Mantle electrical conductivity estimates from geomagnetic jerk observations

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Abstract

The electrical conductivity of the Earth's mantle has been a subject of much debate in the last few years. Induction studies agree mainly in the first 1000 km of the mantle, however in the lower mantle the conductivity is still very uncertain. Experimental studies of mineral physics simulating the conditions of the deep mantle have been performed and results disagree by 3 orders of magnitude depending, for example, on the considered geotherm and composition of the lower mantle. As a complement to mineralogical and induction studies, time variations of the magnetic field in the core can also contribute towards a better understanding of mantle conductivity. Geomagnetic jerks involve abrupt temporal changes in the secular variation of Earth's magnetic field and are believed to be due to motions in the fluid core. In this thesis we use geomagnetic jerk observations to constrain information about the mantle electrical conductivity. There are two possible hypotheses to explain the different occurrence jerk times at the Earth's surface: the first is to consider these differential time delays generated by dynamical processes in the core which do not occur simultaneously; the second is to consider jerks generated instantaneously in the core and the time delays caused by a conducting mantle. In this paper we analyze the second hypothesis so that the geomagnetic field observed at the surface will correspond to a filtered version of the original field generated in the core. The aim of this paper is to obtain information about mantle electrical conductivity by using observations of differential delay times of the 1969 global geomagnetic jerk (Y component). We consider Kuvshinov and Olsen's (2006) model of conductivity of the upper mantle and models of one to three layers below 700 km.

Introduction

The Earth's mantle electrical conductivity reflects chemical and physical properties of the planet's interior, places constraints on core-mantle coupling and controls the transmission of geomagnetic signals from the core to the surface. Despite many mineral physics studies, induction studies and considerations of the transmission of the core magnetic field to the surface, the lower mantle electrical conductivity is still mostly unknown and remains an open question. The aim of this thesis is to obtain constraints on mantle electrical conductivity by using subtle signals that are often globally seen in components of the Earths magnetic field. These signals take the form of rapid variations in the rate-of-change of the field with time, and are termed geomagnetic jerks. Since jerks are time variations of the magnetic field generated in the core, they will pass through the electrical conducting mantle, before arriving at the surface. Consequently, the geomagnetic field observed at the surface will correspond to a filtered version, delayed and smoothed, of the original field generated in the core. This delayed and smoothed version of the input contains information on the filter though which it has just passed, and the challenge is to decode this information in terms of Earth properties.

Method

In view of the fact that the inverse problem is non-linear, we solve it by exhaustive search, where the forward problem is solved repeatedly by changing the electrical conductivity of mantle layers and by calculating the jerk differential delays at the surface. The forward problem can be summarized as: the output (jerk at the surface) is evaluated by the convolution between the input that simulates an impulsive jerk generated simultaneously (at t = 0) at the CMB and the Composite Impulse Response Function (CIRF, see Pinheiro & Jackson, 2008) for each jerk.

We followed seven main steps to solve the inverse problem:

1. Calculation of the Impulse Response Function (IRF) is performed by a modified version of Velímsky and Martinec's [2005] code. The method is based on direct time-integration of the EM induction equation, and uses spatial discretization by spherical harmonics and piecewise-linear finite elements in the lateral and radial direction, respectively;
2. Evaluation of the Composite Impulse Response Function (CIRF) by using the spherical harmonic models of jerk amplitude;
3. Convolution of those CIRFs with the input jerk simulated as a second order impulse in the poloidal field, simultaneous at the CMB and with unit amplitude;
4. Annual mean evaluation of the output jer: since we aim to treat the model in the same way as the data;
5. Fitting of two straight-line segments to the output annual means by the least-squares method: the intersect is defined as the delay time (τ_1) and the error bars in τ_1 are evaluated;

6. Calculation of jerk differential delay times for all locations corresponding to the analysed observatories. The differential delay times ($\Delta\tau_1$) are calculated in relation to the mean delay value (τ_M) negative values are considered as early jerks and positive as late jerks;

7. Comparison of the differential delays of data (Figure 1) with model predictions, by calculating the misfit value that is the measure of how well the differential delays of the model fit the data differential delays:

$$E = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(\Delta\tau_{obs_i} - \Delta\tau_{mod_i})^2}{\beta_{obs_i}^2}} \qquad (1)$$

where N is the number of observatories, $\Delta\tau_{obs}$ are the observed differential delays, $\Delta\tau_{mod}$ the model differential delays and β_{obs} the data error bars in $\Delta\tau_{obs}$.

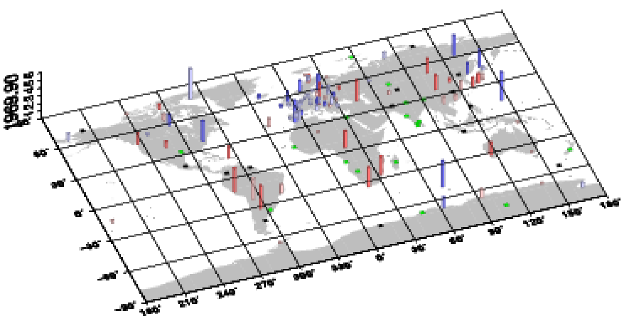

Figure 1: Results for the 1969 jerk occurrence time at the Earth's surface obtained by the L_2 method for Y component of the magnetic field by considering first differences of observatory annual means. The mean occurrence time is shown close to the vertical bar (from 1 to 6 years) which gives the height of the blue and red bars. The red bars represent locations where the jerk appeared later than the mean occurrence time and the blue bars where it appeared earlier. The occurrence time when the bar is red, is given by the sum of the mean occurrence time and the height of the bar in a specific location; while the occurrence time of blue bars in a given location is given by subtracting the mean occurrence time by the height of the bar. Dark red (blue) bars represent locations where the limits of the error bars are later (earlier) than the mean occurrence time and light red (blue) bars where the limits of the error bar are earlier (later). The green squares represent the locations where the jerk was not detected and the black squares where data was excluded.

The Earth's mantle was modelled considering Kuvshinov and Olsen's (2006) 1-D profile for the first 700 km and below that depth four model setups were built:

1. One-layer model with 2200 km thickness;
2. Two-layer model with a bottom layer 1250 km thick and top layer with 950 km;
3. Two-layer model with a bottom layer simulating the D'' (300 km thick) and a top layer (1900 km) simulating the lower mantle;
4. Three-layer model with the lower mantle divided into two layers 950 km thick and D'' with thickness of 300 km.

Results and Discussion

We calculated the misfit values for the one, two and three-layer models, where for all of them we adopt the 1-D electrical conductivity model of Kuvshinov & Olsen (2006) for the first 700 km. For each of the models, some examples of IRFs, differential delays for the locations corresponding to observatories and the misfit, are shown. The differential delay times are calculated in relation to the mean delay, as it was performed in the data analysis. In order to have a basis for comparison between different values of misfit, we define the reference misfit as:

$$E_{ref} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(\Delta\tau_{obs_i})^2}{\beta_{obs_i}^2}} \qquad (2)$$

that is the misfit for an insulating mantle model, where the differential delays are equal to zero ($\Delta\tau_{mod}$=0). For the observatories used in the data analysis of the 1969 jerk, the reference misfit is E_{ref} =1.542 considering our spherical harmonic model (Figure 2A) and E_{ref} =1.456 using the model of Le Huy *et al.* (1998), shown in Figure 2B. This difference is because we exclude observatory locations with amplitudes in the range $\pm$ 1 nT/yr^2. Since the amplitude isolines are different in our and Le Huy et al.'s (1998) model, the excluded observatories will not be the same: 18 locations in Le Huy et al.'s model and 8 locations excluded in our model. In order to quantify how better the minimum misfit model is compared to the reference (insulating mantle), we calculate:

$$\mathcal{E}_{\%} = \left[1 - \left(\frac{E_{min}}{E_{ref}}\right)^2\right] \times 100 \qquad (3)$$

which we call *variance reduction* in an analogous way to that used in seismology.

One-layer model

We calculated the IRFs for a one-layer model varying the electrical conductivity from 1000 S/m to 0.4 S/m. But in order to calculate the differential delays, we need to evaluate first the Composite Impulse Response Functions (CIRFs) for each electrical conductivity model. Each of the CIRFs were convolved with an arbitrary jerk simulated at time t=0 and the result of this convolution is a delayed and smoothed output jerk that we would observe at the Earth's surface. For each electrical conductivity and each jerk morphology model the output jerk will be different.

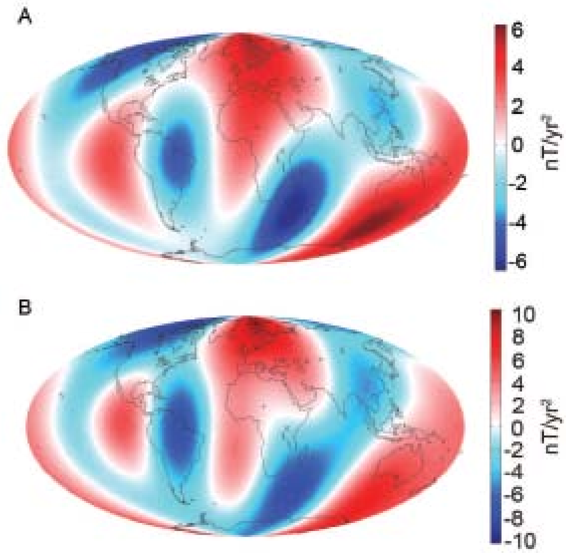

Figure 2: Spherical harmonic models of the 1969 jerk amplitudes calculated in this work (A) with truncation degree L = 5 and by Le Huy et al. (1998) (B) with truncation degree L = 4.

Since we want to treat the model in the same way as the data, annual means were calculated for the output jerk. We selected the same number of data points (7 data points before and after the peak, N=15) for the fitting of two straight line segments, by least-squares. The resulting fitting is shown in Figure 3, for Hermanus and Gnangara observatories and assuming an uniform conducting mantle of 10 S/m and 100 S/m.

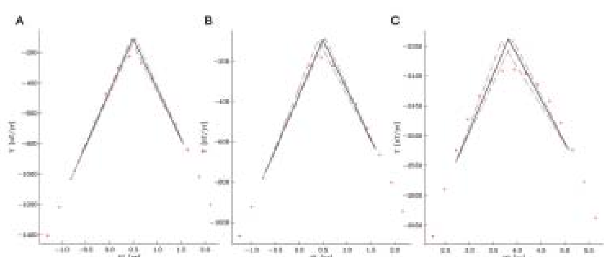

Figure 3: Examples of annual means and the fitting of two straight line segments to the output data (red symbols) at Gnangara observatory (GNA) in Australia (A) and Hermanus observatory (HER) in South Africa (B and C). We used in this example the electrical conducting model of Kuvshinov & Olsen (2006) for the first 700 km depth and below that a layer of 10 S/m (A and B) and 100 S/m (C).

We calculated the misfit values for each of the electrical conductivity models and for the two jerk spherical harmonic models of the 1969 jerk amplitude. In both models, the misfit values increase rapidly with conductivities larger than 20 S/m (see Figure 4) and below that it oscillates in a range smaller than 0.1 of the misfit value (about 1.45 for Le Huy *et al.*'s (1998) model and about 1.55 by using our model). There are only two acceptable conductivity models that are in common when using the two spherical harmonic models: σ = 3 S/m and σ = 5 S/m. By using the model from Le Huy *et al*. (1998) two more models are also smaller than the reference: σ =1 S/m and σ =10 S/m. Consequently, the range of acceptable models in this analysis vary from 10 S/m $\geq \sigma \geq$ 1 S/m.

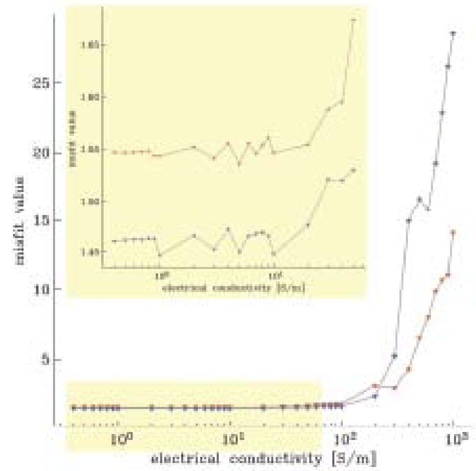

Figure 4: Misfit between the differential delays of the data and model for the one-layer mantle electrical conductivity model. We consider our spherical harmonic model of the 1969 jerk (red symbols) and that taken from Le Huy *et al.* (1998) (blue symbols). The plot in the top left is a zoom of the yellow area in the bigger plot.

Another way to analyse the results is by looking at the differential delays we would observe at the surface for each conducting model. We show the data results (Figure 5A) and examples of differential delays for the minimum misfit models of Le Huy *et al*. (1998) in Figure 5A and our spherical harmonic model in Figure 5B and 5C. Both models are reasonably similar to the patterns early/late seen in the data (Figure 1), however the amplitude of the model differential delays (about $\pm$ 1 month) are more than one order of magnitude smaller than in the data (about $\pm$1.5 year). The delays observed by using our amplitude models are in general larger than those using Le Huy *et al*'s (1998) model. We also looked at the patterns of a highly conducting mantle with 200 S/m (Figure 5C) which does not fit well the patterns early/late observed in the data, but the amplitudes of the differential delays are in better accordance with the data.

Two-layer models

In the first two-layer model we divided the lower mantle into two thick layers of 1250 km (bottom) and 950 km (top) in the low conducting range of 0.457 S/m to 12.346 S/m. The misfit values are shown in Figure 6, considering the two models of jerk amplitudes. Both results show low misfit values in the range of 3 S/m for the top and bottom of the lower mantle.

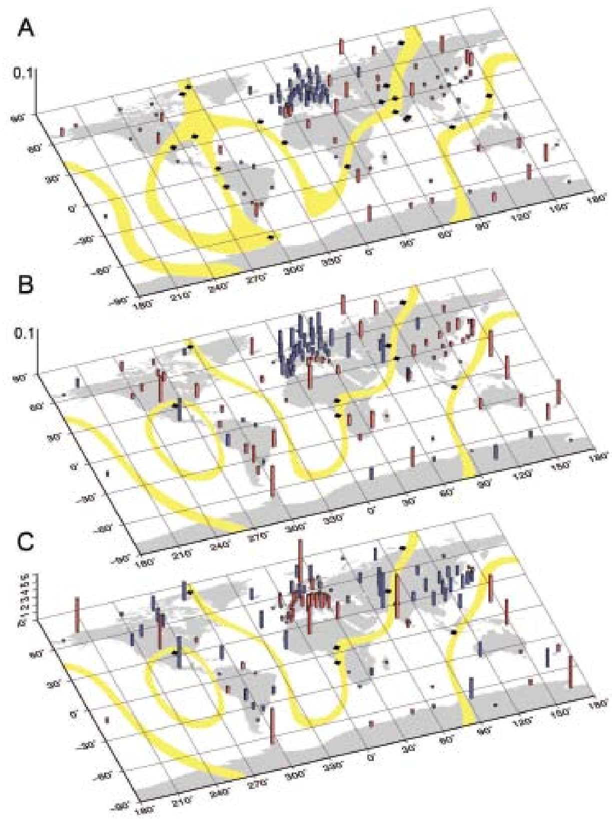

Figure 5: Examples of differential delays for one-layer electrical conductivity model using the spherical harmonic model of Le Huy *et al.* (1998) (A) and the one calculated in this study for mantle electrical conductivities of 1 S/m (B) and 200 S/m (C).

They also present a minima in the left top corner, which correspond to low values of conductivity in the top lower mantle (up to 1 S/m) and high values in the bottom mantle (about 12 S/m that is the maximum conductivity in this simulation). We increase the electrical conductivity values in the second simulation of a two-layer model, but considering a thin mantle layer in the bottom (300 km) that simulates the D''. We want to answer the question whether it is possible to have sensitivity to variations of electrical conductivity in such a thin layer in the bottom of the mantle.

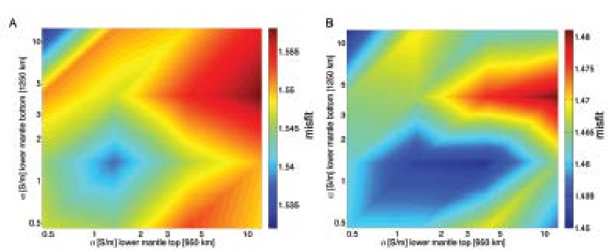

Figure 6: Misfit between the differential delays of the data and model considering our 1969 jerk spherical harmonic model (A) and from Le Huy *et al.* (1998) in B, for the two-layer model with the mantle divided into two layers of 1250 km and 950 km.

Sensitivity to D''

In the second two-layer model the mantle is divided into a top layer with 1900 km thickness and a bottom layer 300 km thick simulating the D''. The first issue to be analysed is whether there is a large difference in the IRF if one varies the electrical conductivity of the simulated D''.

Since this is a non-linear problem, the relationship between the electrical conductivity distribution and delay times does not vary linearly, in contrast to Backus theory. The differential delay times are also going to depend on the spherical harmonic model for the jerk and the mixing of harmonics is able to generate larger differential delays.

We calculated differential delays for models ranging from 111.111 S/m to 0.457 S/m using our spherical harmonic model for the 1969 jerk amplitudes. This result shows that the misfits increase rapidly for electrical conductivities of the lower mantle larger than 10 S/m and that there are two minima in a conductivity of the lower mantle corresponding to 1 S/m and for the D'' about 1 S/m and from about 37 S/m to 100 S/m (in the limits of this simulation range). The most acceptable models are in the range of 1.372 S/m in the lower mantle but there is a high misfit value at a lower mantle with 1.372 S/m and a D'' with 12.346 S/m.

In the second calculation of the two-layer model, we increased the range of the D'' to 1000 S/m and calculated finer variations of electrical conductivity (from steps of three to two), as shown in Figure 7. This result also suggests a low electrical conductivity for the lower mantle of about 1 S/m and a broad range of acceptable conductivities for the D''. Considering this misfit calculation, it seems the data is sensitive to variation in the thick lower mantle, but not sensitive to variations of the D'' electrical conductivity.

In order to analyse the stability of this inverse problem in terms of jerk amplitudes, we calculated misfit values considering the two spherical harmonic models. Considering only some examples it is not possible to prove if the inverse problem is stable or not, but instead we want to address how slightly different results of morphology, for the same jerk event, can influence the results of differential delay times and consequently misfit values for the two-layer model. By comparing those results (Figure 7A and 7B) we conclude that the patterns of low/high misfit are similar by using any of the models. By using our amplitude model for the same jerk, 24 out of 72 misfit values are smaller or equal to the reference while in Le Huy *et al*.'s model only 18 models are acceptable. Both results favoured an electrical conductivity of the lower mantle of 0.977 S/m and a broad range of variation for the D''. However, considering 0.977 S/m as the minimum misfit value for the lower mantle, both models have in common small misfit values for a D'' with 1000 S/m or 125 S/m.

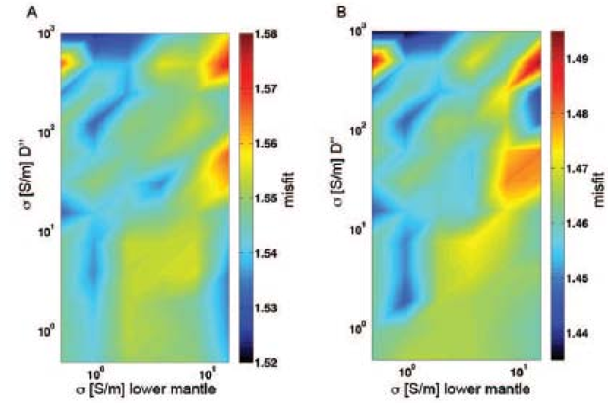

Figure 7: Misfit for the two-layer model considering the 1969 jerk spherical harmonic model calculated in this work (A) and taken from Le Huy *et al.* (1998) in B.

Three-layer models

We also developed models with three layers in order to test whether variations of conductivity in the lower mantle would cause a significant effect in the results of differential jerk delay times, and consequently in the misfit values. We simulated a lower mantle divided into two layers of same thickness (950 km) and D'' with 300 km of thickness. We calculated also this three-layer model for our and Le Huy *et al*.'s (1998) models for the 1969 jerk amplitudes. Each of the plots represent a fixed value of the electrical conductivity in D'' and the misfit is shown as a function of the conductivities of the two lower mantle layers. The patterns of misfit models are similar in both cases and there appears a common minimum misfit for high electrical conductivities of the D'' (Figures 8 and 9).

The minimum misfits, considering our amplitude model, were found to be with a D'' with 111.111 S/m, bottom lower mantle with 0.457 S/m and top lower mantle with 1.372 S/m and another one with the D'' and bottom lower mantle with 12.346 S/m and top lower mantle with 0.457 S/m. By using Le Huy *et al.*'s (1998) model, the minimum misfit values are considering a D'' with 333.333 S/m, bottom lower mantle with 1.372 S/m and top lower mantle with 0.457 S/m and the second with a D'' with 0.457 S/m, bottom lower mantle with 12.346 S/m and top lower mantle with 0.457 S/m. Both minimum misfit results favour a highly conducting D'', however there are also similar misfit values considering some low conducting D'' which shows that it is difficult to constrain the D'' conductivity. If we analyse the intersection of small misfit values by using ours and Le Huy *et al*'s models, we find a consistent range of 12.346 S/m for the D'', 0.457 S/m for the bottom mantle and a variation of 0.457 S/m to 4.115 S/m in the top lower mantle.

Conclusion

The inverse problem was solved, by exhaustive search, considering the same methodology developed in the forward problem: evaluation of IRFs, CIRFs, convolution of the input jerk with the CIRF, generation of annual means of the output signal and fitting of two-straight line segments to this synthetic data.

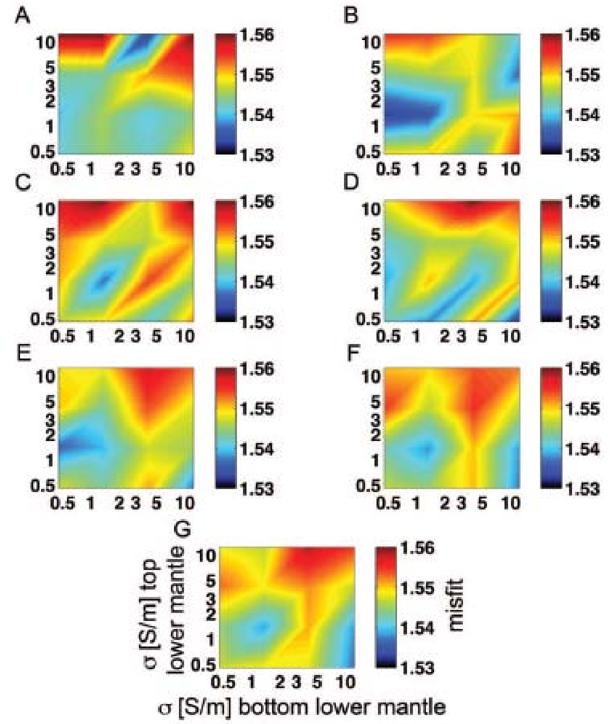

Figure 8: Misfit between the differential delays of the data and model for different mantle electrical conductivity models, considering the spherical harmonic model of the 1969 jerk calculated in this work. This misfit calculation is relative to the three-layer model with a lower mantle divided into two layers of 950 km thick and a D'' of thickness 300 km. The electrical conductivity of D'' is fixed for each misfit plot: in A for 333.33 S/m, in B for 111.11 S/m, in C for 37.04 S/m, in D for 12.35 S/m, in E for 4.11 S/m, in F for 1.37 S/m and in G for 0.46 S/m.

Following this methodology, we calculated the differential delay times in the data and models, from which we evaluated the misfits. The misfit values for all simulations, favour a low conductivity of the lower mantle of about 1 S/m allowing a broad range of conductivities for the D''. We believe that there is not much sensitivity of our data to detect changes in the D'' electrical conductivity.

The patterns early/late jerks in the minimum misfit models were found to be similar to the data analysis, considering the 1969 jerk. However the amplitude of such differential delays is on the order of 0.1 year, while in the data it is about 1.5 year. The minimum misfit models favour the two-layer models with a lower mantle 1900 km thick and a D'' 300 km thick, which gives a variance reduction of about 3.3 \% when using Le Huy *et al.*'s (1998) model and of 2.3 % when using our spherical harmonic model.

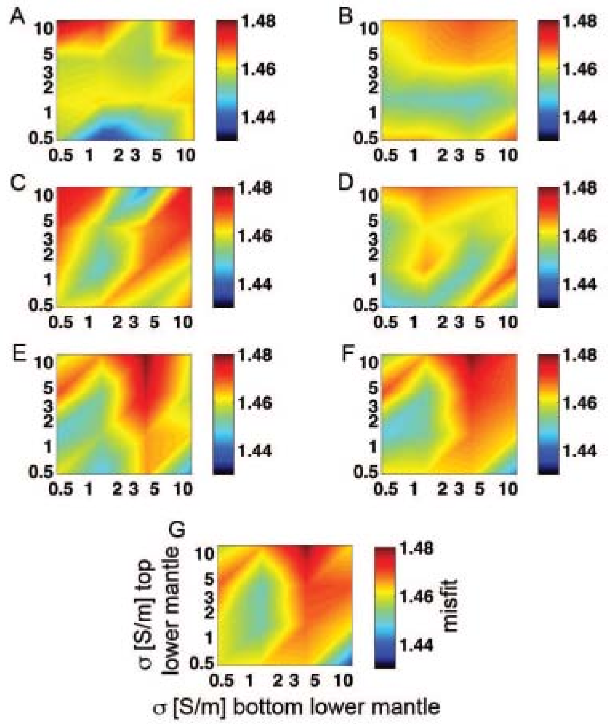

Figure 9: Misfit between the differential delays of the data and model for different mantle electrical conductivity models, considering Le Huy et al.'s (1998) spherical harmonic model of the 1969 jerk. This misfit calculation is relative to the three-layer model with a lower mantle divided into two layers of 950 km thick and a D'' of thickness 300 km. The electrical conductivity of D'' is fixed for each misfit plot: in A for 333.33 S/m, in B for 111.11 S/m, in C for 37.04 S/m, in D for 12.35 S/m, in E for 4.11 S/m, in F for 1.37 S/m and in G for 0.46 S/m.

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