Non linear Bayesian inversion of seismic AVAZ and production data with respect to the fracture aperture and density

Ali Shahraini, University of Bergen, Norway
Morten Jakobsen*, University of Bergen, Norway
Aamir Ali, University of Bergen, Norway

This paper was prepared for presentation during the 11th International Congress of the Brazilian Geophysical Society held in Salvador, Brazil, August 24-28, 2009.

Abstract

This paper proposes a method for characterization of naturally fractured reservoirs by integration of seismic and production data. The method is based on a consistent model for the effective hydraulic and elastic properties of fractured porous media and a (nonlinear) Bayesian method of inversion which provides information about uncertainties as well as mean values. We consider fractured reservoir model as an anisotropic media characterized by unknown fracture density and aperture. Then we look at the problem of characterization as an inverse problem and try to recover the unknown fracture parameters by joint inversion of anisotropic seismic AVAZ data and dynamic production data. A synthetic example is provided to clearly explain the workflow. It shows that the seismic data resolve non uniqueness in inversion and the production data helps to recover the true fracture aperture because production data are more sensitive to the fracture aperture rather than the seismic data.

1. Introduction

Significant amount of oil and gas reserves exist in fractured reservoirs. Proper characterization of fractured reservoirs can increase the ultimate production of the fractured reservoir and contribute to the optimum field development strategy including optimum well locations. How ever due to the anisotropy, heterogeneity and attenuation the characterization of fractured reservoir is a complicated task which needs a multidisciplinary approach. Production data which are available in well locations have good time resolution, in the other hand seismic data have good areal resolution. This is one reason that seismic and production data are complementary to each other. Seismic or production data have different sensitivity to different parameters. For example, seismic anisotropy is not very sensitive to fracture aperture while permeability is influenced by aperture (Liu 2005). Jakobsen, Liu and Chapman (2007a) presented a workflow for estimating the anisotropic permeability of fractured reservoirs from seismic AVAZ analysis. But a significant uncertainty is associated to the estimated permeability due to the fact that the effective permeability tensor of a fractured porous medium is generally much more sensitive to the aperture of the fractures than the effective stiffness tensor. In present work we try to fill the gap by integrating the production data to the seismic data in the inversion problem. We use the same workflow (Figure1) as the (Jakobsen and Shahraini, 2008 a, b) (Ali et al., 2009) for the characterization of fractured reservoir. The main parts of workflow are rock physics modeling, flow modeling, seismic modeling and an inversion method which will be explained in the following parts.

2. The forward problems

Forward modeling consists of using consistent rock physic models to obtain effective mechanical and hydraulic properties of fractured reservoir in order to homogenize the heterogeneous grid blocks (Figure 2) and then using proper fluid flow and seismic models to calculate the production and seismic attributes.
To formulate the forward problem we write:

$$\mathbf{d}=\mathbf{G}(\mathbf{m}) \qquad (1)$$

Here, **m** is a vector of model parameters (fracture density and aperture) and **d** is a vector of observable quantities (seismic AVAZ data and/or production data). The (nonlinear) forward model **G** is based on a combination of the rock physics modeling and tools for fluid flow simulation and seismic anisotropy attribute generation as they will describe in the following section.

2.1 Effective elastic model

The effective stiffness tensor $\mathbf{C}^{d*}$ of the fractured porous medium for the dry case is given by (Jakobsen et al., 2003a):

$$\mathbf{C}^{d*}=\mathbf{C}^{(0)}+\mathbf{C}_1:(\mathbf{I}_4+\mathbf{C}_1^{-1}:\mathbf{C}_2)^{-1} \qquad (2)$$

Here, $\mathbf{C}^{(0)}$ is the background stiffness tensor, $\mathbf{I}_4$ is the identity for second-rank tensors; $\mathbf{C}_1$ is a fourth-rank tensor of first order correction for the effect of isolated fractures:

$$\mathbf{C}_1=\sum_{r=1}^{N}\phi^{(r)}\mathbf{t}_d^{(r)} \qquad (3)$$

Where

$$\mathbf{t}^{(r)} = -\mathbf{C}^{(0)}\left[\mathbf{I}_4 + \mathbf{G}^{(n)}\mathbf{C}^{(0)}\right] \quad (4)$$

Here, $\mathbf{G}^{(n)}$ is a fourth-rank tensor given by the strain Green's function integrated over a characteristic spheroid having the same shape as fractures of type r (see Jakobsen et al., 2003a). $\mathbf{C}_2$ is second order correction for the effects of fracture-fracture interaction:

$$\mathbf{C}_2 = \sum_{r=1}^{N}\sum_{s=1}^{N}\phi^{(r)}\mathbf{t}_d^{(r)} : \mathbf{G}_d^{(rs)} : \mathbf{t}_d^{(r)}\phi^{(s)} \quad (5)$$

Here, $\mathbf{G}^{(rs)}$ is a fourth-rank tensor given by the strain Green's function integrated over a characteristic spheroid having the same aspect ratio as two-point correlation function . (see Jakobsen et al., 2003a)

In order to calculate the effect of fluid-saturation on the effective properties of a fractured porous medium, one can use the anisotropic Gassman relation of Brown and Korringa (1975):

$$\mathbf{S}^* = \mathbf{S}^*_d + \frac{\left(\mathbf{S}^*_d - \mathbf{S}_m\right).\left(\mathbf{I}_2 \otimes \mathbf{I}_2\right).\left(\mathbf{S}^*_d - \mathbf{S}_m\right)}{\varphi^0\left(\mathbf{I}_2.\mathbf{S}_m.\mathbf{I}_2 - 1/\mathbf{k}_f\right) - \mathbf{I}_2.\left(\mathbf{S}^*_d - \mathbf{S}_m\right).\mathbf{I}_2} \quad (6)$$

Here '$\otimes$'denotes dyadic product, $\mathbf{S}_m=(\mathbf{C}_m)^{-1}$, $\mathbf{S}^*=(\mathbf{C}^*)^{-1}$. Figure 3 show the effective stiffness obtained using this model as a function of fracture density and aperture. It shows that the effective stiffness is sensitive to the fracture density but for the aspect ratios less than 0.02 which is the case for the conventional fractures the stiffness is not very sensitive to the fracture aperture.

2.2 Effective permeability model

When a fluid flows in a fractured porous medium the scale-size of pressure variation or the size of a typical grid block in a reservoir simulator is often much larger than the scale size of the fractures so the flowing flow can see only the homogenized or upscaled structure (Figure 2). Here we used a T-matrix based rock physic method (Jakobsen 2007) to upscale the permeability of the medium. We use the result of (Jakobsen 2007) equation (7) to (10) and substitute the inclusion permeability of fractured reservoir, $\mathbf{K}^{(r)}$, with the cubic law ,equation (11) .

$$\mathbf{k}^* = \mathbf{k}^{(0)} + \mathbf{k}_1 \cdot \left(\mathbf{I}_2 + \mathbf{k}_1^{-1} \cdot \mathbf{k}_2\right)^{-1} \quad (7)$$

Here, $\mathbf{I}_2$ is the identity for the second rank tensors, $\mathbf{K}^0$ is matrix permeability. $\mathbf{K}_1$ is first order correction:

$$\mathbf{k}_1 = \sum_{r=1}^{N}\phi^{(r)}\tau^{(r)} \quad (8)$$

where

$$\boldsymbol{\tau}^{(r)} = \left(\mathbf{k}^{(r)} - \mathbf{k}^{(0)}\right)\left[\mathbf{I}_2 - \mathbf{g}^{(r)}\left(\mathbf{k}^{(r)} - \mathbf{k}^{(0)}\right)\right]^{-1} \quad (9)$$

Here, K(r) is a second-rank tensor of effective permeability coefficient for fractures of type r, g(r) is a second-rank tensor given by the pressure gradient Green's function integrated over a characteristic spheroid having the same shape as fracture type r. The second-order correction is:

$$\mathbf{k}_2 = \sum_{r=1}^{N}\sum_{s=1}^{N}\phi^{(r)}\tau^{(r)} \cdot \mathbf{G}_d^{(rs)} \cdot \tau^{(s)}\phi^{(s)} \quad (10)$$

Here, $\mathbf{G}^{(rs)}$ is a second-rank tensor given by the pressure gradient Green's function integrated over a characteristic spheroid having the same aspect ratio as two-point correlation function (see Jakobsen et al., 2003a).

$$\mathbf{k}^{(r)} = \frac{(c^{(r)})^2}{12} \quad (11)$$

$c^{(r)}$ is the fracture aperture. Now the effective transport properties of fractured reservoirs can be viewed as functions of fracture density and fracture aperture.

Figure 4 show the effective permeability obtained using this model as a function of fracture density and aperture. It shows that the effective permeability is sensitive to the fracture density as well as the fracture aperture.

2.3 Seismic modeling

Variations in rock elastic properties are detectable in certain special seismic attributes such as azimuthal variation of amplitude with azimuth, shear wave bi-refringence, and azimuthal variation of propagation velocity for the fractured interval (Will et al., 2005). In this work estimated effective stiffness tensor obtained from rock physic modeling has been used to calculate AVAZ data (Figure 5). Ruger's approximation for HTI media have been used to obtain reflection coefficients**:**

$$R_p^{HTI}(i,\phi) = \frac{1}{2}\frac{\Delta Z}{\overline{Z}} +$$
$$\frac{1}{2}\left\{\frac{\Delta\alpha}{\overline{\alpha}} - \left(\frac{2\overline{\beta}}{\overline{\alpha}}\right)^2\frac{\Delta G}{\overline{G}} + \left[\Delta\delta^v + 2\left(\frac{2\overline{\beta}}{\overline{\alpha}}\right)\Delta\gamma\right]\cos^2\phi\right\}\sin^2 i$$
$$+\frac{1}{2}\left\{\frac{\Delta\alpha}{\overline{\alpha}} + \Delta\varepsilon^v\cos^4\phi + \Delta\delta^v\sin^2\phi\cos^2\phi\right\}$$
$$\sin^2 i\tan^2 i \quad (12)$$

Here Z is P wave impedance, G is shear modulus, α is the P wave velocity, β is the s wave velocity, ε , γ and δ are Thomson's anisotropic parameters , i and Φ are incidents and azimuthal angle respectively.

2.4 Fluid flow simulation

Darcy's law and conservation of mass equations are the governing physics for the fluid flow modeling. Reservoir simulator solves fluid flow equation which is combination of Darcy's law and conservation of mass. We use a reservoir simulator as a black box for fluid flow modeling and we use the effective permeability tensors obtained with rock physic modeling as an input to the reservoir simulator.

3. The inverse problem

The inverse problem consists of estimating the model parameters **m** of the fractures from the reflection coefficient data and production data **d**. In the Bayesian setting, both **m** and **d** are (real-valued) random variables
We use Bayes rule to define a posterior probability density function (PDF) for the model parameter vector **m**, given an observation of the data **d**:

$$p(\mathbf{m}|\mathbf{d}) \propto p(\mathbf{d}|\mathbf{m})p_m(\mathbf{m}) \quad (13)$$

PDFs are used here to express knowledge about the true values of the parameters. In other words, prior and posterior PDFs represent degrees of belief about possible values of **m** (the distribution of fractures) before and after observing the data d. Production and seismic AVAZ (or reflection coefficient) data enters the formulation through the likelihood p(**d**|**m**), which determines the conditional probability density for the observed data given values of the model parameters of the fractures. If the measurement errors are assumed Gaussian, it can be shown that the likelihood function can be written as:

$$p(\mathbf{d}|\mathbf{m}) \propto \exp\left(-\frac{1}{2}(\mathbf{G}(\mathbf{m})-\mathbf{d})^T \mathbf{C}_d^{-1}(\mathbf{G}(\mathbf{m})-\mathbf{d})\right) \quad (14)$$

where the covariance matrix $\mathbf{C}_d$ contains information about measurement errors. The prior probability density for the model parameters, p_m(**m**), is based on information which is found independently of the production and seismic AVAZ data. Assuming that the a priori information on the model parameters is Gaussian, we get:

$$p_m(\mathbf{m}) \propto \exp\left(-\frac{1}{2}(\mathbf{m}-\mathbf{m}_{prior})^T \mathbf{C}_m^{-1}(\mathbf{m}-\mathbf{m}_{prior})\right) \quad (15)$$

where $\mathbf{m}_{prior}$ and $\mathbf{C}_m$ are the priori mean and covariance, respectively. The following solution for the posterior PDF is now obtained from the above equations (based on Gaussian statistics):

$$p(\mathbf{d}|\mathbf{m}) \propto \exp(J(\mathbf{m})) \quad (16)$$

where J(**m**) is the objective function, in the case of uninformative prior J(**m**) is:

$$J(\mathbf{m}) = \frac{1}{2}(\mathbf{G}(\mathbf{m})-\mathbf{d})^T \mathbf{C}_d^{-1}(\mathbf{G}(\mathbf{m})-\mathbf{d}) \quad (17)$$

Numerical integration has been used to obtain marginal distribution of individual parameters:

$$p_1(m_1 \,|\, \mathbf{d}) = \int p(m_2,...,m_N \,|\, \mathbf{d})dm_2...dm_N \quad (18)$$

4. Numerical example and discussion

We consider a vertical fractured reservoir model consists of one production and one injection well in two corners of the reservoir. The fluid flow model is a simple black oil model with 15*15*1 grid blocks in x, y and z directions, respectively (Figure 2). We assume true fracture density and aperture of 0.5 and 0.005 cm, respectively and try to recover the true values using calculated AVAZ (Figure 6) and production data (Figure 7).

Figure 8 shows the result of inversion using just production data. The reason of non uniqueness of the results is that there are two combinations of fracture density and aperture that gives same effective permeability (Figure 4).

Figures 9 and 10 show marginal pdfs for the fracture density and aperture with the joint inversion of seismic and production data.

5. Conclusions and recommendations

By using a simple model of a fractured reservoir which is anisotropic (due to a single set of vertical fractures), we have demonstrated that seismic and production data are complementary. Seismic data not only helps to reduce the uncertainty of results but also helps to solve the problem of non uniqueness of results. In the other hand seismic data are not so sensitive to the aperture and integration of dynamic production data are necessary to estimate the fracture aperture.

The next step may be to use ensemble Kaman filter (EnKF) in order to invert the fracture density , aperture and orientation for a heterogeneous reservoir in which the fracture parameters may vary from one grid block to the another grid block.

References

Aamir, A., A.Shahraini, and M.Jakobsen, 2009, Improved characterization of fault zones by quantitative integration of production and seismic AVAZ data, extended abstract, EAGE meeting in Amsterdam, accepted.

Jakobsen, M., J.A. Hudson, and T.A Johansen, 2003a, T-matrix approach to shale acoustics. Geophys. J. Int., 154, 533-558.

Jakobsen, M., E. Liu, M.Chapman, 2007a, Anisotropic permeability in fractured reservoirs from seismic AVOZ analysis SEG meeting, San Antonio.

Jakobsen, M., 2007, Effective hydraulic properties of fractured reservoirs and composite porous media. Journal of Seismic Exploration, 16, 199-224.

Jakobsen, M. and A.Shahraini, 2008a, Improved characterization of fractured reservoirs by joint inversion of seismic AVAZ and production data. 70th EAGE meeting, Rome, Italy.

Jakobsen, M. and A.Shahraini, 2008b, Anisotropic permeability in fractured reservoirs from joint inversion of seismic AVAZ and production data. Proceedings of the 11th European Conference on the Mathematics of Oil recovery, Bergen.

Liu, E., 2005, Effects of fracture aperture and roughness on mechanical and hydraulic properties of rocks: implication of seismic fracture characterization, J. geophys. Eng., 2(1), 38–47.

Ruger, A., 2002, Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media: Geophysical monograph series. Number 10, SEG.

Will, R., R.Archer, and B.Dershowitz, 2005, Integration of seismic anisotropy and reservoir-performance data for characterization of naturally fractured reservoirs using discrete-feature-network models. SPE 84412.

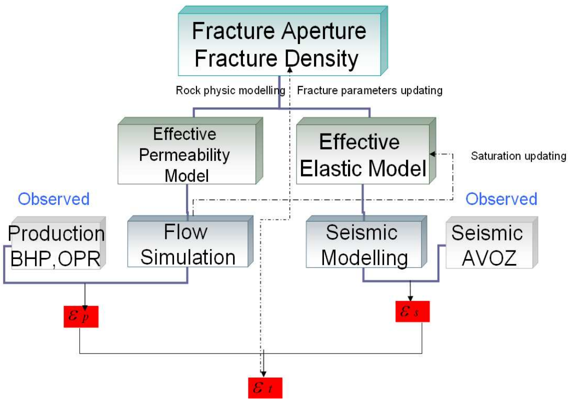


Figure 1: Work flow diagram.

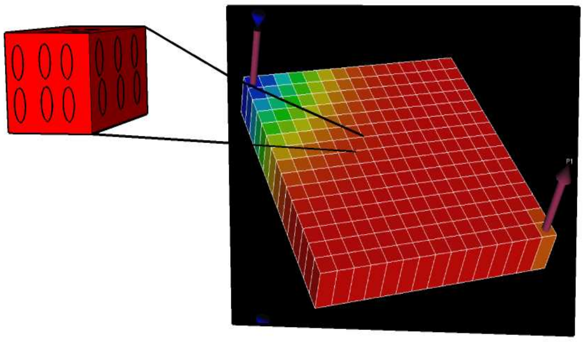


Figure 2: Upscaling- Reservoir model.

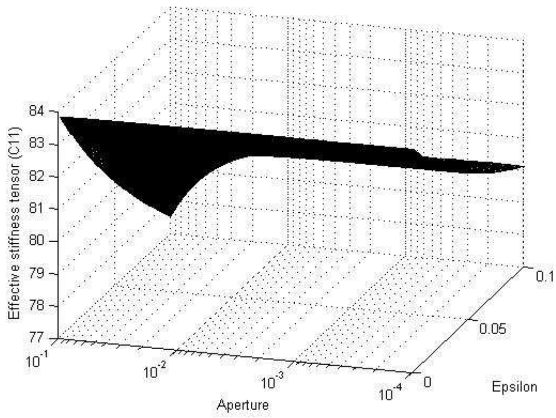


Figure 3: Effective stiffness as a function of fracture density and aperture.

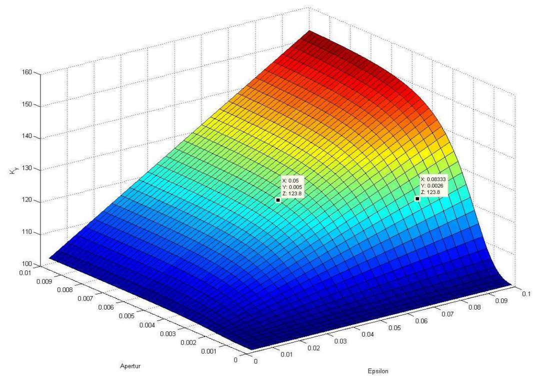
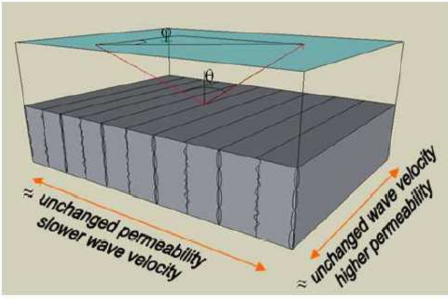


Figure 4: Effective permeability as a function of fracture density and aperture.



$$R_{pp}(\theta, \Phi) = g(\theta, \Phi)$$

Figure 5: AVAZ data, reflection coefficients are Function of polar angle and azimuthal angle.

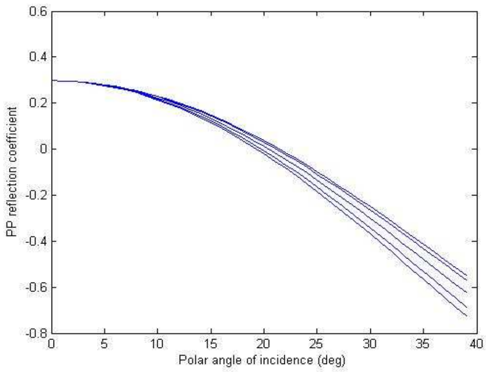


Figure 6: Seismic AVAZ data; PP reflection coefficients vs. polar angle for different azimuth angles.

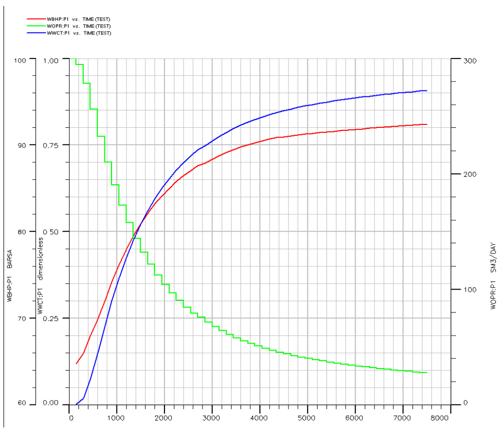


Figure 7: Production data; green curve is WOPR, blue curve is WWC, red curve is: WBHP.

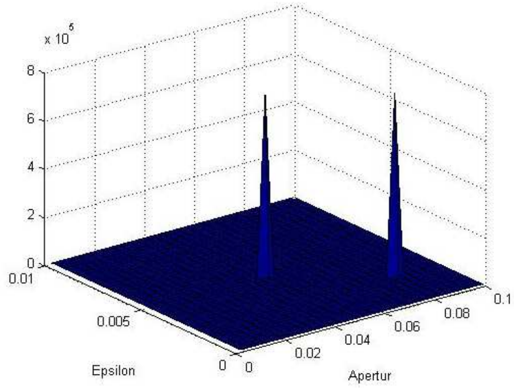


Figure 8: Posterior pdf of fracture density and aperture using production data only.

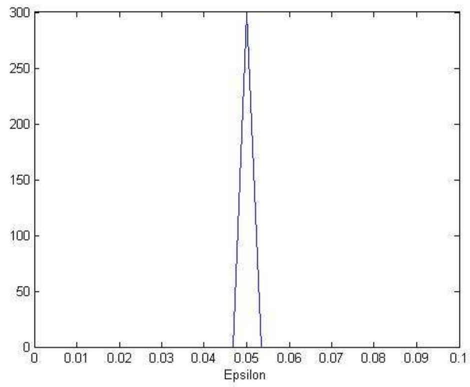


Figure 9: Marginal pdf of fracture density using joint inversion of production and seismic data.

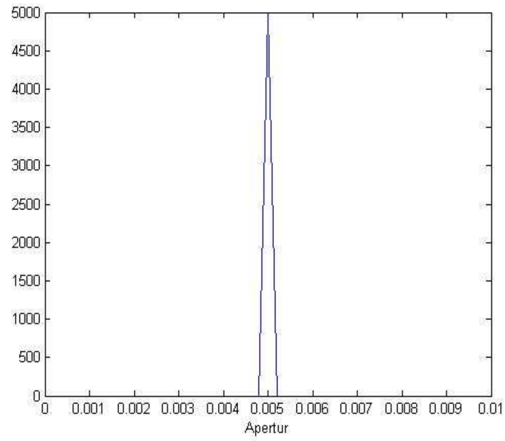


Figure 10: Marginal pdf of fracture aperture using joint inversion of production and seismic data.