Mixed Fourier and wavelet analysis of geomagnetic data

Alberto F. Akel, ON/MCT, Brazil
David Oliva, ON/MCT, Brazil
Andres R. R. Papa, ON/MCT & IF/UERJ, Brazil

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Abstract

We present a mixed Fourier-wavelet statistical analysis of low latitude geomagnetic data. The first step consists in a filtration of daily (and harmonic) components of the signal. We follow by a wavelet decomposition of the remaining signal. We are interested in the possibility of short period studies (a task hardly well achieved by Fourier methods) for general characterization of the geomagnetic signal and, in particular, on the particularities of the signal in periods preceding the occurrence of large geomagnetic disturbances. We discuss some characteristics of our calculations as well as possible extensions and corrections to be taken into account in futures works.

Introduction

The magnetic field that can be measured at the Earth's surface has two major dynamical components: those of internal nature, with very large periods (several years), and those of external or atmospheric nature (on which we will focus our attention), with characteristic periods of several days or less. There are also more or less static components originated at local concentrations of magnetic materials (with geological times, when not under anthropogenic effects, as typical periods). We present in Figure 1 one example of the geomagnetic field dependence during one day (October 14th, 2000).

Several methods have been employed to characterize the geomagnetic signal. Among them we can mention detrended fluctuation analysis (Wanliss, 2005), Fourier analysis (Papa et al., 2006), intermittence analysis (Bolzan et al., 2005), chaos analysis (Barraclough and De Santis, 1997) for the total field, non-linear analysis (Vörös et al, 1994) and stochastic and scaling analysis (Telesca et al., 1999). Large efforts have also been directed to prediction possibility of large geomagnetic disturbances. They include statistical analysis of short (three days) periods previous to storms (Papa and Sosman, 2008) and models for daily variations (Lesur et al., 2005).

At the same time, many attempts have been done of extracting the main characteristics of the geomagnetic signal defining "geomagnetic indexes" that do this in several ways. In this work we examine the Index Dst and Sym-H for the month of October 2000. As for the case of direct measurement, several works have also focused the possibility of prediction for these indexes, or disturbance forecasting of geomagnetic storms based on their study. Among them we mention Dst models (Temerin and Li, 2006), Sym-H statistical studies (Dias and Papa, 2009) and Dst real time forecasting (O'Brien and McPheron, 2000).

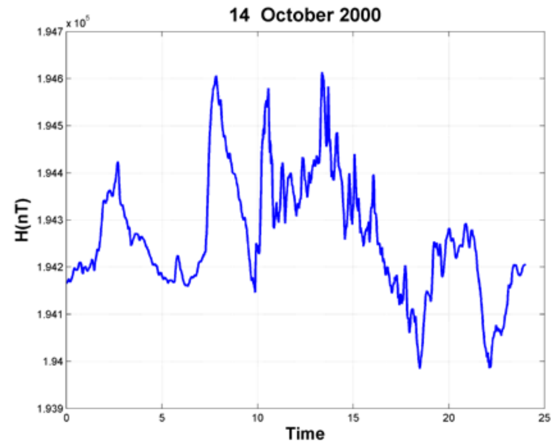

Figure 1- Dendence of the H component of the geomagnetic field during October 14th, 2000 as measured at the Vassouras Observatory. This day was a specially active one.

In Figure 2 we show the dependence of the Dst index for the month of October 2000. This was a relatively quiet month with some active periods around days of numbers 4, 14 and 28. For the present study we have chosen the 14th day.

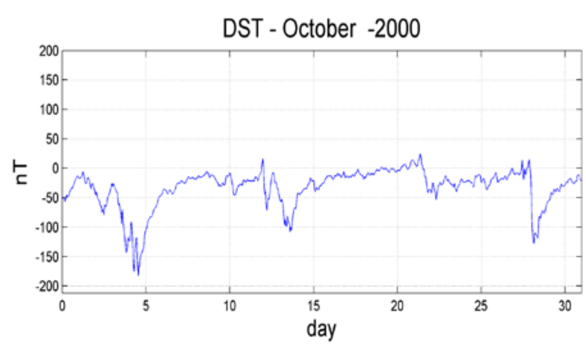

Figure 2 – Dst index for the whole month of October 2000. The data was obtained at the Word Data Center for Geomagnetism, Kyoto, Japan. Note the activity around the day 14, the day we have focused on in this work.

In previous works by our group (see Papa and Sosman, 2008, for example), an extensive use of Fourier transforms have been done. However, Fourier transform are more appropriated for long periods of time (which in our language means, large numbers of experimental points). Some essential characteristics of signals are loosed when we try to use them in short time periods. This was the case, for example, of the double power-law found by Papa et al. (2006) in geomagnetic signals for one month periods. The second regime completely disappears when the period was shortened to three days periods (Papa and Sosman, 2008).

On the other hand, the geomagnetic signal is a highly non stationary one which forces the application of Fourier transforms to be done with a special care and, at the same time, limits its validity in a considerable extent.

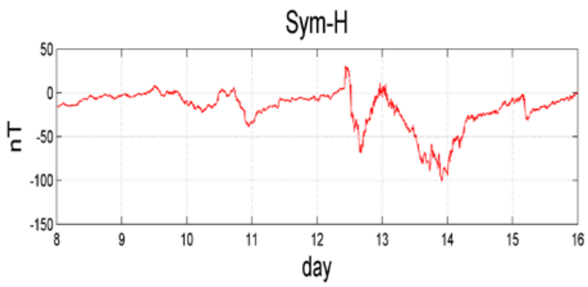

Figure 3 – Sym-H index for October 8-16, 2000. The data was obtained at the Word Data Center for Geomagnetism, Kyoto, Japan. Note again, as in Figures 1 and 2 the great activity around the 14th day.

For all the reasons mentioned above we introduce in this work the use of wavelet transforms to study the geomagnetic signal. This is double welcomed, first, because their capability of analysis in short periods of time (relatively small data sets) and second, because their ability to describe non stationary signals.
The rest of the work is organized in the following way: in the next section (Method of analysis) we breifly describe the procedure used to study the geomagnetic signal of our Vassouras observatory. In the Results and Discussion section the result from the mixed Fourier-Wavelets decomposition as well some considerations on them are presented. Finally we present the Conclusions which include some trends for future works.

Method of Analysis

The method of signal analysis is composed by two relatively simple steps:

1) First, we have decomposed original geomagnetic measurements of the type presented in Figure 1 through the application of the fast Fourier transform (FFT). After that we have filtered the result through an inverse FFT using a second order Butterworth filter (see the work by Papa et al. 2006, to know the way to determine the cutoff frequency of the Butterworth filter and other details). With this we have eliminated direct as well as daily components (and their harmonics). See Figure 4, where an example of the new dependence after the Fourier filtering is shown.

2) Second, we have decomposed through a wavelet transform the already Fourier filtered signal. We used the wavelet transform to decompose the time series into the time and frequency domain. This permits us to study non–stationary signals, as the present case is.

For the wavelet analysis, we choose the mother wavelet as a Gaussian Morlet one, given by

$$\psi(t) = \pi^{-1/4} e^{-iw_0 t} e^{-t^2/2} \quad (1)$$

where w_0 is the frequency parameter that allows one to shift the frequency range to be investigated.

The study was developed for a period of one relatively active day.

Results and Discussion

Figure 4 shows the dependence of the geomagnetic signal (H component) at our Vassouras magnetic observatory for the day October 14th, 2000, as well as the filtered signal obtained after application of a second order Butterworth filter. Note that the red curve shows only low frequency components (all the detailed richness of the blue one was erased through the application of the Butterworth filter).

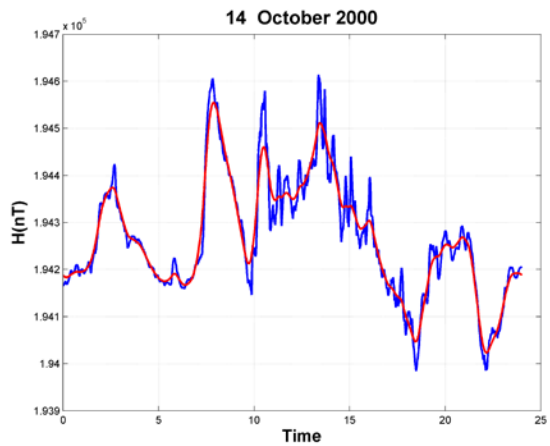

Figure 4 – The same dependence shown in Figure 1 (blue) and the signal after filtering of the higher frequencies (red). On the diference between this two signals was applied the wavelet transform.

The difference between the two signals presented in Figure 4 is shown if Figure 5. It should contain just higher frequency components. On it we have applied the wavelet transform. An alternative to obtain the higher frequency components would be to apply directly a high-pass Butterworth filter to the original signal. However we have preferred to use the original methodology (see Papa et al., 2006, and references therein). Comparing figures 2, 3 and 5 it is clear that all of them show some correlation between equivalent time periods, i.e., period with the higher absolute Dst and Sym-H values coincide with the period of more intense variability of the filtered geomagnetic field.

Figure 6 shows the wavelet decomposition for the signal presented in Figure 5.

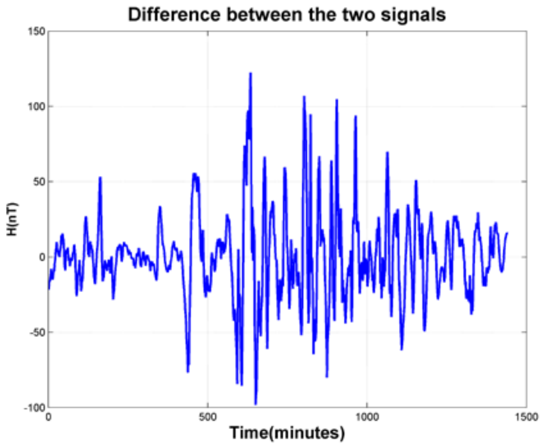

Figure 5 – Difference between the two signals in Figure 4. Note the difference between the scales of vertical axis of both figures. Note also that low frequency components seem to be no more present.

The first feature to be noted in Figure 6 (comparing with Figure 5) is that periods of relative calm (this is, periods where the signal is near zero in Figure 5) are generally poor in frequency, that is the case, for example, for the periods between ~0 and 2 hours, near 4 hours and around 16 hours. Note, however, that there are also periods of high absolute values with no special frequency ranges on them, examples of this are around 9 hours and around 15 hours. The richer frequency ranges are generally associated to periods accomplishing two conditions at the same time: relative large absolute values and the maintenance of these values for long enough periods of time. The last one is the case, for instance, around 7.5 hours and between 10 and 11 hours.

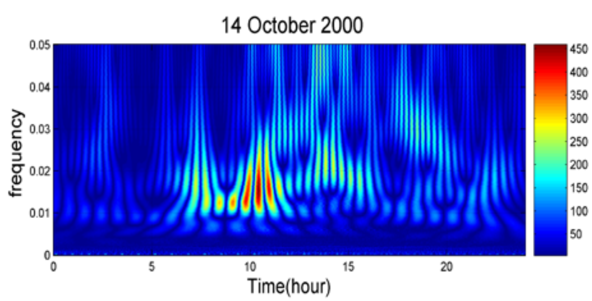

Figure 6 – Wavelet transform of the filtered data for October 14th, 2000. We used a Gauss-Morlet mother wavelet.

Probably those peculiarities are associated to the non stationary character of signal, in periods of rapid variations the signal composition has a few components and there is not enough time to add new of them. In periods of relatively high absolute values with larger duration there is the possibility of new components enter the composition.

Conclusions

We have developed a mixed Fourier-wavelet analysis for geomagnetic signals corresponding to a short period of time (October 14th, 2000) as measured at our Vassouras magnetic observatory. It is clear that the final wavelet transform succeeded in the discrimination of different behaviors corresponding to periods of time as short as a few hours. In this sense it is superior to previous Fourier transforms results for similar signals (where periods of three days are enough to loose some important details of the studied signals). We have found a qualitative correlation between the frequency richness of wavelet transforms and almost instantaneous (from a few minutes to some hours) characteristics of signals that will certainly be explored in the future. We have not attempted to compare in this work, quiet and active periods, nor to establish correlations between disturbed periods and their precursors. The explicit analysis of Dst and Sym-H indexes through wavelets also stayed out of the main scope of the present work. However, it is now clear the discrimination capability of the mixed analysis and both problems should be the aim of coming works.

Acknowledgments

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