2D SOC model for one of the main geomagnetic disturbances' source: flares

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Abstract

We introduce a simple model for solar flares, one of the main sources of geomagnetic disturbances. We have obtained power-laws for the first return time of flare avalanches which could serve as the fingerprint of a critical state at the base of such phenomena and, given that we have not introduced a fine tune mechanism, of self-organized criticality. We analyze the possible connection of these power laws with power laws already found by our group in geomagnetic disturbances distributions. We also present some limitations of our model as well as possible extensions and corrections to be taken into account in futures works.

Introduction

The magnetic field that can be measured at the Earth's surface has several major components: those of internal nature (on which we are not interested on the present work), those of external nature (on which we will focus our attention) and those of atmospheric sources (in a great part driven by external causes also). The most intense source of external character is the activity at the surface of the Sun known as flares.

Flares are the sudden releases of energy supposed to come from the violent reconnection of magnetic loops at the Sun's surface (Bershadskii and Sreenivasan, 2003).

The energy distribution function of flares, as well as the distribution functions for fluency and duration, follows power laws of the form:

$$f(E) = A.E^{-(1+\alpha)} \qquad (1)$$

In Equation (1), $f(E)$ is the distribution function (the probability of finding a flare with energy higher than E), A is a proportionality constant and α is the exponent. For the energy α = 0,45, for fluency (of course, substituting the energy by the fluency and A by a different constant) α = 1,75, and for duration (with the correspondent changes in Equation (1)), α = 0,6 (Podlazov and Osokin, 2002).

However, the exponent of the distribution depends on the nature of flares: with coronal mass ejection (CME) or without coronal mass ejection (Yashiro et al., 2006).

The distribution of first return times (i. e., of time periods elapsed between a flare of a given intensity and another flare of similar intensity) also follows power laws.

Experimental evidence of self-organized criticality (Bak et al., 1987) in solar particle events, a close to flares matter, was very recently published by Xapsos et al. (2006). They used daily and monthly fluencies of solar protons measured by the IMP-8 and GOES satellites during a period of 28 years to demonstrate long-term correlation of events, fractal characteristics and power law distribution functions for fluency magnitudes and waiting times (related to first return times). The main conclusion of the work was that it is not possible to predict the time occurrence and magnitude of solar particle events within narrow limits. However, the Sun surface is a system sufficiently complex as to include other types of mechanism which could let the prevision problem as an open one.

Probably, the first attempts of modeling the dynamical behavior of solar flares were done by Lu and Hamilton (1991) and Lu et al. (1993). They model the active region as a cubic lattice with the magnetic field attached to each cell. When the difference between the field in a cell and the average field of the six nearest neighbors exceeded some predefined threshold, they considered the cell as "unstable" and the difference distributed by the seven cells with some energy release. The model evolved to a self-organized critical state with distribution functions of flares obeying power laws with exponents very close to actual ones.

An alternative approach was proposed by Podlazov and Osokin (2002). They introduced a two dimensional self-organized critical model of eruption process based on a new concept: magnetic elements. Their approach allowed to describe eruptive processes at the solar atmosphere in a clearer fashion and to simulate easily their basic properties. The model yielded a power law distribution function for flare energies in good agreement with experimental observations. In a few words the rules for the model are: i) the attachment of magnetic elements of opposite sign and equal absolute value to two randomly selected cells; ii) then, if the presence of elements of the opposite happens in the selected cells, one of them (also randomly selected) is annihilated with the incoming element. The absolute value of annihilating elements is reduced in 1 (one) and if it becomes zero it is simply eliminated; iii) this annihilation causes an outward disturbance wave simulated by taking out of the cell all the remaining elements releasing them in a randomly selected cell among its eight nearest neighbors cells. These releases could cause new annihilations in the modified cell. This annihilation avalanche is associated to flares. If at some step there are no new annihilations the model returns to step i).

One of the motivations for the present work is the connection between solar events and the magnetic field connection with the Earth (Nitta et al., 2006) as well as power laws already found in distributions functions of geomagnetic fields (Papa et al., 2006; Papa and Sosman, 2008).

Here we profit of all the accumulated experience mentioned before. However, our model follows lines quite different from theirs. The rest of the paper is organized as follows: In the next section (The Model) we present the model. In the Result and Discussion section we present the results of our simulations as well as a discussion on these results and their possible connection with previous works on the statistics of geomagnetic measurements. Finally, we present our conclusions and some possible trends for future works.

The Model

We simulate the Sun surface and the magnetic structures on its surface by an LxL square lattice, where L=100 in all cases. This gives 10^4 nodes on each simulation or that we are looking for this quantity of equally spaced points at the Sun's surface. To each of these positions we have initially given a random value between 0 and 1 to simulate the accumulated magnetic energy at each of the simulated positions. We have looked then for the highest value through the whole system and changed it and its four nearest neighbors by new random values. With this we simulate a more or less continuous energy flux to the Sun surface (these is the reason to pick the highest value) and the reconnection between neighbor cells (the change of the cell and its neighbors). At the same time, the assignment of new random values, to the highest and its neighbors, works as a continuous release of energy out of the system (which is also necessary to the establishment of SOC).

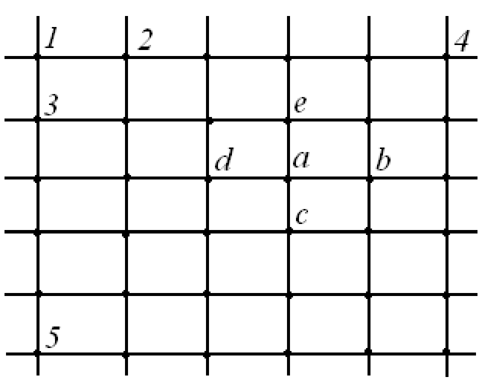

Figure 1 – Schematic representation of a hypothetical 6x6 square lattice with periodic boundary conditions (only first neighbors). The neighbors of node 1 are nodes 2, 3, 4 and 5 (note the peculiarity for nodes 4 and 5). For the node a the neighbors are b, c, d and e.

Constructed in the way we have done, our model qualifies as a Bak-Sneppen one (Bak and Sneppen, 1993). The Back-Sneppen model probably is the simplest model presenting self-organized criticality, i.e., the tendency to a stationary state without necessity of a fine tuning (similar to the necessary tuning of temperature in usual phase transitions).

The Bak-Sneppen model is a general model that has found applications in a large number of fields among which we can mention evolution (Bak and Sneppen, 1991), the brain (da Silva et al., 1998), the cosmic ray spectrum (Stenkin, 2005) and X-rays bursts at the Sun surface (Bershadskii and Sreenivasan, 2003).

Many scientific efforts have been devoted to characterize the Bak-Sneppen model from several points of view. Examples of them are: its correlations from detrended fluctuation analysis (Ma et al., 2005), damage spreading on it (Bakar and Tirnakli, 2008), and its behavior under reduction to near zero dimension (Dorogovtsev et al., 2000).

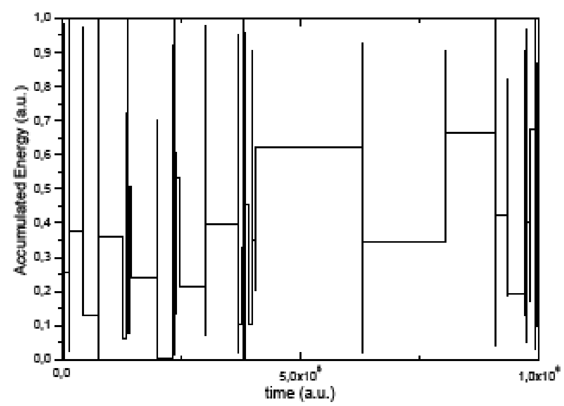

Figure 2 – Consecutive values of a single accumulated energy in 10^6 steps. There are periods of extreme activity alternated with periods of absolute calm.

This process is repeated several times (usually between 10^6 and 10^8 times) to obtain a stationary distribution of the quantities we are interested in. We have used periodic conditions (see Figure 1) in our square system (which means that we are simulating an essentially spherical surface by a thoroidal one).

Note that within the simplified model here introduced it should not be expected a detailed description of the system, but just some specific details and, in particular, the class of universal behavior displayed by the real system that the model represents.

Results and Discussion

Beginning with an arbitrary distribution of accumulated magnetic energy at each node, the subsequent activity will be completely uncorrelated in space and time. But as times goes by (and then, the mean accumulated energy decreases as a consequence of selecting and changing the highest) it will be more and more likely that near neighbors are consecutively changed. After a transient the system reaches a steady state characterized by a step-like distribution for the accumulated energies and a threshold E_c for the distribution of the higher barriers. The distribution of higher energies vanishes (not shown) at and below the self-organized threshold.

Figure 2 shows the values of an arbitrarily chosen node. All nodes present similar behaviors. If a period of great activity is expanded the picture seems to be of the same fashion just in another scale time (fractal character).

In Figure 3 we show the first return time distribution function obtained from our model. It follows a power law with exponent around -0,97. The important fact is the existence of a power law. At this moment it has not much sense to compare with real flares because depending on the established energy threshold for them the exponent changes in a large range of values.

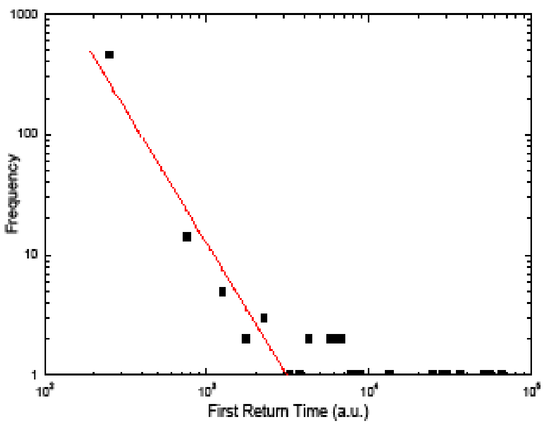

Figure 4 – First return time distribution function. The slope of the straight line is -0,97 and its error 0,23.

We have obtained so far a single power law regime for each simulation. However, there are previous results indicating the existence of different power law regimes depending on the concomitant existence of coronal mass ejection or not (Yashiro et al., 2006). There are also previous results indicating the existence of double regime power laws in the frequency distribution of time periods between events of the same intensity for geomagnetic disturbances (Papa et al., 2006). The notable fact is that the double regime distribution is a continuous curve indicating a single phenomenon at the base of both regimes. Probably there exists a limit for the energy that can be released without CME. This limit should be very close to the minimum energy of flares with relevant CME (coronal mass ejection that allow their detection by us). The introduction of a dead time for flares (i.e., a time during which it is not possible the release of new flares because of the damage caused at the corona by CME) would probably lead to power laws with different exponents. However, the inclusion of both types of phenomena in a single simulation necessarily requires more elaborate models.

Conclusions

We have introduced a two dimensional self-organized critical model to simulated solar flares, i.e., one of the main causes of magnetic disturbances at the Earth's surface. We have obtained power law distributions for several relevant quantities, similar to the results of experimental works on flares. This could imply that the Sun surface could be in a critical state where the greater possible "avalanche" is limited just by the size of the system (i.e. the size of the Sun). We have not attempted to introduce a memory in the model or in other words, to simulate the time elapsed between the occurrence of a flare at a given position and the next flare at the same position probably associated to the local damage caused by coronal mass ejections. This should be one of the elements to be introduced in future works.

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