3D True-Velocity Radon Filter in perspective

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Abstract

In this paper I discuss alternatives for implementing a true velocity filter based on Radon transforms in common and future surface seismic geometries. A procedure for the discrimination of reflections based on their velocities is feasible if we consider, for instance, a properly chosen group of common midpoint families. This can be made in 2D or 3D. New acquisition geometries characterized by a more regular sampling of offsets and azimuths provide common midpoint families that allow for true velocity discrimination. The use of a group of such well sampled common midpoint families would introduce criterias of consistency that could improve velocity determination in a noisy environment.

Velocity Indeterminacy in 2D

Historically, velocities are estimated from surface seismics in common midpoint (CMP) gathers. Assuming a locally plane set of reflectors underneath, a hiperbolic approximation for the dependence of observed arrival time and offset is used. It is well known (Levin, 1971) that velocities estimated from the observed normal moveout (NMO) of reflections on dipping plane horizons are increased according to the formula,

$$\mathrm{v}_{app} = \frac{\mathrm{v}_{true}}{\cos(\theta)} \quad , \tag{1}$$

in a time-offset relation as,

$$t = \sqrt{t_0^2 + \frac{\chi^2}{\mathrm{v}_{app}^2}} \tag{2}$$

where v_{app}, v_{true}, θ, t, t_0, and χ are, respectively, the stacking or apparent velocity, the true velocity of the media, the dip of the reflector, the arrival time, the intercept time at zero offset and the offset.

Equation (1) points to the indeterminacy of the medium or true velocity in a CMP since hyperbolical moveout is a function only of the apparent velocity . However, as dips can be estimated from at least two neighboring CMPs, it is possible to overcome this indeterminacy if a group of CMPs are considered. Keeping the same order of approximation as in (2), the time-offset curve can be rewrited for a small set of neighboring CMPs as,

$$t = \sqrt{\left[t_0 + \frac{2x\sin(\theta)}{\mathrm{v}_{true}}\right]^2 + \frac{\chi^2\cos(\theta)^2}{\mathrm{v}_{true}^2}} \tag{3}$$

where, x stands for the CMP coordinate and t_0 now is the time at zero offset and at $x = 0$. Generally, besides the requirement of small values of χ, equation (3) is expected to hold only in a narrow vicinity of x where θ is estimated. Reflectors' curvature limitates the vicinity to be considered. Moreover, if velocity vary laterally there may be observed events with time-offset relationships that depart too much from (3).

Assuming a smooth lateral variation of velocity, for sufficiently greater values of t_0 or after normal moveout correction, one can take the dependence of t with respect to χ and x respectively as parabolic and linear. Assuming also that time-offset curves vary locally as a hyperbola shifted as a function of t_0 only (see figure 1), an extended Radon

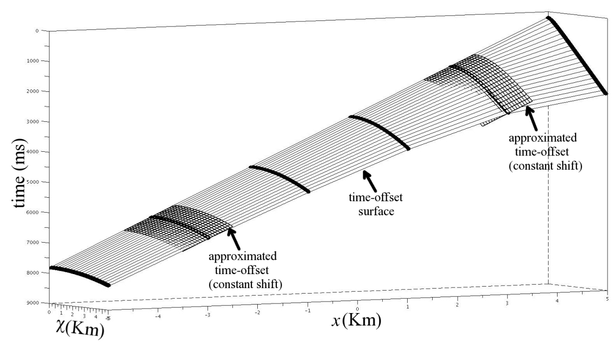

Figure 1: A comparison between the constant velocity reflection time-offset surface and shifted hyperbolas curves in the (t,χ,x) domain. This shows the errors in the local (multigroup) Radon mapping as suggested by expression (3). It can also be seen that the approximation tends to be better for small offsets and deeper events.

transform, which is parabolic with respect to the offset and linear with respect to the CMP position, could be used to map such an event in a (τ,α,p) space where most of the energy would be gathered around the point[1] (τ_0,α_0,p_0) given by (de Oliveira et. al., 2007),

$$\tau_0 = t_0 \quad , \quad \alpha_0 = \frac{1}{2t_0}\left(\frac{1}{\mathrm{v}_{true}^2} - \frac{p_0^2}{4}\right) \quad , \quad p_0 = \frac{2\sin(\theta)}{\mathrm{v}_{true}} \ . \tag{4}$$

Here, α and p are proportional respectively to the parabolic and linear moveouts. The dependence of α to v_{true}, t_0, and p, as suggested above generates a surface in the (τ,α,p) domain that can be used to discriminate events by their true velocity (see figure 2). Velocities increase when one moves from "outside" to "inside" the surface drawn in figure 2.

[1] If the shifted hyperbola approximation is poor, α is a function of t_0 and the event mapping is more complicated.

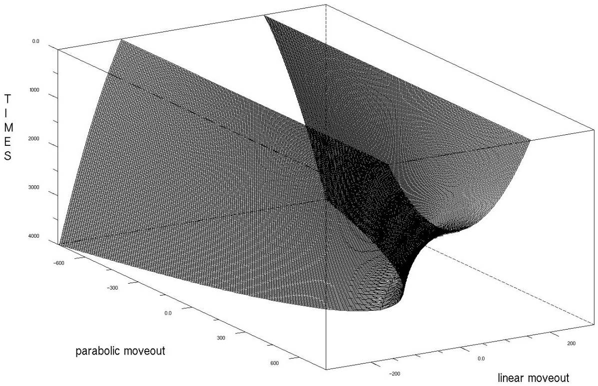

Figure 2: A constant velocity surface in the (τ, α, p) domain given by $\alpha = \alpha(\tau, p, \mathrm{v})$ as written in (4) letting p vary.

Velocity Indeterminacy in 3D (narrow azimuth)

Equation (3) above is valid in 2D. In a more general configuration, equation (3) turns into,

$$t = \sqrt{(t_0 + \mathbf{p}\cdot\mathbf{x})^2 + \frac{\chi^2}{\mathrm{v}_{true}^2} - \frac{(\boldsymbol{\chi}\cdot\mathbf{p})^2}{4}} \qquad (5)$$

with,

$$\mathbf{p} = \frac{2\sin(\theta)}{\mathrm{v}_{true}}\mathbf{e} \quad , \qquad (6)$$

$\mathbf{e}$ a 2D (surface) unit vector in the dip direction, and $\boldsymbol{\chi}$ a 2D offset vector.

In 3D, the relationship between apparent velocity (and dip) with the true velocity depends on the azimuth of the line of acquisition. Considering narrow azimuth surveys with offsets parallel to the main line of acquisition ($\boldsymbol{\chi}\cdots\boldsymbol{x} = \chi x$), an extension of the technique proposed above for the 3D case demands a 3D set of neighboring CMPs. The corresponding time-offset 4D surface, analogous to what is seen if figure 1, is not easy to represent. However, the relationship between $(t, \chi, \boldsymbol{x})$ and $(\tau, \alpha, \boldsymbol{p})$ domains are mathematically easy to derive under the same type of approximations used in expression (4),

$$\alpha = \frac{1}{2t_0}\left(\frac{1}{\mathrm{v}_{true}^2} - \frac{\|p\|^2}{4}\right) \quad , \qquad (7)$$

where $\|p\|^2$ is the magnitude of the vector $\boldsymbol{p}$, and the discrimination with respect to the true velocity is possible.

Velocity Indeterminacy in 3D (wide azimuth)

Equation (5) holds, in the hyperbolical range of approximation, for regular surface seismics as well as for new well-sampled (many offsets and azimuths) surveys. It can be seen from figure 3 that in a 3D CMP it is possible to determine true velocities since there is an azimuth where $\mathrm{v}_{app} = \mathrm{v}_{true}$, which corresponds to aquisition perpendicular to the dip direction and implies the minimum apparent velocity for a given event. On the other hand, as discussed in the previous section, a group of neighboring CMPs would still exhibit the dip as in the 2D case and this could be used to raise the indetermination of the true velocity as well.

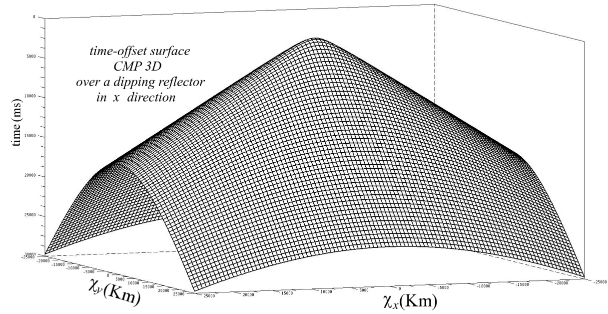

Figure 3: A constant velocity surface $t = t(t_0, \boldsymbol{\chi}, \boldsymbol{x})$ in a CMP 3D over a dipping reflector. Note that the moveout in the χ_y direction is not affected by the dip.

Again, a properly designed Radon transform can help discriminating events by their moveouts. Mathematically, this Radon transform could be written as,

$$\Psi(\tau, \boldsymbol{\alpha}, \mathbf{p}) = \int_{-\infty}^{\infty} d\mathbf{x}^2 \int_{-\infty}^{\infty} d\boldsymbol{\chi}^2 \psi(\tau + \textstyle\sum_i \alpha_i \chi_i^2 + \mathbf{p}\cdot\mathbf{x} \, , \, \boldsymbol{\chi}, \mathbf{x}) \quad . \qquad (8)$$

Expression (8) is based on the assumption that a paraboloid that osculates the 3D time-offset curve shown in figure 3 at the origin would have the form,

$$t = \tau + \alpha_1 \chi_1^2 + \alpha_2 \chi_2^2 \quad , \qquad (9)$$

with $\alpha_{1,2}$ and $\chi_{1,2}$, respectively, the parameter for the parabolic moveout and the offset in the direction 1 or 2. A straightforward computation yields the following relation,

$$\alpha_j = \frac{1}{2t_0}\left(\frac{1}{\mathrm{v}_{true}^2} - \frac{p_j^2}{4}\right) \quad \text{for} \quad j = 1, 2, \qquad (10)$$

which can be used to discriminate events based on their true-velocity as in previous cases.

Summary and Comments

In this paper I discuss the perspectives of true-velocity discrimination based on the information about dip available in small groups of common mid-point families of surface seismic data. It was shown that a more precise determination of dip is possible only in 3D and that there is an intrinsic redundancy if a rich or wide azimuth acquisition is considered. The discussion was made in close connection with Radon transforms as a means to classify events by their linear moveout, associated with refletor dips, and parabolic moveout, associated with the apparent velocities in a first order (small offset) approximation.

References

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