Model rays for depth-to-time conversion

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Abstract

Seismic traces of time-migrated sections have its amplitudes distributed along image rays when properly converted to depth. Image rays and its corresponding orthogonal lines, constitute a coordinate system, in such a way that time-to-depth conversion problems can be addressed as coordinate transformation problems. In depth-to-time conversion, vertical lines in depth are mapped into curves in time. Because of the analogy with image rays in depth, we suggest the term model rays to refer to these curves in time. We present two differential equations which permit to trace model rays and its corresponding orthogonal lines. We implement two approaches to trace model rays, one based on paraxial ray-tracing theory and the other on wavefront construction by Huygens principle. Finally, we demonstrate how to use model rays to correlate seismic data with well information.

Introduction

Hubral (1977) introduces the concept of image rays to connect time-migrated sections with depth images. Image rays are largely used in the industry to convert seismic maps to depth.

Cunha and Filpo (2001) apply the concept of image rays to convert amplitudes of time-migrated sections using a coordinate system transformation approach.

The problem of tracing image rays using velocity model in time is addressed in Cameron, Fomel, and Sethian (2008), and also in Iversen and Tygel (2008). Portugal, Filpo and Vicentini (2009) apply the concept of model rays for time-to-depth conversion of velocity models in time.

Synthetic seismograms are widely used to match seismic data and well log information. Lateral velocity variation around the bore-hole produces misfit of synthetic seismograms and trouble in wavelet estimation. Traditionally, the misfit problem is overcome by stretching and squeezing the synthetic trace, while wavelets are better estimate by using correlating window instead single traces. The correct positioning of synthetic traces along model rays significantly reduces the need of stretching and squeezing, and it permits the use of smaller correlating windows for wavelet estimation.

Time-to-depth conversion – Image-ray coordinate system

Consider a 2-D time-migrated section represented by $\mathrm{A}(\xi,\tau)$, where ξ represents the midpoint coordinate and τ is the zero-offset hyperbola time, being the hyperbola associated with a diffractor point in depth. Connection between points of time-migrated section and its corresponding diffractor points is given by the image-ray starting at the corresponding midpoint location. Each seismic trace of the time-migrated section is associated with a specific image ray, while each isochron line is related to the wavefront perpendicular to all image rays in a specific traveltime. Image rays e wavefronts constitute an orthogonal coordinate system, which can be used to convert a time-migrated section $\mathrm{A}(\xi,\tau)$ into a depth section $\mathrm{A}(x,z)$. Practical aspects of time-to-depth conversion by image-ray coordinate system transformation are presented in Cunha and Filpo (2001).

In summary, time-to-depth conversion can be viewed as a coordinate system transformation, where amplitudes distributed in a rectangular grid in a Cartesian coordinate system in time are mapped into another rectangular grid in a curvilinear coordinate system in depth.

Depth-to-time conversion – Model-ray coordinate system

In the absence of caustics, depth-to-time conversion is the inverse of the coordinate transformation described in the previous section, in such a way that a rectangular grid in depth is mapped into a curvilinear one in time.

Consider a 2-D depth-migrated section. Each vertical line of this section, i.e. a depth seismic trace, is mapped into a curve in the time-migrated domain. This curve plays in depth-to-time conversion an analogous role of the image ray in time-to-depth conversion. Due to this analogy, we suggest the term model ray to refer to this curve.

Figure 1 illustrates the model ray concept. Each point of the vertical line $x = x_\lambda$ in depth is mapped into the curve λ in time, which represents the model ray associated with the depth seismic trace located at x_λ . Diffraction curves generated by diffractor points with horizontal coordinate x_λ have its apex distributed along the model ray λ . The connection between each apex and its corresponding diffractor point is done by a specific image ray. For example, the point M in depth generates a diffraction curve whose apex is located at the point N with midpoint coordinate ξ_Ω .The image ray Ω starting at ξ_Ω reaches the point M at the time τ_Ω .

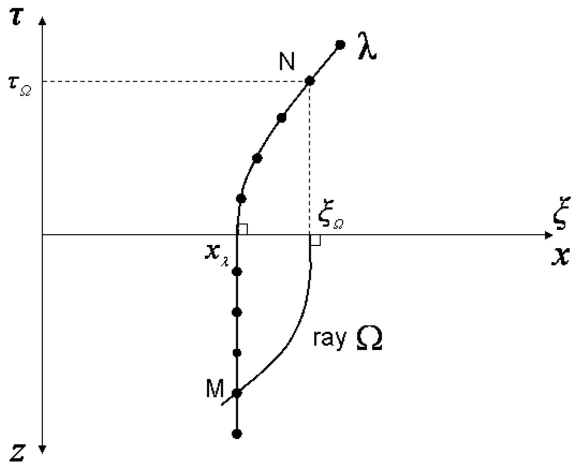

Figure 1: Model ray concept: Points of a vertical line in depth are mapped into the curve λ in time, which is the model ray associated with that vertical line.

In summary, depth-to-time conversion can be implemented as a coordinate system transformation, where rectangular grids in depth become curvilinear in time, being vertical lines mapped into model rays, while horizontals are mapped into "same depth" curves which play the role of "model wavefronts" in time.

Dual equations

Some of the properties described in the previous section evidence the existent dualism between time-to-depth and depth-to-time conversion. Besides these properties, the dual behavior is observed in two differential equations:

$$v^2\left(\frac{\partial z}{\partial \xi}\right)^2 + \left(\frac{\partial z}{\partial \tau}\right)^2 = v^2 \qquad (1)$$

and,

$$v^2\left(\frac{\partial \tau}{\partial z}\right)^2 + \left(\frac{\partial \xi}{\partial z}\right)^2 = 1. \qquad (2)$$

Equation (1) is related to the eikonal equation and it governs the propagation of "model wavefronts" in time, it is expected that model-rays are its characteristics curves.

Equation (2) is associated with the Huygens principle and act as the equation (3) of Sava and Fomel (2001), permitting the construction of "model wavefronts" in time.

Model rays from paraxial ray tracing

As model rays are connected with image rays, it is possible to trace them by finding, for each point, the corresponding image ray.

Figure 2 shows how the paraxial ray theory is applied to solve the two-point ray tracing problem that arises in the determination of image-rays initial conditions. For each point M of a vertical line in depth, the problem consists of determining the takeoff point O, where the image ray that reaches M starts. This two-point ray-tracing problem is solved using an iterative approach suggested by Cervený, Klimes and Psencik (1988), where the takeoff point is updated by the equation

$$\vec{x}(O,O_0) = \mathbf{Q}(O_M, O_0)^{-1}\,\vec{x}(M,O_M). \qquad (3)$$

The matrix $\mathbf{Q}(O_M, O_0)$ is obtained by dynamic ray tracing with plane-wave initial condition. It is related to the ray that starts vertically at O_0 and ends at O_M located at the same depth level of M.

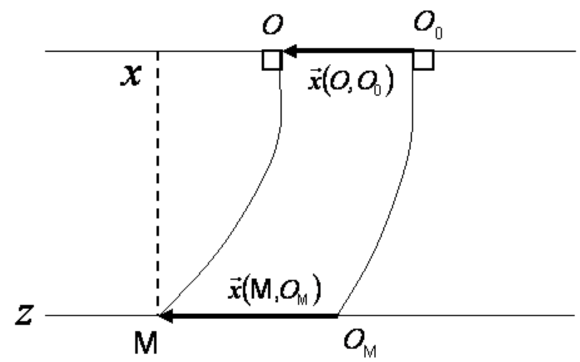

Figure 2: Two-point ray tracing procedure for determining image-ray takeoff points.

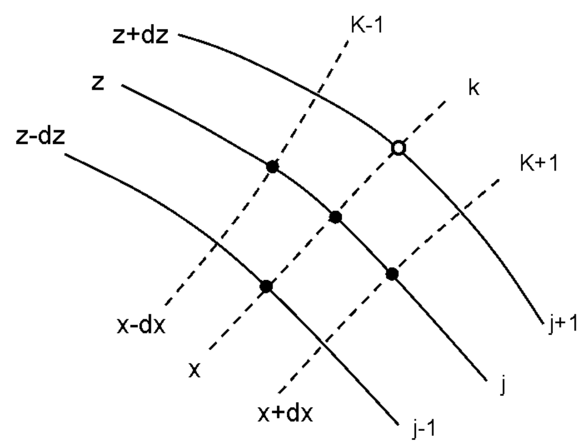

Figure 3: "Model-wavefront" construction using Huygens principle. The white point, located at the position with index (j+1,k), is given by a finite-difference operator involving two preceding wavefronts (j and j-1) e two neighbors rays (k-1 and k+1).

Model rays by Huygens Principle

Based on the method of Huygens wavefront tracing presented by Sava and Fomel (2001), we develop an algorithm to construct "model wavefronts" and model rays in time. In our approach, equation 2 is associated with a circle, and it produces an extra equation when derived in relation to the horizontal coordinate x, which is the model ray parameter in 2-D case. The resulting differential equation system is solved by finite difference to determine “model wavefront” coordinates in time. In this finite-difference approach, each point depends on differences measured along two preceding “model wavefronts” as Figure 3 shows.

Application Example

Synthetic seismograms are frequently used to match seismic data with seismic extracted from wells. Usually, they are generated by convolving a seismic wavelet extracted from the time-migrated section with a reflectivity function given by a sonic log. Synthetic seismograms are displayed together seismic data after that some stretching and/or squeezing are applied to adjust them to the time-migrated section. The main goal of this article is to improve the synthetic seismograms matching avoiding artificial deformation, which is achieved by the use of model rays.

In general, the matching of synthetic seismograms with seismic data is performed by inserting the synthetic trace in the position where the well is located, in such a way that for vertical wells, synthetic traces are inserted in the bore-hole position. We propose the use of model rays in both creation and insertion of synthetic traces in time-migrated section.

Figure 4 shows the seismic model used for generating seismic data and simulation of well information. The model is composed by sixteen homogeneous layers separated by fourteen plane interfaces with small dips and one curved interface at the top, simulating a slope geological model. The lateral velocity variation imposed by the first layer promotes the bending of image rays. Consequently, model rays are curved and well matching along vertical paths should be degraded.

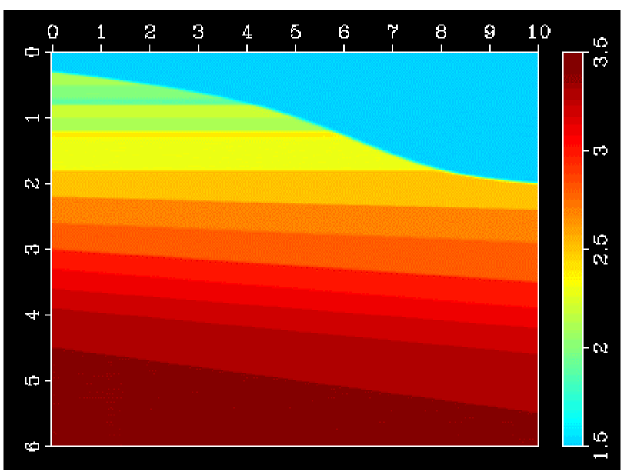

Figure 4: Seismic model simulating a slope geological model.

In order to generate an appropriated seismic dataset to exemplify the use of model rays in synthetic seismograms matching, a zero-offset section (Figure 5) is generated by Kirchhoff modeling.

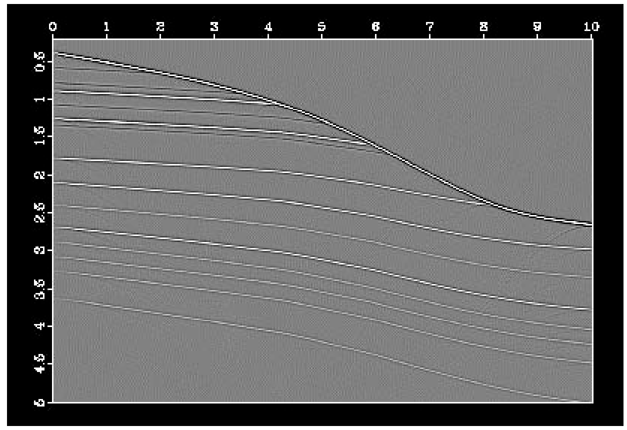

Figure 5: Zero-offset section generated by Kirchhoff modeling.

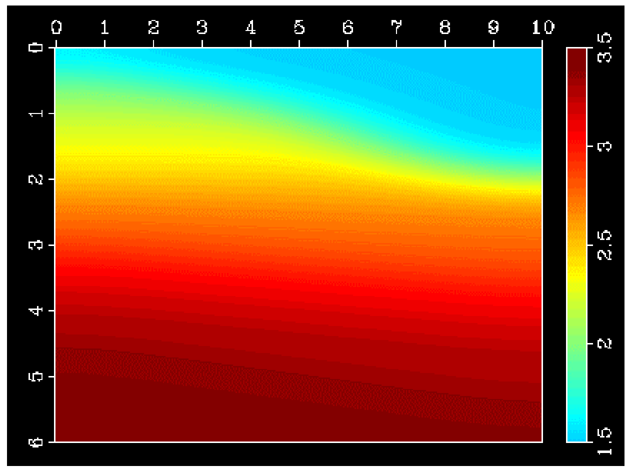

Figure 6: Smoothed velocity model used to compute RMS velocity along image rays, and to trace model rays by a paraxial and Huygens approaches.

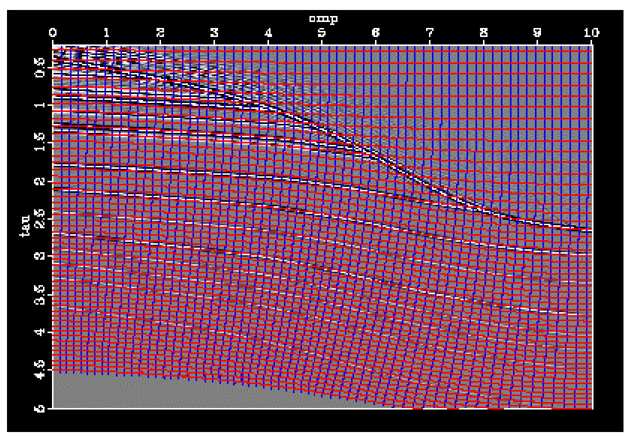

Figure 7: Time-migrated section with superposition of model rays and wavefronts.

The zero-offset section is migrated by a Kirchhoff time-migration algorithm with a RMS velocity field computed along image rays, which are traced in the smoothed velocity model presented in Figure 6. The same velocity mode is used to trace wavefronts and model rays by the Huygens approach. Figure 7 shows a superposition of model rays, wavefronts and time-migrated section. Also, model rays are computed by the paraxial ray tracing approach with similar precision in the results.

Synthetic seismograms are computed by convolving a Ricker wavelet with the reflectivity function computed for a simulated vertical well located at x=5.1 km. Here, the reflectivity function is determined in two different manners. In the first case, it is computed in a traditional way, i.e, traveltime is computed along the vertical path which coincides with the well trajectory. In the second case, traveltime is computed along model rays. Indeed, we trace image rays with their initial position determined by the paraxial ray-tracing approach. Also, the hybrid display, that shows the synthetic seismogram together the time-migrated section, is prepared in two different manners. In the first case, the synthetic trace is directly inserted in the well location. In the second case, the synthetic trace is inserted sample by sample along the model ray that starts at the bore-hole location.

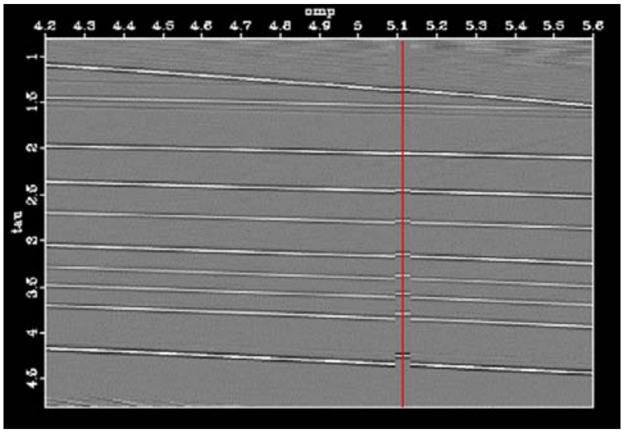

Figure 8: Detail of time-migrated section with synthetic seismogram vertically inserted in the well position.

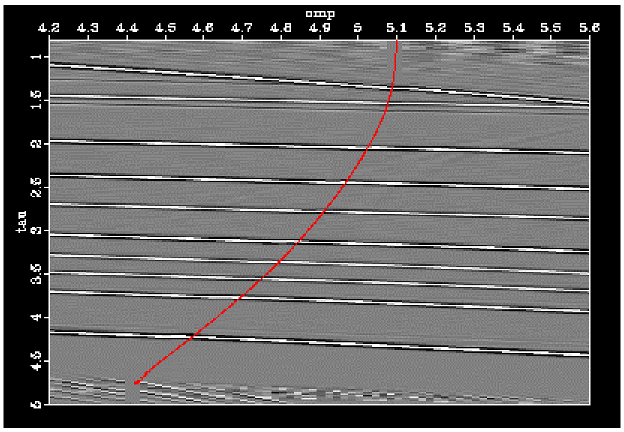

Figure 9: Detail of time-migrated section with synthetic seismogram inserted along the model-ray starting at the well position.

Figures 8 and 9 show zooms of the generated hybrid sections for the traditional and model rays approach, respectively. In both Figures, red lines indicate the place where the synthetic seismograms are inserted. Observe the perfect matching of model-ray synthetic seismogram and the time-migrated section, while the traditional synthetic seismogram is clearly discordant with the time-migrated section.

Conclusions

We introduce the concept of model rays which are virtual trajectories in time-migrated domain. Model rays present a strict relationship with image rays and play an important role in depth-to-time conversion problems. We present two different approaches to trace model rays in lateral varying velocity media and two basic equations that govern the hypothetical movement of “model” wavefronts in time. Finally, we present a practical application of the concept of model rays in seismic exploration.

For simplicity, we present 2-D formulas and examples, but all equations and concepts presented in this article are directly extended to 3-D.

Acknowledgments

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