\alpha+i\omega$. Using the constructed "windows" gives a significant effect for the embedding of theoretical spectra into the calculated ones and has a little more considerable character, than the smoothing of theoretical spectra, leading to reduction of the discontinuous characteristics, presented at these spectra, see Mitrofanov, Priimenko and Missagia (2007(a)).



(a)



(b)

Figure 2

Therefore, Figure 4 presents the results of the calculation of the two-dimensional spectra using the synthetic seismograms with $\alpha=0{,}01$ (a), the results of the calculation of the theoretical spectra for the corresponding model without the use of "windows" (b), and using special "windows" (c). We note that bands of frequencies ω and ν were constructed using the recommendations proposed in Mitrofanov, Priimenko and Missagia (2007(b)). It gave the opportunity to provide the maximum embedding of the calculated spectra into the theoretical ones. It is visible that besides "suppression" of the discontinuous features in the theoretical solution of the problem, the procedure allows us to draw essentially together both types of the spectral characteristics (calculated and theoretical) in areas of absence of discontinuities, even for the indicated small values of α. There is an essential distinction in the structure and the form of the theoretical and calculated spectra in the case of not using special "windows". A convergence of the spectral characteristics obtained in the various ways is observed with the increasing of α. But even for biggest values of this parameter use of smoothing "windows" allows us to receive considerably better coincidence for theoretical and calculated spectra. Figure 5 shows the spectra, corresponded to the spectra presented in Figure 4 with $\alpha = 5$.

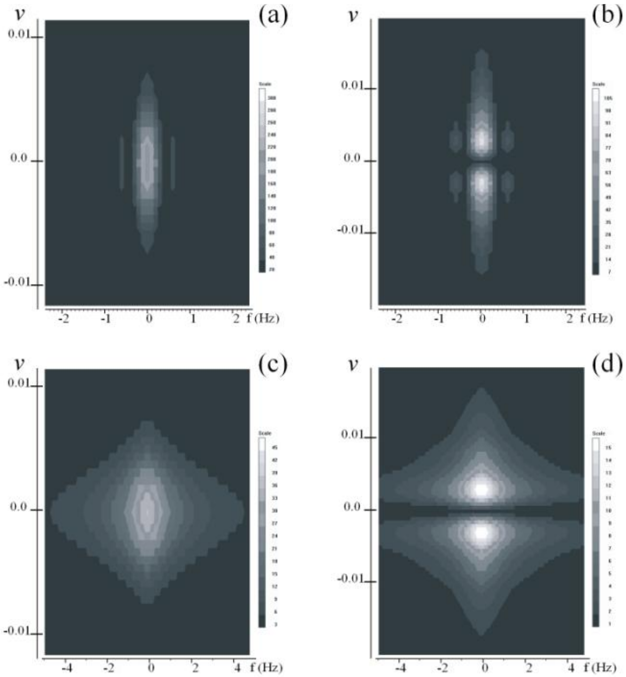
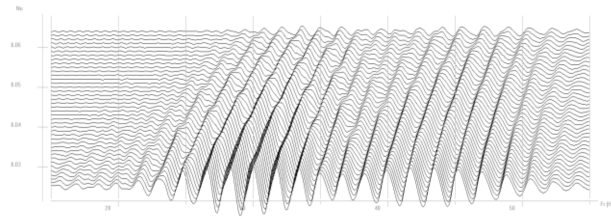


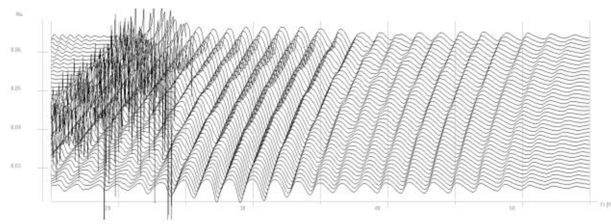
Figure 3

It is necessary to consider one more feature which can have a big influence on the solution of the inverse problem. It is connected with the characteristics of orientation of the real sources that usually are not considered in the theoretical solution. This feature of the real data can also be eliminated by using the procedure of the wave field decomposition.

Figure 6(a) presents spectra of the PP-wave reflected from a thin-layers object. The top amplitude spectra are the results of the corresponding calculation on the selected intervals of the synthetic seismogram. Next spectra (from top to lower parts in the figures given) represent the theoretical spectral characteristics corresponding to the thin-layers object, which were multiplied on the spectrum of a wavelet, considered as a theoretical approximation to the spectrum of the signal reflected from the subject. The third row of the spectra (b) represents the difference between two first spectra. It is visible that the difference between these spectra can reach substantial values (in certain cases up to 58%). The value of the target functional, constructed on the basis of the specified spectra, is $0,31245$. Such value is not acceptable, since similar values are obtained in the case of the erroneous model.



(a)



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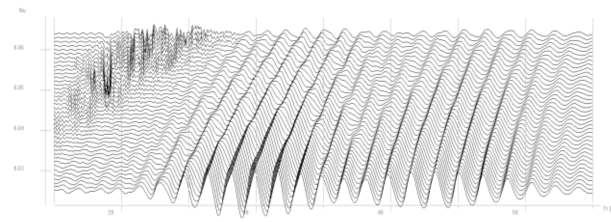
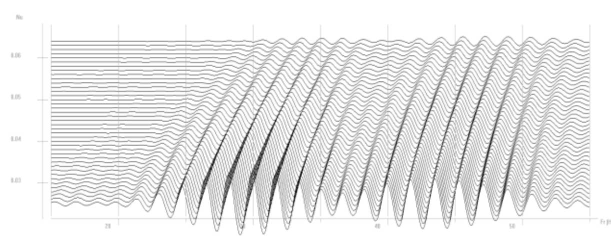
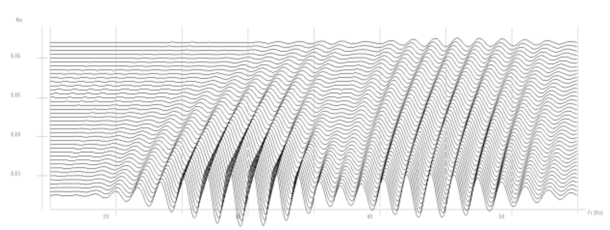


(c)

Figure 4



(a)



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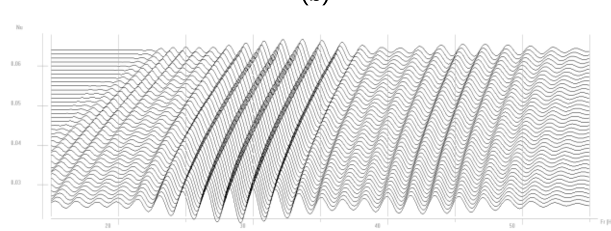

(c)

Figure 5

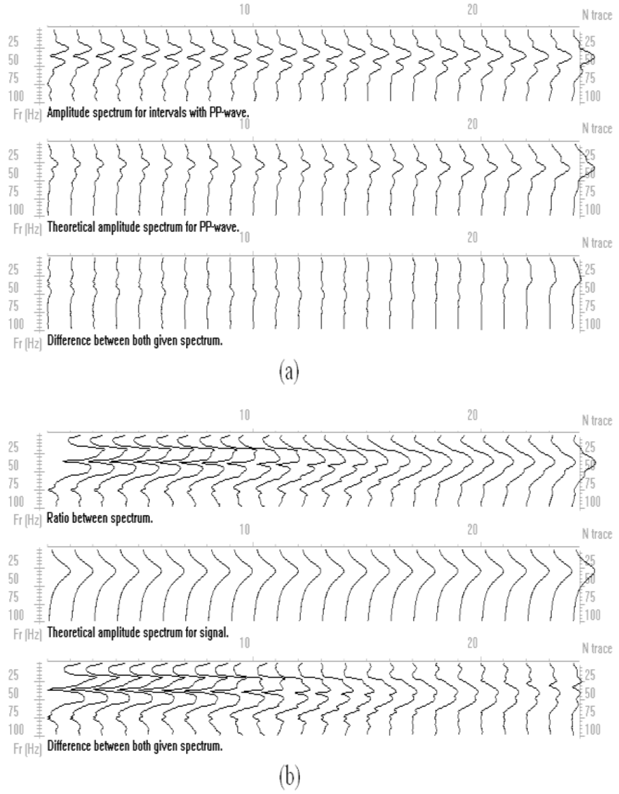

Figure 6

Figure 6(b) demonstrates significant distinctions between the theoretical and calculated spectra. Here the top part of the figure represents relations between the calculated (or observed) spectra and the theoretical spectral characteristics of the object. By definition, they should correspond to the spectral characteristics of the wavelet (shown in the middle part of the figure). However, in practice we have a considerable distinction between this relation and the spectrum of the wavelet. The value and the structure of this distinction are shown in the lower part of the figure.

Using the simplified decomposition procedures, which specify the form of the incident impulse which directly steps up to the thin-layers object, allows us to improve the quality of the calculated spectra. The results presented in Figure 7(b) confirm such conclusion. As a result, we can define more precisely the smooth spectrum of the incident impulse. However, despite the improvement of the quality of estimations of the spectral characteristics, there is a regular component, which changes the form of the incident impulse depending on the removal of a source-receiver or from the incidence angle of the corresponding wave on target object.

The nature of such variations is connected with the directional radiation pattern, used in the complete solution (embedded source of the centre of expansion type). It is clear for the physical reasons, and its manifestation in the synthetic data demands using more difficult models of decomposition which can consider such changes. The changes, obtained as the result of the directional pattern estimation in the seismograms, corresponding to various target objects, are shown in Figure 8. It is visible that for the objects near the depth of bedding of the target horizons we obtain near characteristics of the directional radiation pattern, see Figures 8(a) and 8 (b). The changing of the target horizon and its depth leads to the change of the corresponding directional radiation patterns, see Figure 8(c).

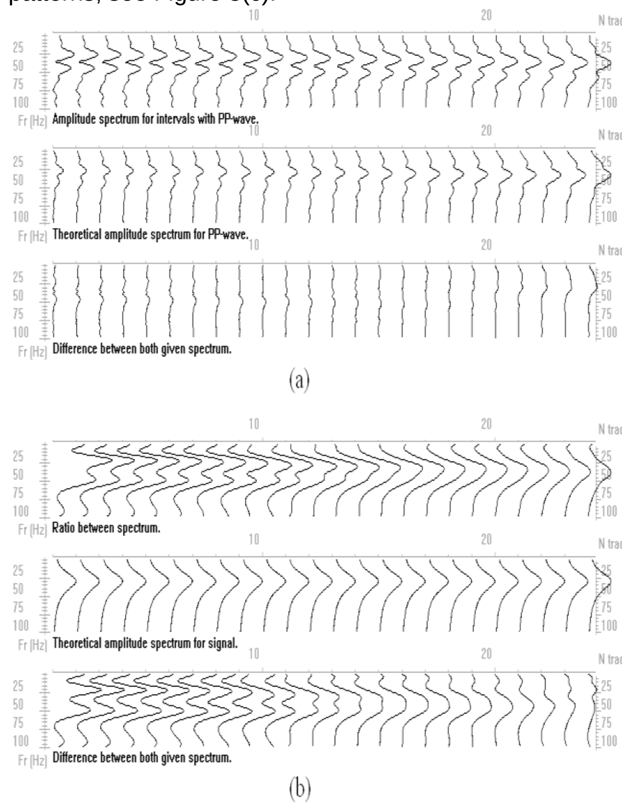

Figure 7

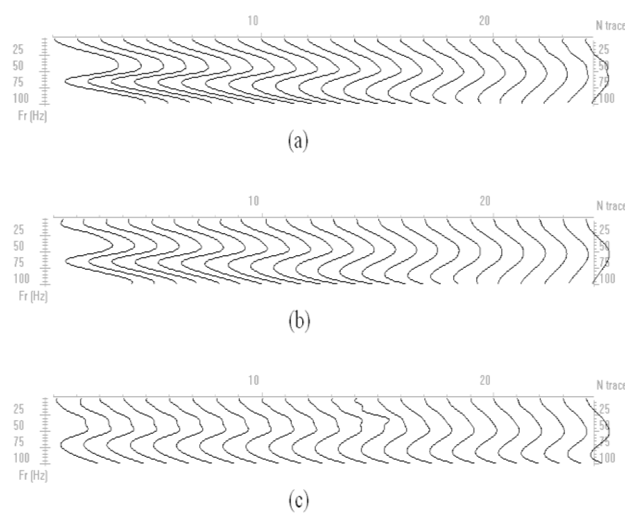

Figure 8

Thus, in the real experiment when the directional radiation pattern is unknown and can vary for various sources, its account in the decomposition procedures is required. It should be noted, that in our experiments the taking into account of such characteristic allowed us to provide the coincidence of calculated and theoretical spectra, closing to the ideal. In this case the functional value is decreased from $0,31245$ to $0,01427$.

Another important aspect of the solution of inverse seismic problems within the spectral domain is using the full information on spectra of the observed waves. Often in practice, there is only used the amplitude component of the spectrum, excepting the phase component of the solution. For this purpose some additional assumptions about zero or minimum phase spectrum of the observed signals are proposed. The following result illustrates the significance of the phase component for the solution of the inverse problem in the case of thin layer models.

Consider a model of the target object composed with three thin elastic layers, see Figure 9(a). The total thickness of the object is equal to $20\mathrm{m}$, the thickness of each layer and the corresponding values of V_p, V_s, ρ are shown by the solid line at the figure. A multi-component seismogram was calculated for this model using the algorithm proposed in Mitrofanov, Priimenko and Missagia (2007(a)), see Figure 9(b). In accordance with the linearized solution there were selected time intervals (dotted line) contained signals, corresponded to the converted wave reflected from the target object. After that, using the selected signals, there were calculated the phase and amplitude spectra for the band $[10\mathrm{Hz}, 60\mathrm{Hz}]$. Such band is typical one in the real data processing. The calculated spectra were used as an input data for the nonlinear estimation algorithms of the elastic parameters of the target object. In addition, there was considered (as an initial approximation) a model of the target object with larger thickness (30m) and with altered parameters in the second layer. Modification of the layer thicknesses and the parameters, mainly with the decreasing of V_s, was correct from the geological point of view, because there were supposed the presence of a deposit with these parameters in the layer. The altered structure of the model and its parameters are indicated in Figure 9(a) by the dotted line.

The results, obtained using the amplitude spectrum only, has a poor quality, see Figure 9(c). In this case we cannot estimate correctly either the true layer thicknesses or the total thickness, also there is an essential instability in the definition of V_p in the second layer. These features of the obtained solution are quite explicable taking into account the information, contained in the amplitude spectrum of converted wave in a limited band, which gives the chance to define the structural parameters of layers and the basic elastic characteristic of the P-wave at a lesser degree.

Using the phase component essentially improves the accuracy of the reconstruction of the structural parametres of thin-layers object (in the experiment the error made size barely exceeding the value of 1m for the upper boundary of the second layer), and also it helps to define precisely all the elastic parametres, see Figure 9 (d). Improvement of the solution is connected by that the phase component reacts to the change of the structural parametres (thickness) more sharply. Therefore including the phase component in the target functional improves essentially its properties.

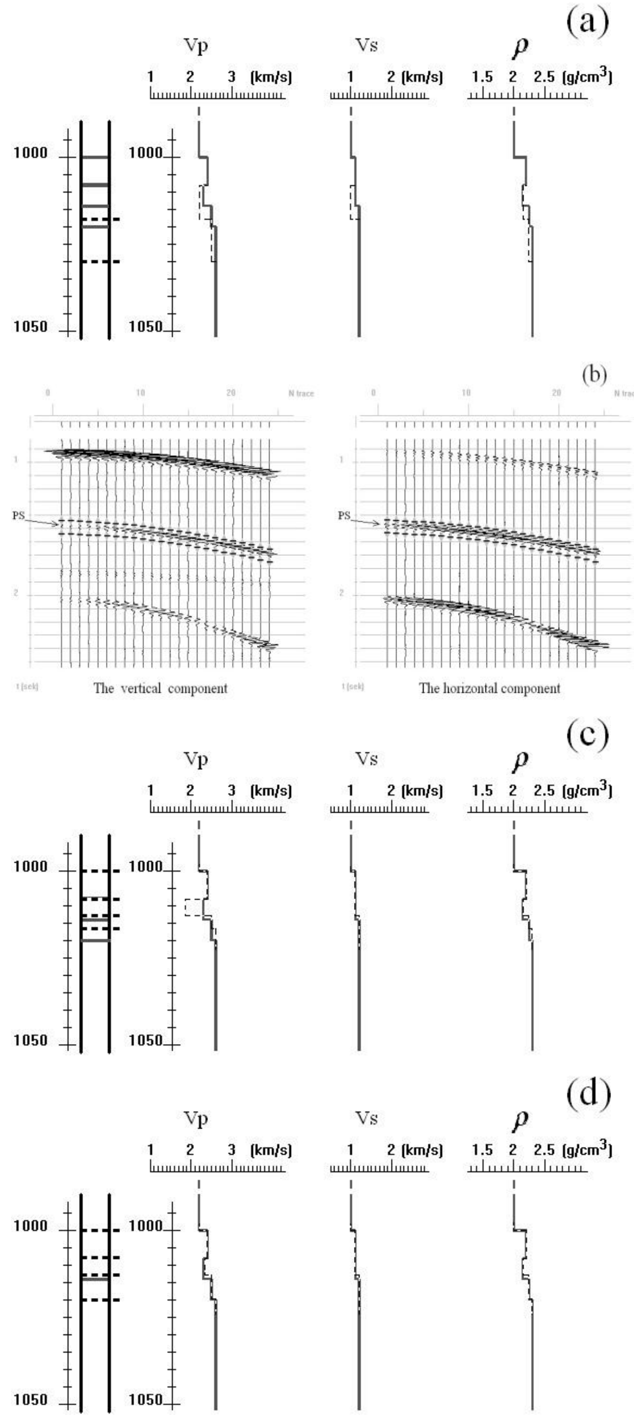

Figure 9

Furthermore the phase spectrum gives some additional information concerning variations of the elastic parameters of all layers that eliminates non-uniqueness of the solution. Thus, the phase spectrum, despite its seeming less stable in comparison to the amplitude spectrum, can improve the accuracy of the solution to the inverse dynamic problem for thin-layer objects.

Conclusion

Analysis of the embedding of the theoretical solutions into observed data shows that the existence of effective algorithms for the inverse problems solution is not enough for their successful practical application. In practice it is required to solve various problems related to the embedding of the theoretical solutions into observed data. Using, as an example, the inverse dynamic problem for thin-layer elastic models, we show how on the basis of the multilevel decomposition of the observed wave fields and the form of seismic signals, the observed data can be transformed to initial information for the solution of the inverse problem related to local target objects of the real media. Taking into account the features of real observation systems is important for the theoretical solution of such problems. It allows us to satisfy the conditions for the embedding of calculated spectra into theoretical ones.

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