A method for water velocity estimation

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Abstract

It is known that the velocity of propagation of sound waves in the water can vary over time. For a 3D seismic survey, where data is acquired over the same location but at different dates, this implies that the same reflection point will registered at different times. The first step to take this effect into account in seismic processing is to measure this velocity. This article presents a 3D tomographic method that directly estimates the water velocity. Using picked water bottom reflection and an initial depth and velocity model, it is shown that good results are obtained inverting only two parameters: the variation of the propagation velocity and a constant variation of the reflector depth in relation to the initial model. The method is tested with real data. The results show that the velocities are well estimated.

Introduction

The propagation velocity of sound in water (water velocity, for short) is a function of some physical properties of the sea water, especially temperature and salinity (Sheriff, 1991). As these can vary seasonally, so does the water velocity. In a 3D seismic survey, it is not unusual adjacent sail lines to be acquired in different dates. If in the interval between shooting these lines (which can be days, weeks or even months) the water velocity has changed, then the recorded times of the same reflection points will be different. If not taken into account in seismic processing, these time differences degrade the quality of processes such as stacking and migration (Wombell, 1996).
The time shifts associated with water velocity variations are typically a deep water problem. The magnitude of the time shift is proportional to the distance the wave travels in the water. So, the deeper the water, the greater the time shift. Also, as a consequence, the larger the offset, the greater the time shift. In general, before taking the trouble to compute the water velocity and applying it to the data, one has to study if the magnitudes of the time shifts are large enough to adversely affect the data.
Some methods have been presented that measure the water velocity or the corresponding reflector time shift, and use this information to correct the data to a constant water velocity medium. The most straightforward method is to estimate the water velocity using a NMO based velocity analysis (Wombell, 1996). But, where the sea bottom is dipping, this estimation will be wrong by a factor of $1/\cos\theta$, where θ is the dip angle. Furthermore, if the sea bottom is complexly structured, the moveout will not be hyperbolic in the CMP domain, making the velocity analysis more uncertain. There are methods that obtain the velocity in an indirect way (Fried & MacKay, 2002). Their method is based on the minimization of the time differences of the water bottom reflection projected to zero offset. It requires data overlap from different sail lines, a condition which is not always satisfied.
The method here presented is based on a 3D tomographic inversion of velocity. It uses as input the measured times of the water bottom reflection for a sail line, and an initial 3D model of the sea bottom and an initial water velocity. The algorithm computes updated water bottom and velocity models that minimize the difference between the measured and modeled travel times. It uses data from a single sail line, so it does not require data overlap. As the modeled travel times are computed via ray tracing through the model, it naturally takes into account the structure and dips of the water bottom.

Tomographic inversion of the water velocity

Let me now formally state the problem. I will assume for now that data from only one sail line is used. Let t be travel time of the seismic wave between the source (s) and the receiver (r), reflected at the water bottom.

$$t = T_{s,r}\,(\vec{m})$$

where T is a non-linear functional that depends on the source and receiver position and $\vec{m}$ is a model vector that describes the medium where the wave propagates:

$$\vec{m} = [z(x,y), v_s(x,y)]^T$$

where $z(x,y)$ describes the water bottom surface and $v_s(x,y)$ describes the water velocity of the wave associated with the source (s).
As it is usual in tomography, I will use the slowness, $s = v^{-1}$, instead of velocity, because the travel time is linear in this parameter. So,

$$\vec{m} = \left[z_1, z_2, \dots, z_{n_z}, s\right]^T = [\vec{z}, s]^T$$

The problem to be solved is the following: given a set of k measured reflection times $(\tau_i, i = 1, \dots, k)$, each time characterized by its source and receiver, (s_i, r_i) we want to estimate the model $\vec{m}$, in such a way that the functional $\phi(\vec{m})$ below is minimum.

$$\phi(\vec{m}) = \sum_{i=1}^{k}\left(T_{s_i,r_i}(\vec{m}) - \tau_{s_i,r_i}\right)^2$$

The functional $\phi(\vec{m})$ can be minimized by the Gauss-Newton algorithm, which can be briefly described as follows (Aster, Borchers, & Thurber, 2005):

1. Given an initial model m^0
2. Solve the following system for $\Delta\vec{m}$:

$$\vec{J}(\vec{m}^k)^T\vec{J}(\vec{m}^k)\Delta\vec{m} = \vec{J}(\vec{m}^k)^T(\vec{T}(\vec{m}^k) - \vec{\tau})$$

3. $\vec{m}^{k+1} = \vec{m}^k + \Delta\vec{m}$
4. Go back to step 2, until $\|\Delta\vec{m}\|_2 < \varepsilon$

In the algorithm above, $\vec{J}$ is the Jacobian :

$$J_{i,j} = \frac{\partial T_i}{\partial m_j}, \quad i = 1, \dots, k \quad m_j = z_1, z_2, \dots, z_{n_z}, s$$

The parameter ε is a small number that controls the end of the iterations.

The computation of the modeled times $T_i(\vec{z}, s)$ must be done numerically. The partial derivative $\frac{\partial T_i}{\partial s}$ can be easily computed. The modeled time $T_i(\vec{z}, s)$ can be writen as:

$$T_i(\vec{z}, s) = s \times d_i(\vec{z})$$

where $d_i(\vec{z})$ is the distance source-reflector-receiver. Therefore,

$$\frac{\partial T_i(\vec{z}, s)}{\partial s} = d_i(\vec{z})$$

A simple solution can be obtained if we consider that the initial depth model can only vary constantly in the vertical direction. In this case, there will be only one depth parameter to be estimated, the constant depth displacement of the water bottom surface, Δz. The problem of inverting a large, sparse matrix can, in this way, be reduced to inverting for only two parameters. The Jacobian to be computed is then:

$$J = \begin{bmatrix} \frac{\partial T_1(\vec{z}, s)}{\partial z} & \frac{\partial T_1(\vec{z}, s)}{\partial s} \\ \vdots & \vdots \\ \frac{\partial T_k(\vec{z}, s)}{\partial z} & \frac{\partial T_k(\vec{z}, s)}{\partial s} \end{bmatrix}$$

The variation of the model to be solved is:

$$\Delta m = \begin{bmatrix} \Delta z \\ \Delta s \end{bmatrix}$$

The partial derivative $\frac{\partial T_i(\vec{z}, \vec{s})}{\partial z}$ could be computed using finite difference methods, for example. But, there is a better way. The variation in the reflection time due to a small variation in the depth of the reflector is given by (Stork & Clayton, 1991):

$$\Delta t = \frac{2\,\Delta h \cos\theta}{v}$$

where Δt is the time variation in the reflection time, Δh is the variation in the reflector position in the direction normal to its surface, θ is the angle between the ray and the normal to the surface (i.e. , the reflection angle), and v is the propagation velocity above the surface (i.e., the water velocity).

The Δt above is written as a function of Δh. To write it as a function of Δz, the vertical depth variation, it is easy to show that

$$\Delta h = \Delta z \times \cos\alpha$$

where α is the angle between the normal and the vertical direction. Therefore, the derivative $\frac{\partial T}{\partial z}$ in the Jacobian can be computed as

$$\frac{\partial T}{\partial z} \cong \frac{\Delta t}{\Delta z} = \frac{2\ \cos\alpha\ \cos\theta}{v}$$

Real data example

I applied the method developed in the previous section to a real 3D survey, acquired in the deep waters off the Brazilian coast. The water bottom is more than 2000 m deep.

The first step in the inversion sequence is to define the initial depth model. From a previously time migrated data, I picked the water bottom reflection time. I applied a simple constant velocity, vertical ray, time-to-depth conversion to create the depth model.

I chose 17 sail lines, each with approximately 250 shots, to estimate their water velocities. In each line, approximately 570,000 traces were processed. Figure 1 shows the shot location map of these sail lines. Note that they have a varied degree of overlap among them. As each line is processed independently, this overlap or lack of it does not matter.

Choosing the initial velocity model is the second step in the inversion, and the easiest one. I chose 1500 m/s as the initial velocity

The third step in the processing is the picking of the water bottom reflection for each trace. This is the most challenging step in the method. I started applying a phase filter to convert the wavelet to zero-phase and then picked the highest amplitude. The high quality of the data and the absence of significant noise allowed for great confidence in the picking.

For computing the travel distance between source, reflector and receiver, needed to build the Jacobian, I used numerical ray-tracing

Having all the necessary input, the velocity and depth shift estimation were done for each of the 17 sail lines. Two iterations of the Gauss-Newton algorithm were done, but convergence was achieved in the first iteration. Table 1 shows the results obtained.

Sail Line	Δz (m)	V (m/s)
101	-7.7	1499.4
102	-7.0	1500.0
103	-7.1	1497.4
104	-7.4	1498.6
105	-8.0	1500.8
106	-7.9	1496.3
107	-7.5	1496.9
108	-7.1	1497.8
109	-6.7	1499.4
110	-6.6	1500.0
111	-7.0	1499.9
112	-6.9	1499.7
113	-6.0	1500.9
114	-7.7	1499.8
115	-6.6	1500.7
116	-7.4	1500.1
117	-5.7	1500.1

Table 1: Estimated depth variation and velocities for each of sail lines.

The results were checked with two different methods: the analysis of the residuals and migrating the data with the estimated velocities. The time residuals (r_i) are the difference between the modeled times and the picked times for each trace i.

$r_i = T_i(\vec{m}^k) - \tau_i$, k=0 or 2 for the initial and last model

Figure 2 shows the histogram of the residuals for 4 of the lines processed. In all of them (and also in the other lines, not shown here) more than 90% of the samples are between -2 and +2 ms. The peaks of the distributions are approximately 0 ms. The possible sources of errors are approximations in the modeling, assuming constant water velocity, vertically and laterally, errors in the source and receiver coordinates, errors in the source and cable depths and errors in the picking of the water bottom reflection. Nevertheless, given that the sample interval is 4ms, that most of the residuals are less than this, and the simplicity of the inversion, I think the results are very good.
The second method of checking the results is to use the estimated velocities to migrate the data. To do this I used a Kirchhoff 3D depth migration algorithm, modified to take into account the water velocity associated with each sail line. I first chose a CRP whose migration aperture area included data from sail lines with the greatest water velocity variation among them. This CRP was first migrated with one velocity for all lines, equal to the average of water velocities of the lines that were in its aperture area. Then it was migrated with the modified Kirchhoff algorithm, with each trace being migrated with the water velocity of its sail line. The results are shown in figure 3. It is clear that the CRP migrated with the velocity appropriate for each sail line is the best result.

Figure 4 shows the result of stacking the migrated CRP's for the inline Y=1875 m with one velocity for all data and with the estimated velocities for each sail line. Again, the resulting water bottom reflection shows a much uniform appearance in the later than in the former. Nevertheless, the overall structure of the reflector is the same.

Conclusion

The method presented in this article successfully estimated the water velocity. This conclusion was reached applying the estimated velocities in a modified Kirchhoff migration algorithm and checking for the residual move-out in the CRP's.
Unlike other methods it naturally takes the water bottom dip into account and it does not require data overlap.

Acknowledgement

I would like to thank Petrobras for permission to present this article. My thesis advisor, Prof. Webe Mansur, always gave me constant support in my research. Finally I would like to thank my colleagues at Petrobras, especially Carlos Cunha and João Boechat, for their help.

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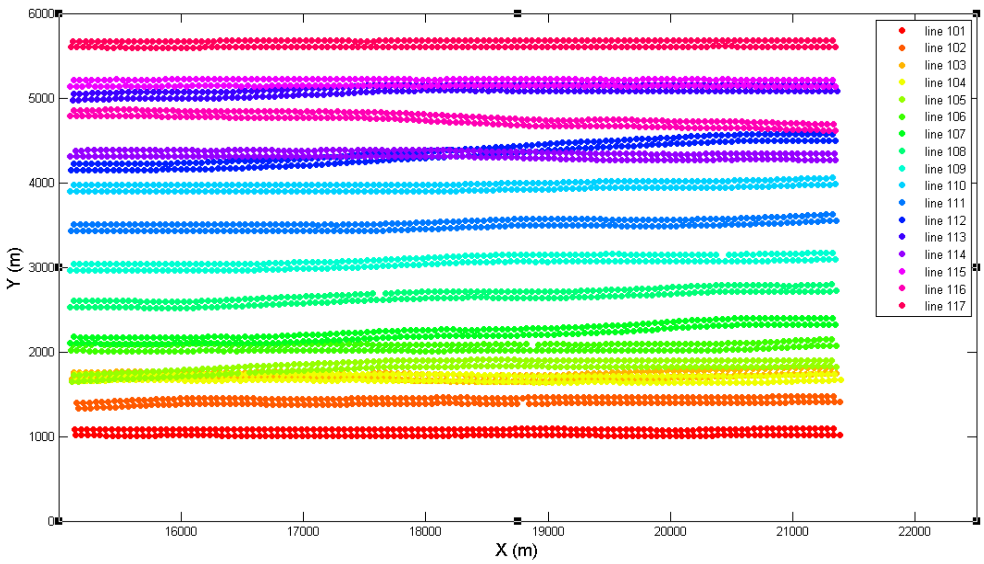

Figure 1 : Shot point map

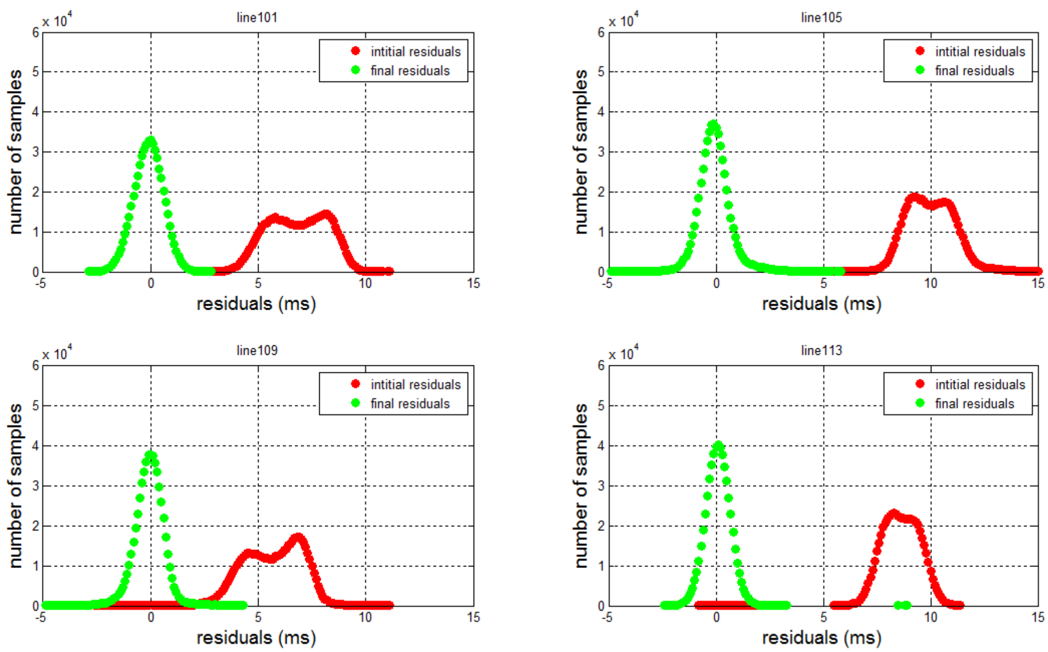

Figure 2 : Histogram of the residuals

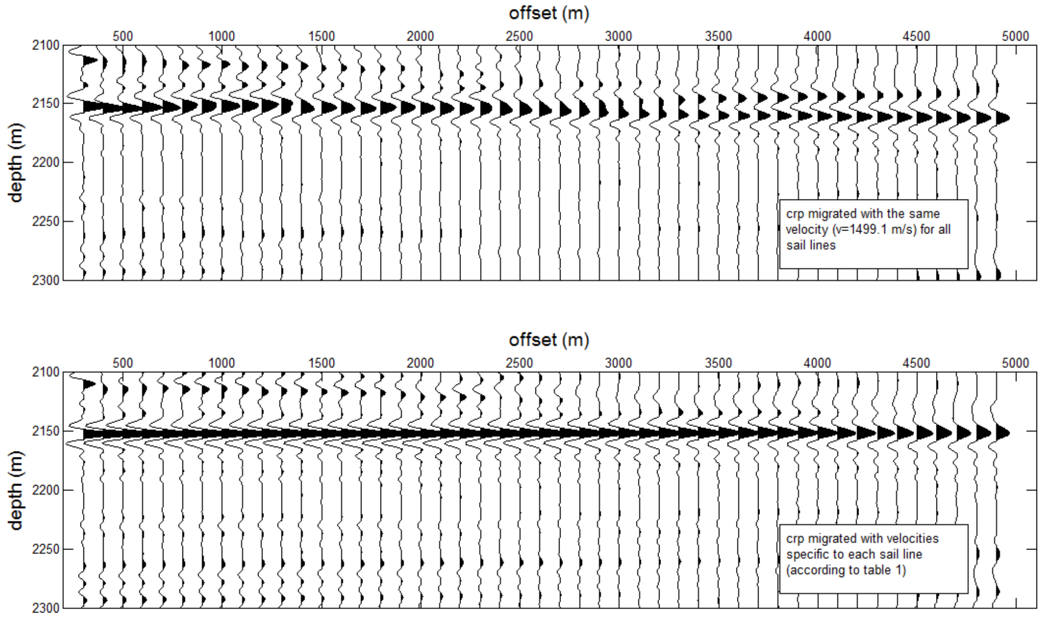

Figure 3 : CRP at (X=17000,Y=1875) migrated with the same velocities for all sail lines (above) and with velocities specific for each sail line (below). Note the reflections appearing at the bottom of the crp migrated with line-specific velocities.

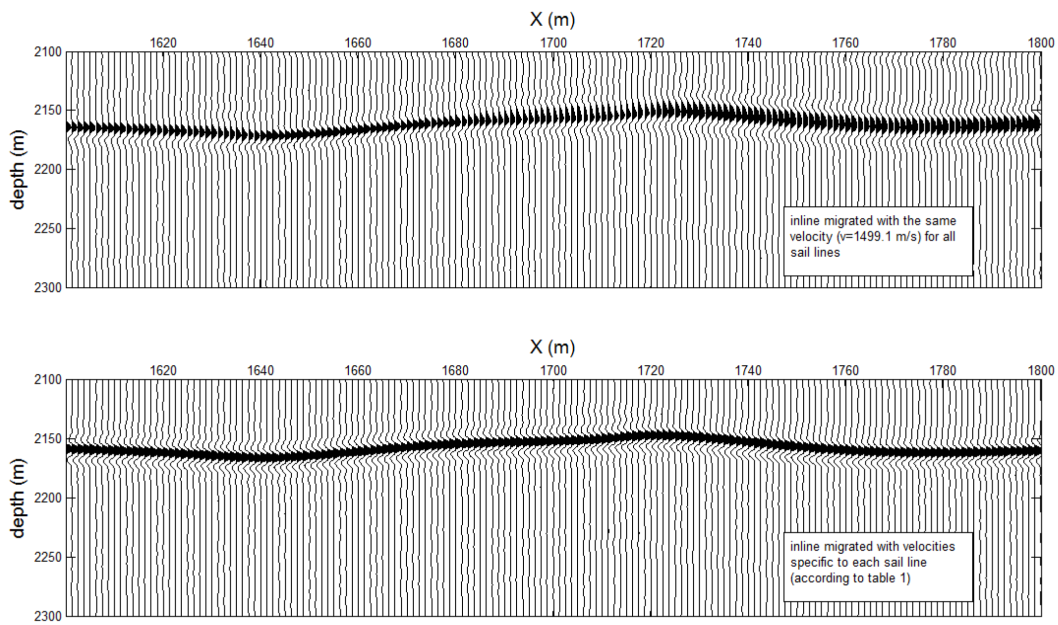

Figure 4 : Inline (Y=1875) migrated with the same velocity (v=1499.1 m/s) for all sail lines (above) and with velocities specific to each sail line, according to table 1 (below).