Gaussian beam modified true-amplitude diffraction stack migration: Application to Marmousi dataset

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Abstract

The Gaussian Beam (GB) approach is a (zero order) complex paraxial ray theory, that is a harmonic solution of the wave equation in ray centered coordinates. It has been considered a more physical representation of the wavefield than the standard ray theory. The regularity in the description of the wavefield, as well as its high accuracy in some singular regions of the propagation medium, transformed the use of GB's into a strong alternative to seismic modeling and imaging problems. In this paper we make use of the flexibility in imaging of the true amplitude prestack Kirchhoff depth migration, in addition to the regularity of the wavefield representation by superposition of GB's, to develop a new true-amplitude prestack depth migration. As a way of controlling, in a very stable way, some quantities used in the construction of the beams, we consider some information based on the Fresnel volume elements (Fresnel zone radius) around the reflection point in depth and its counterpart, the projected Fresnel zone. The result is a robust and stable prestack depth migration algorithm able to efficiently determine the plane wave reflection coefficients and depth seismic images in complex geological medium.

Introduction

In the recent years we have found in the geophysicists literature many studies about the superposition of paraxial Gaussian beans as an alternative solution of the seismic wave equation (Müller (1984), Cěrvený (2000), and Popov (2002)). The Gaussian beam method advantages, e.g. the regularity of the wavefield determination even in presence of caustics or shadows zones, have also attracted the attention of people that work with seismic data processing. The possible combination of the flexibility of the Kirchhoff type migration and the robustness of the Gaussian beam approach has motivated the proposition of some Gaussian beam based migration alternative methods (Hill, 1990; Hill, 2001; Albertin et al., 2004; Ferreira and Cruz; 2004; Gray and Bleistein, 2009).

In the present paper we propose a new derivation of the true-amplitude prestack depth migration, by using the Gaussian beam superposition integral to describe the seismic wavefield in the true-amplitude migration operator. Our approach is based on the representation of the seismic wavefield vector by a Gaussian beam superposition integral (Cěrvený, 2000), that permits an analytical solution in locally arbitrary observation coordinates. By using this result in the true-amplitude prestack depth migration operator given by Schleicher et al. (1993), we consider it is a sufficient representation of the seismic wavefield. Through the stationary phase method, we determine the modified weight function used for obtain asymptotically an estimation of the plane wave reflection coefficient and a high resolution seismic imaging. In this case there is no limitation about acquisition configuration, since our algorithm is built as a modification of the general true-amplitude migration method presented by Schleicher et al. (1993).

In comparison with the result of Sun et al. (2000), our approach can be interpreted as a beam stack migration operator with the sources and receivers patches specified by the integration domain of the Gaussian beam superposition integral. In the stationary phase situation the central point of the Gaussian beam stack domain coincides with the critical point of the asymptotic analysis of the true-amplitude migration integral. Due to one seismic trace can participate in multiple adjacent supergathers, the Gaussian beam stack migration formalism here presented is more general than the beam Kirchhoff migration approach presented by Sun et al. (2000). Differently of other migration approaches, we take care about constraints of the Gaussian beam superposition integral domain, which is limited to the maximum width of a projected Fresnel zone (Hubral et al., 1993 and Schleicher et al., 2004). For common-offset configuration we show the relationship between the weight function derived by Albertin et al. (2004) and the obtained by our approach.

The proposed algorithm is applied to synthetic seismic data in two examples: the first is a common-offset seismic imaging of the Marmousi dataset; and the other case is an example of estimation of the plane wave reflection coefficients for a synclinal model.

3-D Gaussian beam method

In case of a 3-D compressional point source, we consider a congruence of rays with the ray parameters given by the spherical coordinates (θ,φ), with origin at the source position. Each central ray is defined by $\vec{r}=\vec{r}_o(s,\theta,\varphi)$, being s the ray arc length. In the vicinity of the central ray $\vec{r}_o(s)$, the paraxial Gaussian beam approximation of the principal component of the seismic wavefield is given by (Popov, 2002):

$$\mathbf{u}(s,q_1,q_2;\omega)=\frac{\vec{t}}{\sqrt{v_p\rho_o\det\mathbf{Q}}}\exp\left\{i\omega\left[\tau_o(s)+\frac{1}{2}\sum_{j,k=1}^{2}M_{jk}q_jq_k\right]\right\}, \tag{1}$$

where (q_1,q_2) are the ray centered coordinates associated with a right-hand Cartesian system of the plane perpendicular to the central ray. The complex number $i=\sqrt{-1}$, ω is the angular frequency, $\vec{t}=d\vec{r}_o(s)/ds$ is the unity tangent vector along the central ray, $v_p=v_p(s)$ and $\rho_o=\rho_o(s)$, and $\tau_o(s)$ are the P-wave velocity, the density and traveltime along the central ray. The M_{jk}, with $j,k=1,2$, are elements of the 2x2 matrix defined by:

$$\mathbf{M}=\mathbf{P}\mathbf{Q}^{-1} \quad , \tag{2}$$

being the 2X2 matrices $\mathbf{P}$ and $\mathbf{Q}$ solutions of the dynamic ray tracing systems (Popov, 2002 and Cěrvený, 2000), that under complex initial conditions present the following properties: 1) $\det\mathbf{Q}\neq 0$ for arbitrary s; 2) the complex valued matrix $\mathbf{M}$ is symmetrical, i.e. $\mathbf{M}^T=\mathbf{M}$; 3) The imaginary part of $\mathbf{M}$ is positive-defined. The Gaussian beam method can be used in more general cases, i.e. not only for point source, if the properties of the high frequency wavefield are preserved, by considering that initial data ϕ_o are specified on a known surface.

Gaussian beam superposition integral

Let us specify a bundle of reflection central rays by using ray parameter coordinates $\boldsymbol{\gamma}=(\gamma_1,\gamma_2)$, starting and ending on the measurement, in general, curved surface Σ_S. A source and a receiver on Σ_S has the position vectors $\mathbf{x}_s$ and $\mathbf{x}_g$ with respect to a global Cartesian system. In the paraxial vicinity of the central rays, the primary reflection P wavefield is calculated in a reference point $\mathbf{x}_o$ by the superposition integral over paraxial Gaussian beams (Popov, 2002) ,

$$\mathbf{U}(\mathbf{x}_o,\omega)=\iint_{D_B}d\gamma_1 d\gamma_2\phi_o(\gamma_1,\gamma_2)\mathbf{u}(s,q_1,q_2;\gamma_1,\gamma_2,\omega)\cdot \tag{3}$$

The superposition integral (3) depends on the chosen Gaussian beam free parameters and initial amplitudes. Müller (1984), Cěrvený (2000), Popov (2002) gave optimized criteria to efficiently calculate the seismic wavefield by means of the superposition of paraxial Gaussian beams.

A more suitable version of integral (3) for migration and seismic inversion problems, in frequency domain, is given by (Cěrvený, 2000),

$$\mathbf{U}(\boldsymbol{\xi}=\boldsymbol{\xi}_o,\omega)=\iint_{D_B}d\gamma_1 d\gamma_2\Phi(\gamma_1,\gamma_2)\times \mathbf{u}_o(\gamma_1,\gamma_2)\exp[i\omega T(\boldsymbol{\xi}_o,\boldsymbol{\xi}_b)]\,. \tag{4}$$

The single parameter vector $\boldsymbol{\xi}=(\xi_1,\xi_2)$ is used to specify acquisition points with respect to local 2-D Cartesian systems at the start and end points of a central ray, obeying the following relationships:

$$\mathbf{x}_s(\boldsymbol{\xi})=\mathbf{x}_{so}+\boldsymbol{\Gamma}_S\Delta\boldsymbol{\xi} \quad \text{and} \quad \mathbf{x}_g(\boldsymbol{\xi})=\mathbf{x}_{go}+\boldsymbol{\Gamma}_G\Delta\boldsymbol{\xi}, \tag{5}$$

where $\mathbf{x}_s(x_s,y_s)$ and $\mathbf{x}_g(x_g,y_g)$ are Cartesian coordinates at the source and receiver positions, in the vicinity of the fixed pair ($\mathbf{x}_{so}$, $\mathbf{x}_{go}$). The difference $\Delta\boldsymbol{\xi}=\boldsymbol{\xi}-\boldsymbol{\xi}_o$. The 2x2 constant matrices $\boldsymbol{\Gamma}_s$ and $\boldsymbol{\Gamma}_r$ are specified according to the selected acquisition geometry, which can be a null, $\mathbf{0}$, or identity, $\mathbf{I}$, matrices.

The exponential factor $T(\boldsymbol{\xi}_o,\boldsymbol{\xi}_b)$ is the complex second order paraxial reflection traveltime, which is described as function of the parameter vector $\boldsymbol{\xi}=\boldsymbol{\xi}_b$, being $\boldsymbol{\xi}=\boldsymbol{\xi}_o$ a fixed coordinate vector, also called the central point of the Gaussian beam superposition integral domain, and it is expressed by:

$$T(\boldsymbol{\xi}_o,\boldsymbol{\xi}_b)=\tau_R(\boldsymbol{\xi}_b)-\mathbf{p}_s(\boldsymbol{\xi}_b)\cdot\boldsymbol{\Gamma}_S(\boldsymbol{\xi}_b-\boldsymbol{\xi}_o)+ \mathbf{p}_g(\boldsymbol{\xi}_b)\cdot\boldsymbol{\Gamma}_G(\boldsymbol{\xi}_b-\boldsymbol{\xi}_o)-\frac{1}{2}(\boldsymbol{\xi}_b-\boldsymbol{\xi}_o)\cdot\mathbf{H}_R(\boldsymbol{\xi}_b)(\boldsymbol{\xi}_b-\boldsymbol{\xi}_o). \tag{6}$$

The time $\tau_R(\boldsymbol{\xi}_b)$ is the reflection traveltime of the central ray that starts and ends at the source and receiver position coordinates $\mathbf{x}_s(x_s,y_s)$ and $\mathbf{x}_g(x_g,y_g)$, respectively, within the superposition integral domain D_B. The vectors $\mathbf{p}_s$ and $\mathbf{p}_r$ are the projections of the slowness vectors of the central ray at source and receiver 2-D local Cartesian coordinate systems. The paraxial distance vector $\boldsymbol{\delta}=(\boldsymbol{\xi}_b-\boldsymbol{\xi}_o)$ is assumed to be sufficiently small. The 2X2 complex-valued matrix $\mathbf{H}_R$ has a real part with components given by the second order derivatives of the reflection traveltime on relation to the paraxial distance vector $\boldsymbol{\delta}$, providing the shape of the Gaussian beam; and its imaginary part is chosen arbitrarily in a way to define the Gaussian beam width.

In the equation (4) the weight function Φ compensates for using the standard zero-order ray-theory complex-valued amplitude of the primary reflection wavefied vector, $\mathbf{u}_o$, and it is determined by applying an asymptotical analysis (Bleistein, 2008) or by simultaneous diagonalization of complex-valued quadratic form method

(Červený, 2000), and equating the result to the zero-order term of the ray series. To obtain the solution of integral (4), the integration variables (γ_1, γ_2) should be transformed into the variables $\boldsymbol{\xi}_b = (\xi_{b1}, \xi_{b2})$, at the emergence point of the central ray on the acquisition surface, by using the jacobian:

$$d\gamma_1 d\gamma_2 = (\cos\alpha_S \cos^2\alpha_G)^{-1}[|\det \mathbf{N}_{SG}(\boldsymbol{\xi}_b)|]d\xi_{b1}d\xi_{b2}, \qquad (7)$$

where α_S and α_G are the start and the emergence angles of the central ray at the source $S(\boldsymbol{\xi}_b)$ and receiver $G(\boldsymbol{\xi}_b)$ on the acquisition surface. The 2X2 matrix $\mathbf{N}_{SG}$ can be expressed by:

$$\mathbf{N}_{SG} = \mathbf{N}_{RS}{}^T \boldsymbol{\Lambda}^{-T} \mathbf{H}_P \boldsymbol{\Lambda}^{-1} \mathbf{N}_{RG}. \qquad (8)$$

The $\mathbf{N}_{RS}$ and $\mathbf{N}_{RG}$ are non-symmetrical 2X2 matrices with components given by the second order mixed-derivative of the reflection traveltime with respect to the source-reflection point and to the receiver-reflection point coordinates, respectively. The 2X2 matrix $\boldsymbol{\Lambda}$ depends on the acquisition configuration, being expressed by:

$$\boldsymbol{\Lambda} = (\mathbf{N}_{RS}\boldsymbol{\Gamma}_S + \mathbf{N}_{RG}\boldsymbol{\Gamma}_G). \qquad (9)$$

The 2X2 matrix $\mathbf{H}_P$ is the well known projected Fresnel zone matrix (Schleicher et al., 2004), that corresponds to the projected first Fresnel zone on the acquisition surface, which coordinates satisfy the relationship:

$$|\boldsymbol{\delta}^T \mathbf{H}_P \boldsymbol{\delta}| \le \frac{\mathrm{T}}{2}. \qquad (10)$$

By equation (10) we establish the limits of the projected Fresnel zone on the acquisition surface given by the maximum paraxial distance $\boldsymbol{\delta} = \boldsymbol{\delta}_F$, being T the relevant period of the seismic wavefield.

The integral (4) is now rewritten as follows:

$$\mathbf{U}(\boldsymbol{\xi} = \boldsymbol{\xi}_o, \omega) = \iint_{D_B} d\boldsymbol{\xi}_b (\cos\alpha_S \cos^2\alpha_G)^{-1} |\det \mathbf{N}_{SG}(\boldsymbol{\xi}_b)| \Phi(\boldsymbol{\xi}_b) \times \mathbf{u}_o(\boldsymbol{\xi}_b) \exp[i\omega T(\boldsymbol{\xi}_o, \boldsymbol{\xi}_b)]. \qquad (11)$$

By applying the method of simultaneous diagonalization to the Gaussian beam superposition integral (11), and assuming that for high-frequencies ω, the main contributions come from the vicinity of the ray passing through the position vector $\boldsymbol{\xi}_o$ (Červený, 2000) , that is assumed to be situated in a regular region, we obtain:

$$\mathbf{U}(\boldsymbol{\xi}_o, \omega) \approx (2\pi/\omega)[\cos\alpha_S \cos^2\alpha_G]^{-1} |\det \mathbf{N}_{SG}(\boldsymbol{\xi}_o)| \times \Phi(\boldsymbol{\xi}_o)[-\det \mathbf{M}_\Delta(\boldsymbol{\xi}_o)]^{-1/2} \mathbf{u}_o(\boldsymbol{\xi}_o) \exp[i\omega\tau_R(\boldsymbol{\xi}_o)]. \qquad (12)$$

By equation (12) we can see that the seismic wavefield is asymptotically approximated by the product of quantities calculated during the dynamic ray tracing and specified at the position vector $\boldsymbol{\xi}_o$. The complex valued 2X2 symmetrical matrix $\mathbf{M}_\Delta(\boldsymbol{\xi})$ is a function that can be defined in many ways, e.g. see (Červený, 2000) and Popov (2002), with the conditions: $\Im\{\mathbf{M}_\Delta\}$ is a positive definite matrix and $\det \mathbf{M}_\Delta \neq 0$. This relevant matrix can be determined by second order derivatives of the traveltime function calculated at $\boldsymbol{\xi}_o$.

We choose for the Gaussian beam superposition integral a modified version of the weight found by (Červený, 2000), that is now expressed as function of the coordinates $\boldsymbol{\xi}_b \in D_b$:

$$\Phi(\boldsymbol{\xi}_b) = \cos\alpha_G[-\det \mathbf{M}_\Delta(\boldsymbol{\xi}_b)]^{1/2} |\det \mathbf{N}_{SG}(\boldsymbol{\xi}_b)|^{-1}. \qquad (13)$$

3-D Gaussian beam modified true-amplitude diffraction stack migration

For simplicity, in the next results we consider only horizontal plane acquisition surface, without losing of the geometrical generalizations. Following Schleicher et al. (1993), the weight modified diffraction stack is the appropriate operator to obtain a true-amplitude seismic prestack depth migration from finite-offset data.

Based on the definition of a stack surface, so-called Huygens surface, given by:

$$\tau_D(\boldsymbol{\xi}, M) = \tau(S, M) + \tau(G, M), \qquad (14)$$

where $S = S(\boldsymbol{\xi})$ and $G = G(\boldsymbol{\xi})$ are source and receiver points, in the migration aperture A. $\tau(S,M)$ and $\tau(G,M)$ denote the traveltimes from S to the subsurface point M, and from G to M, respectively. The traveltime τ_D represents for each point M the diffraction stack surface along which the seismic data is weighted summed, being the summation mathematically expressed in frequency domain by (Schleicher et al.,1993):

$$V(M, \omega) = -\frac{i\omega}{2\pi} \iint_A d\boldsymbol{\xi} w(M, \boldsymbol{\xi}) U(\boldsymbol{\xi}, \omega) \exp[-i\omega\tau_D(\boldsymbol{\xi}, M)]. \qquad (15)$$

In the true-amplitude Kirchhoff prestack depth migration (K-PDM), by considering a high-frequency approximation $(\omega >> 1)$, the $V(M, \omega)$ is given approximately by an asymptotic evaluation of the integral (15), being the reflector image built by positioning the output of the integral in the chosen depth point M (Schleicher et al. (1993) and Bleistein (1987)).

By means of the zero-order ray theory, Schleicher et al. (1993) obtained the appropriate weight function $w(M, \boldsymbol{\xi})$, at the stationary point $\boldsymbol{\xi}^* \in A$, by applying the stationary phase method to the diffraction stack integral (15). As result they received the plane wave reflection coefficients of the image points at the subsurface. In the case $\boldsymbol{\xi}^* \notin A$, the asymptotic evaluations of (15) are contributions of the boundaries of the aperture, when the gradient of the phase function does not vanishes, being necessary to be used some taper function in the vicinity of the migration aperture border.

Instead of the zero-order of the ray series, we consider in the equation (15) the seismic wavefield $U(\boldsymbol{\xi},\omega) = |\mathbf{U}(\boldsymbol{\xi},\omega)|$ is well represented by the Gaussian beam superposition integral (11), resulting in the new formalism:

$$V(M,\omega) = -\frac{i\omega}{2\pi}\iint_A d\boldsymbol{\xi} w_b(M,\boldsymbol{\xi})\exp[-i\omega\tau_D(\boldsymbol{\xi},M)]\times$$
$$\iint_{D_B} d\boldsymbol{\xi}_b(\cos\alpha_S\cos^2\alpha_G)^{-1}|\det\mathbf{N}_{SG}(\boldsymbol{\xi}_b)|\Phi(\boldsymbol{\xi}_b)\times.$$
$$u_o(\boldsymbol{\xi}_b)\exp[i\omega T(\boldsymbol{\xi},\boldsymbol{\xi}_b)]. \quad (16)$$

In equation (16) we have two double integrals, the external has the same mean of the classic true-amplitude Kirchhoff prestack depth migration, i.e. a weighted stack along the Huygens surface in the aperture A, but now using the new weight function w_b for considering also effects of the Gaussian beam stack. It is the innermost operator that represents the Gaussian beam stack, i.e. the weighted stack of the paraxial seismic data in the Gaussian beam stack domain D_B. It is important to emphasize that the position vectors $\boldsymbol{\xi} \in A$ define points in the migration aperture, while the $\boldsymbol{\xi}_b = \boldsymbol{\xi} + \boldsymbol{\delta}$ with $\boldsymbol{\xi}_b \in D_B$ specify points in the Gaussian beam stack domain. In order to fix the important integration domain D_B we consider only $\boldsymbol{\xi}_B$ coordinates that pertain to the domain given by relation (10), i.e. in the proposed Gaussian beam stack process we consider only observed wavefields within the projected Fresnel zone $(|\boldsymbol{\xi}_b - \boldsymbol{\xi}|) \le \boldsymbol{\delta}_F$. The aperture A is considered without taper function.

By using the same methodology proposed by (Cĕrvený, 2000) for calculating the Gaussian beam superposition integral, and using the weight function (13), the migration operator (16) becomes:

$$\widehat{V}(M,\omega) \approx \frac{-i\omega}{2\pi}\widehat{W}(\omega)\iint_A d\boldsymbol{\xi} w_b(M,\boldsymbol{\xi})[v_s(\boldsymbol{\xi})p_{sz}(\boldsymbol{\xi})v_g(\boldsymbol{\xi})p_{gz}(\boldsymbol{\xi})]^{-1}\times$$
$$\left(\frac{2\pi}{\omega}\right)\mathrm{u}_o(\boldsymbol{\xi})\exp\{-i\omega[\tau_F(\boldsymbol{\xi},M)]\}.[$$
$$(17)$$

In the equation (17) the function $\widehat{W}(\omega)$ is included to permit the source wavelet effects, and the phase function $\tau_F(\boldsymbol{\xi},M) = \tau_D(\boldsymbol{\xi},M) - \tau_R(\boldsymbol{\xi})$ is useful for the asymptotic analysis. The pairs (v_s, v_g) and (p_{sz}, p_{gz}) are the P-wave velocities and the vertical components of the slowness vectors at source and receiver positions, respectively. The seismic amplitude is described by the ray theoretical approximation:

$$\mathrm{u}_o(\boldsymbol{\xi}) = \frac{C_t R_c}{G_s}, \quad (18)$$

where C_t, R_c and G_S are the total loss due to the transmissions, the plane-wave reflection coefficient at the reflection point R and the normalized complex-valued geometrical spreading factor, respectively.

Assuming the Hessian matrix $\mathbf{H}_F$ of the second-order Taylor series expansion of τ_F is nonsingular, i.e. $\det\mathbf{H}_F \ne 0$, at the critical point $\boldsymbol{\xi} = \boldsymbol{\xi}^*$, and considering a high-frequency situation $(\omega >> 1)$, the result of the 2-D stationary phase method (Bleistein, 1987) applied to (16) is:

$$V(M,\omega) \approx \widehat{W}(\omega) w_b(\boldsymbol{\xi}^*, M)\frac{[v_s v_g p_s p_g]^{-1} C_t R_c}{G_S\sqrt{|\det\mathbf{H}_F|}}\times$$
$$\left(\frac{2\pi}{\omega}\right)\exp[i\omega\tau_F(\boldsymbol{\xi}^*,M) - \frac{i\pi}{2}(1 - Sgn\,\mathbf{H}_F/2)], \quad (19)$$

being the “Sgn” function called signature and defined by

$$Sgn(\mathbf{H}_F) = \mathrm{sgn}(\lambda_1) + Sgn(\lambda_2), \quad (20)$$

with $\mathrm{sgn}(x) = \pm 1$, for $x > 0$ or $x < 0$, and λ_1, λ_2 the real nonzero eigenvalues of the matrix $\mathbf{H}_F$.

In order to obtain as result the reflection coefficient we need the weight function:

$$w_b(\xi^*, M) = \left(\frac{\omega}{2\pi}\right)[v_s v_g p_s p_g] G_S\sqrt{|\det\mathbf{H}_F|}\times$$
$$\exp[\frac{i\pi}{2}(1 - Sgn\mathbf{H}_F/2]. \quad (21)$$

As a corollary of the above result given by equation (21) the equation (19) reduces in time domain to

$$V(M, t=0) = \begin{cases} R_c W(t), \cdots M = R \\ 0, \cdots M \ne R \end{cases}, \quad (22)$$

For considering there is not loss energy due to transmissions, $C_t = 1$, this is the output of the true-amplitude Kirchhoff Gaussian beam prestatck depth migration (KGB-PSDM) of the seismic data, for a specific point $M = R$ in the earth macro-model.

Comparison with other results

By starting from the Kirchhoff scattering theory, Albertin et al. (2004), (equation 15), presented a Gaussian beam prestack depth migration operator, which provides as result a true-amplitude migration. In order to compare our result with the Albertin's weight function, we rewrite the weight function (21) of this paper by considering that there is no caustic along the ray trajectories. By inserting the definitions of the relative geometrical spreading and of the Hessian matrix $\mathbf{H}_F$ found in Schleicher et al. (1993), (equations B-16 and B-22) in equation (21) we have:

$$w_b(\xi, M) = \frac{\omega v_s v_g p_{sr} p_{gr} h(\xi, M)}{2\pi \,|\, \nabla \tau_s + \nabla \tau_g \,|\, A_s(M,S) A_g(G,M)}. \tag{23}$$

In the equation (23) $h(\xi, M)$ is the well-known Beylkin's determinant; the vectors $\nabla \tau_s$ and $\nabla \tau_g$ are the gradients of the traveltime functions of the two rays that start at the source and at the receiver, respectively, evaluated at the selected depth point M ; and A_s and A_g are the corresponding amplitude factors along the two ray branches.

By comparing the two results (equation 23 above and equation 15 of Albertin's paper) we have:

$$w_b \equiv v_s v_g w_a \, , \tag{24}$$

where w_a is the weight function found by Albertin and co-authors in their paper, for a fixed slowness. In our result the weight function does not depend on the horizontal slowness parameter, because we does not use the slant stack procedures, i.e. the data is transformed direct from time to depth.

Examples

The proposed migration algorithm is applied to the well-known Marmousi data set (Versteeg, 1994). It is a 2-D synthetic data set for a complex structure model, based on increasing listric faults in the center of model, starting from a salt complex structure on the bottom and ending near the top surface.

In the Figure 1 we have the result of our approach applied to the data section with offset 200 m, and for comparison we show in Figure 2 the result of the Kirchhoff migration applied to the same data set. We can see that the three faults in the center of model is better imaged by our approach. This is also true for the reflectors on the right of the most right fault. It is also observed that our approach is more efficient for imaging reservoir structures on the bottom part of the model.

In order to test the efficiency of the proposed true amplitude Kirchhoff Gaussian beam migration, we apply it to a set of synthetic seismic data, corresponding to a constant velocity layer over a half-space separated by a smooth curved synclinal interface. In Figure 3 we have the values of reflection coefficients obtained by our approach (green) and by the true-amplitude Kirchhoff prestack depth migration (K-PSDM). In Figures 4 and 5 we have the migrated seismic data obtained by the KGB-PSDM and K-PSDM, respectively. Both results, in this case, presented a good performance and a light difference in the bottom part of synclinal.

Conclusions

We proposed a new true-amplitude migration algorithm that is a modification of the traditional Kirchhoff prestack depth migration. For that we use a beam stack before migration based on a Gaussian beam superposition integral. The result of our endeavor is a more stable and robust true amplitude prestack depth migration operator, by using weights functions obtained from paraxial ray theory, by attenuating the beam stack border effects. It was successful applied to two set of synthetic seismic data: 1) Prestack depth imaging of the Marmousi data set; and 2) an example of estimation of plane wave reflection coefficients in a synclinal model.

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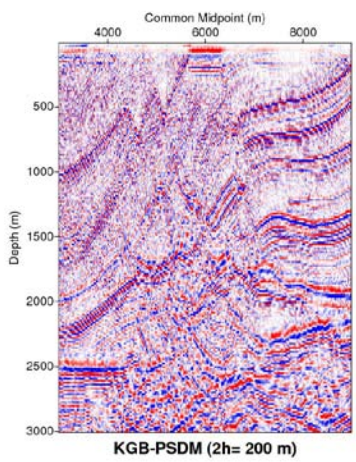

Figure 1:Kirchhoff Gaussian beam prestack depth migration (KGB-PSDM) applied to Marmousi data set with common-offset 200 m. AGC=100%.

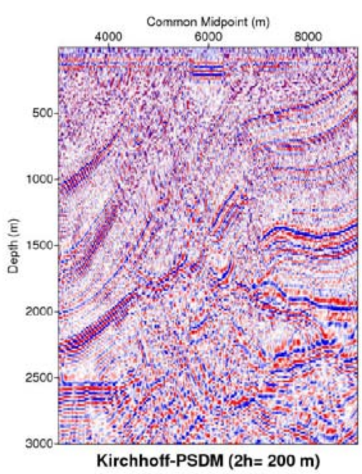

Figure 2:Kirchhoff prestack depth migration (K-PSDM) applied to Marmousi data set with common-offset 200 m. AGC=100%.

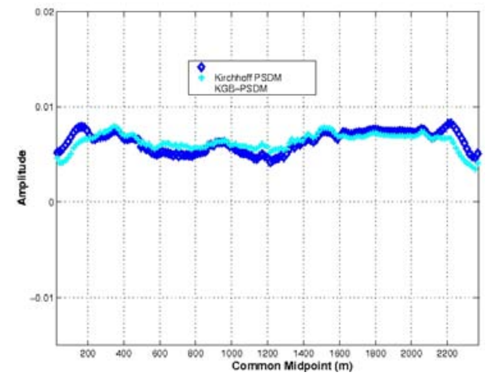

Figure 3: Plane wave reflection coefficients of synclinal model by using the KGB-PSDM (green) and K-PSDM (blue).

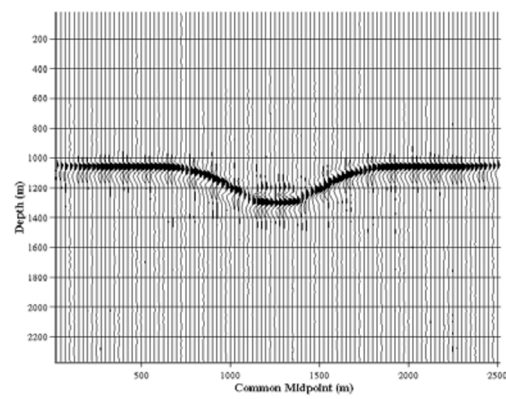

Figure 4: Prestack depth migration of the synclinal model seismic data, obtained by the KGB-PSDM.

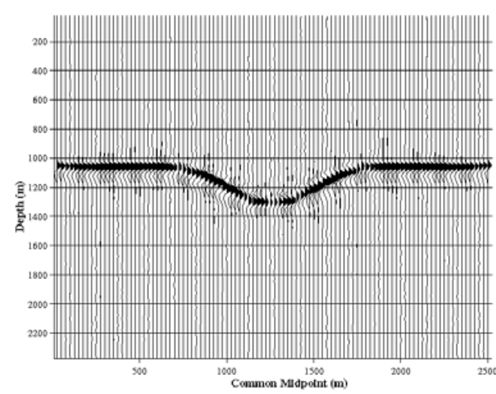

Figure 5: Prestack depth migration of the synclinal model seismic data, obtained by the K-PSDM.