The EGM08 Model and SRTM Data for the Gravimetric Geoid Using Voronoi/Delaunay Discretisation

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Abstract

We used the recent EGM08 geopotential model complete to degree and order 2,159, in a remove-compute-restore (RCR) method to compute the geoidal undulations for the State of Rio de Janeiro, Brazil. Two discretisation methods by pointwise numerical integration were applied, in order to bypass the usual gridding process. They are based on both Voronoi/Delaunay structures. In Voronoi technique, original data are used and preserved, if they have a regular distribution, and not requires filling blanked areas. Target area is subdivided into a set of polygonal cells, which each hold an original gravity anomaly. Delaunay scheme uses a tesselation over the area, subdividing it into triangles, creating a triangulated irregular network - TIN. If data points have a regular spatial distribution, the triangles' vertices hold the original values and no gridding is also required. The difference here is that mean gravity anomalies are interpolated for the barycenters from the corresponding triangles' vertices. Despite of this interpolation, it is done locally and it is limited to each figure. As the geoidal undulations were computed for the triangles' vertices, Stokes function singularity is gone. For the terrain corrections and indirect effect we used a 6-arcsec resolution DTE. The EGM08 geopotential model and the topographical data result in a improvement in the geoid modeling. We compared both Voronoi/Delaunay schemes with the classical pointwise numerical integration technique.

Introduction

Despite the techniques for computing the gravimetric geoid by Stokes (1849) method, the target area is partitioned into grid elements. The geoidal undulations are computed at these cells, and the gravity anomalies are interpolated into a regular grid. Thus, modified data are used instead of the original ones. Also, gridding usually requires excessive manual/computational effort, and may generates spurious information. In this work, Voronoi/Delaunay structures (Aurenhammer, 1991), are applied which bypass the usual gridding process. These approaches are reported in Santos and Escobar (2004). As we computed the geoidal undulations at the grid nodes and not at the cell centers for the classical method comparison, Stokes function singularity has gone. The most recent EGM08 geopotential model and topographical data derived from Shuttle Radar Topography Mission – SRTM were used in the computation. These data were structured for the brazilian region by the National Institute for Space Research - INPE, Brazil (INPE, 2008).

Method

The Stokes technique was applied for the local geoid determination by pointwise numerical integration, which the target area was partitioned according to Voronoi/ Delaunay tesselations. The suitable format for the discrete Stokes integral, which yields the geoid-ellipsoid distance at a point, is given by

$$N_{\text{Stokes}} = \frac{1}{4\pi R\gamma}\sum_{i=1}^{n}\Delta g_i S(\psi_i) a_i \,, \tag{1}$$

where R is the earth mean radius, γ is the normal gravity at the ellipsoid surface, and Δg_i is the gravity anomaly at the data point. The subscript relates to the i-th polygon/triangle in the integration. The term Δg_i is the Helmert residual gravity anomaly, and ψ_i is the spherical distance between integration and computation points. The spherical Stokes function $S(\psi_i)$ is given by (Heiskanen and Moritz, 1967)

$$S(\psi_i) = \left(\frac{1}{\vartheta}\right) - 4 - 6\vartheta + 10\vartheta^2 - \left(3 - 6\vartheta^2\right)\ln\left[\vartheta(1+\vartheta)\right], \tag{2}$$

where $\vartheta = \sin(\psi/2)$.

According to the Voronoi scheme, each original data point represents its cell. We considered the spherical distance between data points, where the geoidal undulations are computed for the integration. On Delaunay scheme, gravity anomaly at each cell is the mean value, weighted for the respective distances vertex-to-barycenter. The geoidal undulations are computed at the triangles' vertices (data points), and the spherical distances are related to the barycenters.

On both the schemes, clustered data are removed outside a given radius, in order to avoid singularities in Stokes function and/or generation of irregular polygons. In Voronoi scheme, when a data point is coincident with an integration point, Equation (1) is replaced by (Heiskanen and Moritz, 1967)

$$N_o = \frac{\Delta g_o \psi_o}{\gamma}, \quad (3)$$

where Δg_o is the residual Helmert anomaly at the point, and ψ_o is the mean spherical distance point-polygon sides. In Delaunay scheme, we adopted a procedure that avoids the Stokes function singularity, i.e., the computation points are just on the triangles' vertices and not inside them.

As the RCR technique was used (Denker and Wenzel, 1987; Strang van Hees, 1986), the geoidal undulation may be given by (Sideris and She, 1995),

$$N = N_{\text{Stokes}} + N_{\text{EGM08}} + N_{\text{Ind}}, \quad (4)$$

where N_{EGM08} was computed with the EGM08 model coefficients (Pavlis et al., 2008).

For the computation of the residual component N_{Stokes}, residual anomalies were derived by subtracting EGM08 gravity anomalies from local Helmert anomalies according to the Second Helmert Condensation Method (Lambert, 1930), corrected for the atmospheric mass effect. The indirect effect is computed using SRTM elevation data.

The EGM08 model and topographical dataset

The most recent Earth Gravitational Model 2008 - EGM08, complete to degree and order 2,159 was applied (Pavlis et al., 2008). Its expansion yields a 5-arcmin horizontal resolution, although it contains spherical harmonic coefficients extending to degree 2,190 and order 2,159. EGM08 model was developed by National Geospatial-Intelligence Agency - NGA, and combines terrestrial gravity anomalies with GRACE mission data, and gravity data derived from altimetric satellite. It was presented at the 2008 General Assembly of the EGU (Pavlis et al, 2008).

A 6-arcsec resolution DTE data was used from National Institute for Space Research - INPE, Brazil. These data was raised for the whole Brazil area as the TOPODATA project (INPE, 2008). It combines local elevation data with topographical data from Shuttle Radar Topography Mission - SRTM. Sixteen sheets in scale 1:250,000 from TOPODATA project was used for the terrain corrections and indirect effect computation.

Results

Results from Voronoi/Delaunay discretisation techniques were compared with the classical numerical pointwise integration. Data includes 1,940 terrestrial gravity points, filled out with 491 Geosat anomaly data with 5-arcmin resolution on the oceanic area. In sites with no gravity information, gridded data derived from Bouguer anomalies were used. This computation summed up 430 points to the dataset. Figure 1 shows the dataset used.

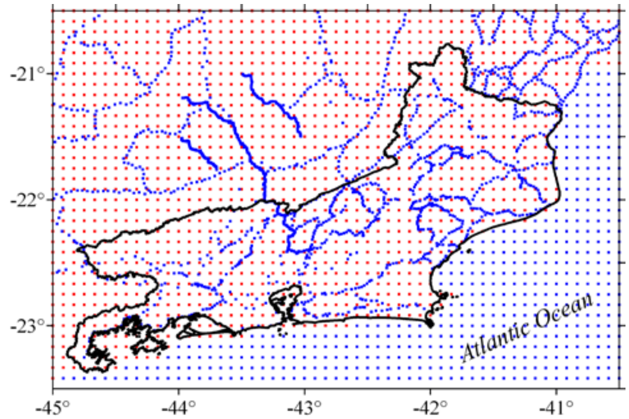

Figure 1: Gravity data distribution (terrestrial/oceanic - blue; Bouguer derived - red).

Topographical relief varies between 0 m and 2,821 m (Agulhas Negras peak), which mean height is 740 m. Figures 2 and 3 show the geoidal undulations and gravity anomalies computed with EGM08 model, respectively. EGM08 gravity data are in agreement with the local gravity anomalies (Figure 4). A comparative test between these data is discussed in Escobar (2008).

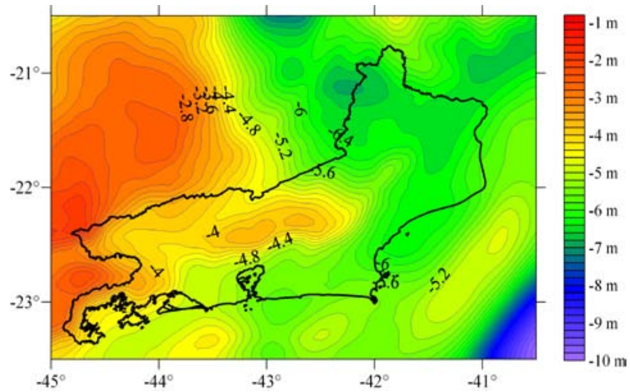

Figure 2: EGM08 geoidal undulations.

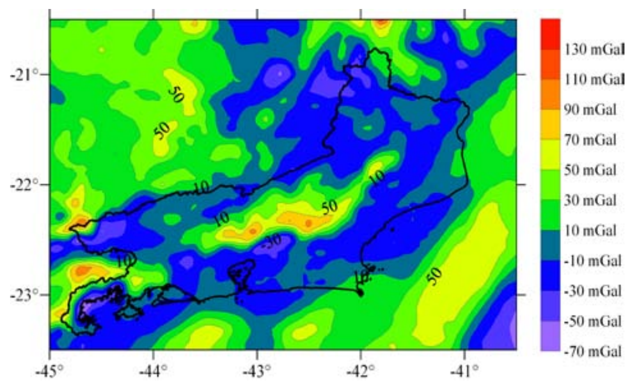

Figure 3: EGM08 gravity anomalies.

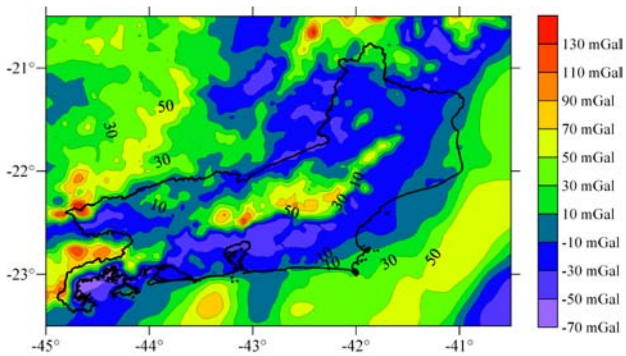

Figure 4: Helmert gravity anomalies.

The results from Voronoi/Delaunay schemes were compared with the classical pointwise numerical integration. The area was subdivided into 3,146 Voronoi polygons, and 6,116 triangles. For the classical technique the area was partitioned into 1,980 constant geographic cells. The differences between Voronoi/Delaunay and classical schemes are exhibited in Figures 5 and 6, respectively.

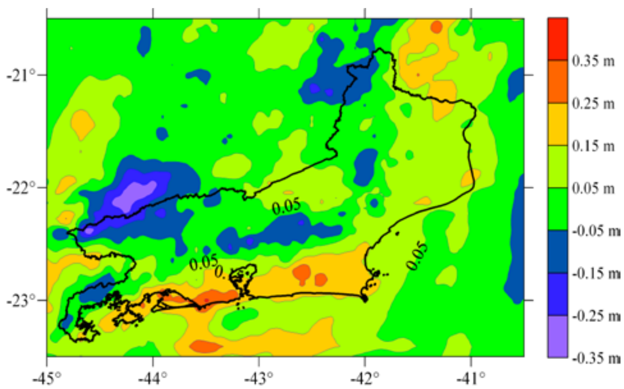

Figure 5: Differences between classical and Voronoi techniques for the geoid.

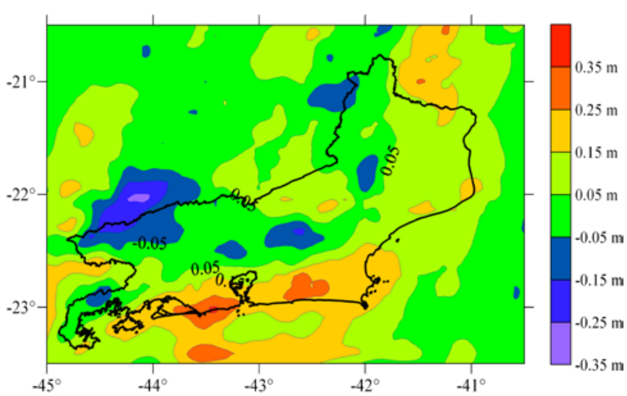

Figure 6: Differences between classical and Delaunay techniques for the geoid.

The differences are due to the inclusion of local data for Voronoi/Delaunay schemes, which are "smoothed" in the gridding process for the classical technique. Thus, Voronoi/Delaunay schemes seem to be more reliable than the classical technique.

Geoidal undulations are presented in Figures 7 and 8, respectively, for Voronoi/Delaunay schemes. The results point out an agreement between the techniques.

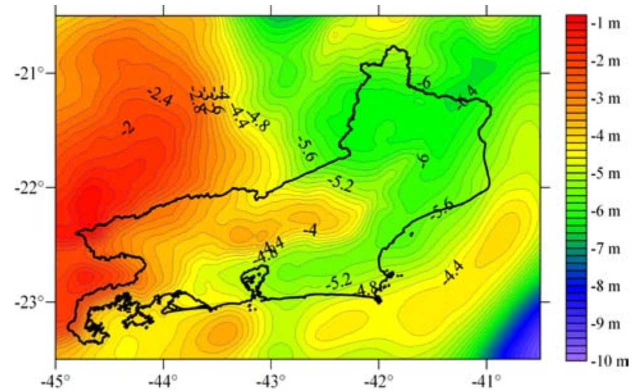

Figure 7: Geoidal undulations according to Voronoi scheme.

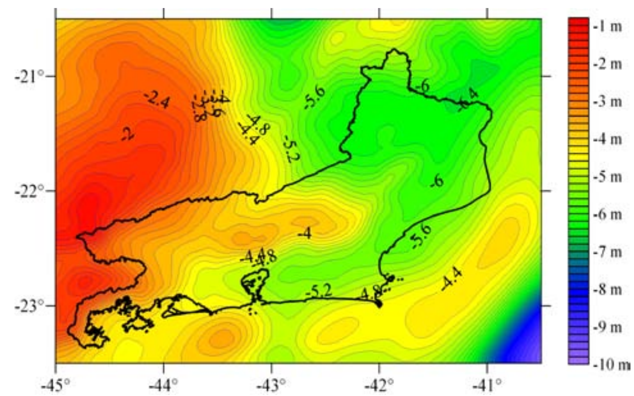

Figure 8: Geoidal undulations according to Delaunay scheme.

Conclusions

The EGM08 geopotential model contributed to the results, as well as agreed with the local data - either the gravity anomalies or the geoidal undulations. The results points out the convenience of EGM08 model for the geoidal computing. As well, the topographical data derived from SRTM also improves the computation, either the terrain corrections / indirect effect, and the derived gravity anomalies.

Both Voronoi/Delaunay structures proved to be straighforward methods for computing the residual geoid component. As remarked in Santos and Escobar (2004), the schemes presented results alike to each other, in comparison with the classical integration method, if the data distribution is homogeneous and dense enough. Also, they have the advantage of avoiding a gridding step. If there are blanked regions in the studied area, local and gridded data can be merged, thus exploring the best of these kinds of data. This flexibility seems to be the main advantage of using those structures, which are very simple and convenient for the geoidal computing. In addition, areas to be mapped can take any shape or data configuration.

Acknowledgments

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References

Aurenhammer F., 1991. Voronoi diagrams - A survey of a fundamental geometric data structure. ACM Computing Surveys, 23, 3, 345-405.

Escobar I.P., 2008. Modelo geopotencial EGM08, testes realizados no Estado do Rio de Janeiro. In: II Simpósio Brasileiro de Ciências Geodésicas e Tecnologias da Geoinformação. Recife, Sept 8-11, 2008. In Portuguese.

Heiskanen W.A. and Moritz H., 1967. Physical Geodesy. San Francisco, W.H. Freeman, 364 pp.

INPE, 2008. TOPODATA Banco de dados geomorfométricos do Brasil. (Online at http://www.dpi.inpe.br/topodata/acesso.php). In Portuguese.

Lambert, W.D., 1930. The reduction of observed values of gravity to sea level. Bull. Géod., 26, 107-181.

Pavlis N.K., Holmes S.A., Kenyon S.C. and Factor J.K., 2008. An Earth Gravitational Model to Degree 2160: EGM08. Geophysical Research Abstracts, 10, EGU2008-A-01891, 2008.

Santos N.P. and Escobar I.P., 2004. Discrete evaluation of Stokes's integral by means of Voronoi and Delaunay structures. J. Geod., 78, 6, 354-367.

Sideris M.G. and She B.B., 1995. A new, high-resolution geoid for Canada and part of the U.S. by the 1D- FFT method. Bull. Géod., 69, 92-108.

Stokes G.G., 1849. On the variation of gravity on the surface of the Earth. In: Mathematical and Physical Papers, Vol. II, New York, 131-171 (from the Trans. of the Cambridge Philos. Soc., Vol. VIII, 672-695.)