Mixed phase inverse filter and deconvolution

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Abstract

An important problem in seismic data processing is to improve the temporal resolution and to estimate the white reflectivity series (spiking deconvolution). Minimum phase filter is normally applied like conventional procedure in the deconvolution of seismic data. The deconvolution sometimes produces a poor result because the natural wavelet estimate is non minimum phase. This paper shows an analysis procedure with mixed phase filter deconvolution. The main propose to show is how to obtain mixed-phase inverse filters and the corresponding wavelets estimated from a minimum-phase wavelet, It filter is decompose in causal and anti-causal components, also was tested several inversed filter in the data with a parameter in the filter estimation. Finally we tested several wavelets estimated and filters for the deconvolution.

Introduction

Conventional procedure assumption in the filters design and wavelet estimation for the deconvolution of seismic data is: the reflectivity is a random process like white noise, the wavelet is stationary and minimum phase, the signal to noise ratio is large, the autocorrelation function(ACF) of the seismic trace is similar to the wavelet . The z-transform minimum phase wavelet has all the roots outside of the unit circle in its polynomial, and a optimal filter is design to solving the normal equations with the Levinson algorithm, it normal equations also is called Yule-Walker equations. if the wavelet is mixed phase, the filter produces a poor result, if the wavelet is know an optimal pulse shaping filter is applied to the data, but in many cases the wavelet is not know, the mixed phase wavelet has roots inside and outside of the unit circle, solving to problem Porsani and Ursin (1998) used the extended Yule Walker equations where ACF whit lag α on the main diagonal of the filter equations, now the minimum phase filter have a short component , it short filter is converted in a maximum phase component and convolved with a long filter that is minimum phase, resulting a mixed phase filter. For several value α we calculated the roots of the Z-transforms polynomial of the short filter and also selecting combinations of roots placing inside the unit circle, in several tested filter , the filter good is the that result with the perfect spike. We applied the algorithm in synthetic seismic data.

Mixed-phase inverse filter

The seismic wavelet of mixed phase is:

$p_t = \{p_o, ..., p_N\}$ with Z-transform

$$P(Z) = \sum_{t=0}^{N} p_j Z^j \qquad (1)$$

$P(Z)$ has no zeros on the unit circle, *B* zeros inside the unit circle and *N-B* zeros outside the unit circle, also has two component, minimum phase component *A(Z)* and maximal phase component $Z^B B(Z^{-1})$, the pulse is:

$$P(Z) = p_0 A(Z) Z^B B(Z^{-1}) \qquad (2)$$

p_0 is the first sample, where:

$$A(Z) = 1 + a_1 Z + ... + a_{N-B} Z^{N-B} \qquad (3)$$

$$B(Z^{-1}) = 1 + b_1 Z^{-1} + ... + b_B Z^{-B} \qquad (4)$$

Now, the seismic wavelet of minimum phase is:

$$P(Z) = \tilde{p}_o A(Z) B(Z) \qquad (5)$$

Together of the components, *A(Z) and B(Z)* are minimum phase and $\tilde{p}_o$ is the first sample. So the equation 2 and (5) is:

$$P(Z) = \tilde{P} \frac{Z^B B(Z^{-1})}{B(Z)} \qquad (6)$$

We show that the mixed-phase wavelet in the equations (6) is the minimum phase wavelet convolved with an all-pass filter.

The inverse filter of the mixed phase pulse is:

$$H(Z) = \frac{1}{P(Z)} = \frac{Z^{-B}}{p_0 A(Z) B(Z^{-1})} \qquad (7)$$

$$H(Z) = \frac{1}{p_0} C(Z) Z^{-B} D(Z^{-1}) \qquad (8)$$

Where *C(Z)* and *D(Z)* are the inverse of *A(Z)* and *B(Z),* it are two component, the causal and ant causal component, now with the equations (7) and (8)

$$H(Z) = \widetilde{H}(Z) \frac{B(Z)}{Z^B B(Z^{-1})} \qquad (9)$$

We show that the mixed-phase inverse filter in the equations (9) is the minimum phase inverse filter convolved with an all-pass filter.

Mixed-phase filter and change of phase

The mixed phase inverse filter can be computed by solving the EYW equations (extended Yule Walker) with different lags of the ACF on the diagonal of the matrix (Porsani M.J. and Ursin B. 2000), we solve the EYW equations to obtain a decomposition of minimum phase wavelet into a short minimum phase wavelet component of length α and a longer component also with minimum phase, it show that the short wavelet component is:

$$b_t^B = \frac{1}{p_0} \widetilde{p}_t * c_t^\alpha \qquad (10)$$

c_t^α is the component of the minimum phase inversed filter, where α=0 is the minimum phase inverse filter associate to Wiener-Levinson and b_t^B is the Dirac delta.

The Z-transform is:

$$B^\alpha(Z) = 1 + b^\alpha{}_1 Z + ... + b_\alpha^\alpha Z^\alpha \qquad (11)$$

$$B^\alpha(Z) = \prod_{j=1}^{\alpha} \left(1 - \frac{Z}{r_j}\right) \qquad (12)$$

We can to reverse $B^\alpha(Z)$, now is ant causal and of maximal phase , so there is α roots inside of the unit circle , now we can also change the quantity the roots and placing some inside of the unit circle and to vary of phase with $B^B(Z)$ roots.

$$B^B(Z) = \prod_{j=1}^{B} \left(1 - \frac{Z}{r_j}\right) \qquad (13)$$

The equation (13) is applied in the inverse mixed phase filter:

$$H(Z) = \widetilde{H}(Z) \frac{B^B(Z)}{Z^B B^B(Z^{-1})} \qquad (14)$$

And also the mixed phase wavelet estimation is :

$$P(Z) = \widetilde{P}(Z) \frac{Z^B B^B(Z^{-1})}{B^B(Z)} \qquad (15)$$

Simulations with mixed-phase inverse filter

The figure 1 shows the real minimum phase wavelet that has its Z-plane (Z-Transform) with all the roots outside of the unit circle(Robinson E.A. and Treitel S) and it has real and complex roots, complex roots always occurs complex conjugate pair ,also the figure 2 shows the mixed phase wavelet and its Z-plane (Z-Transform) but the roots is inside and outside of the unit circle .

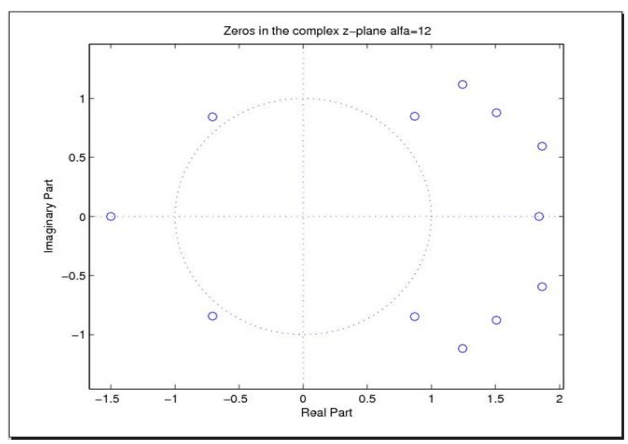

Figure 1 Z-plane of minimum phase wavelet with roots outside of the unit circle

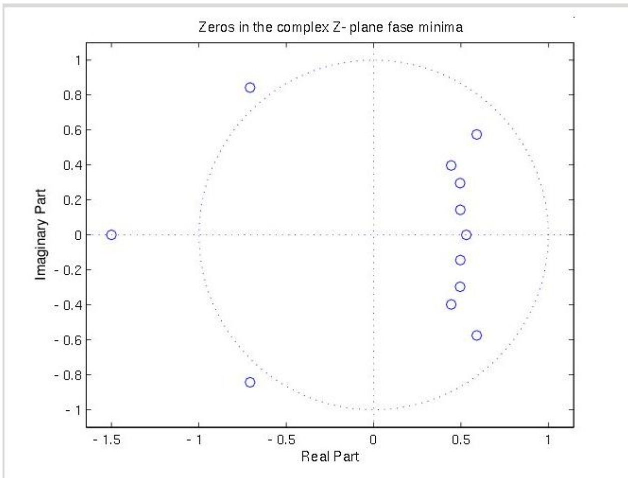

Figure 2 Z-plane of mixed phase wavelet with roots outside and inside of the unit circle

The figure 3 shows the application of a filter of minimum phase. The filter performance very good in the minimum phase wavelet, remember that the real minimum phase wavelet has all the roots outside of the unit circle, also the inverse filter is causal and minimum phase, the result is a perfect spike, but the deconvolution with the filter of minimum phase and the mixed phase wavelet to produces a poor result because it has two component the causal and ant causal component, the filter of minimum phase only has a causal component, the mixed phase and minimum phase wavelet has the same ACF.

The figure 4 show the perfect result of the deconvolution with a filter of mixed phase (equation 8), now it has two component, causal and ant causal, the figure 4 show a perfect spike.

The figure 5 show the performance of the equation 10. We calculated the inverse filter with the Extended Yule-Walquer equation and obtained C^{α}, it is the component of the minimum phase filter, after is convolved with the minimum phase wavelet, the result is a short component of the minimum phase wavelet , if the value is α=0, the result is the minimum phase inverts filter associated to Wiener-Levinson or Yule-Walquer (figure 5), the result is a Dirac delta. In the figure 6 and figure 7, if the value is α=6 and α=9 the component of the minimum phase inverts filter C^{α} is observed, also is observed the component of the minimum phase wavelet.

The figure 8 show that for α=12, that also correspond to roots number of the minimum phase wavelet, the filter C^{α} is a perfect spike and the component B^{α} of the minimum phase wavelet is the minimum phase wavelet.

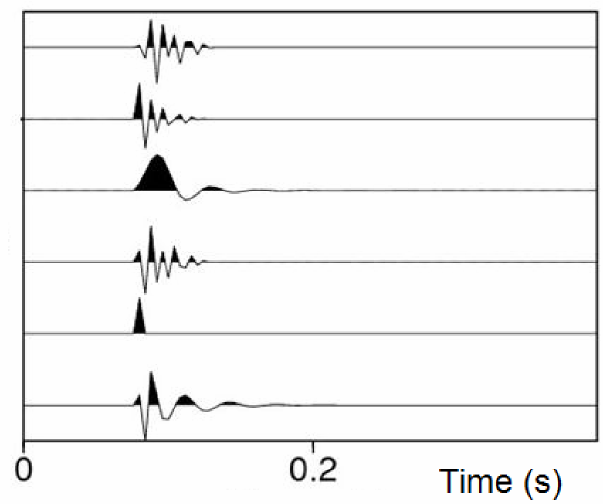

Figure 3 the order of top to bottom is: the mixed phase wavelet, the minimum phase wavelet associated, the ACF, the minimum phase filter, the pulse deconvolved (perfect spike) and the poor result deconvolved with the mixed phase wavelet.

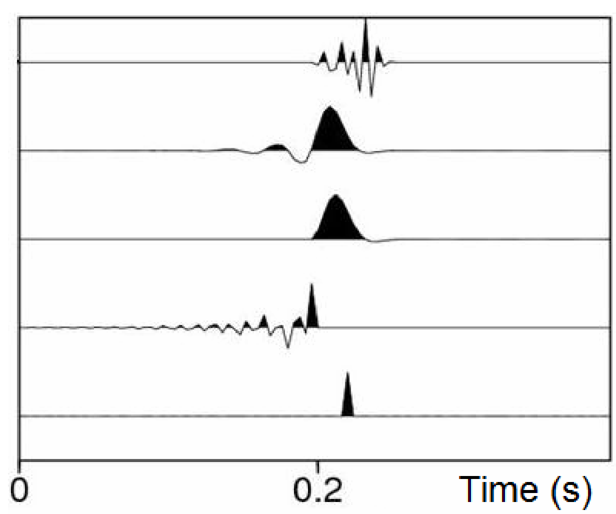

Figure 4 the order of top to bottom is: the mixed phase wavelet, the mixed phase inverse filter, the minimum phase filter component associated, the maximal phase filter component associated, the pulse deconvolved with the mixed phase inversed filter(perfect spike).

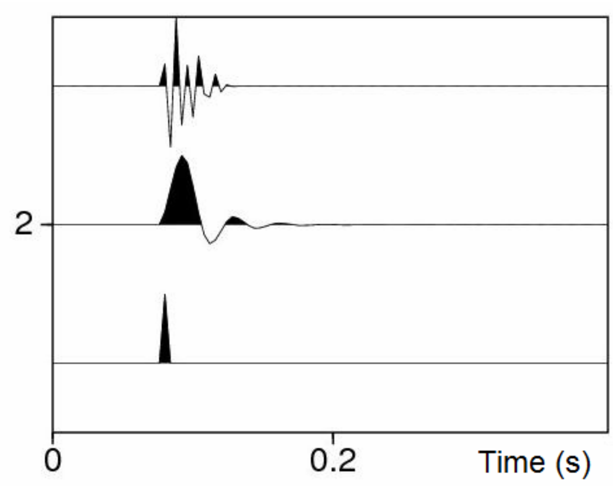

Figure 5 the order of top to bottom is: the minimum phase wavelet associated, the component minimum phase filter of the extended Yule-Walker equations, the result of the convolution that is B^{α},a perfect Spike.

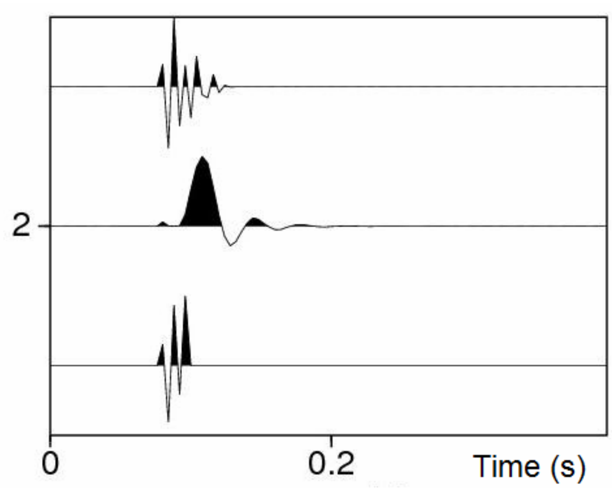

Figure 6 the order of top to bottom is: the minimum phase wavelet associated the component minimum phase filter of the extended Yule-Walker equations, the result of the convolution that is B^{α}.

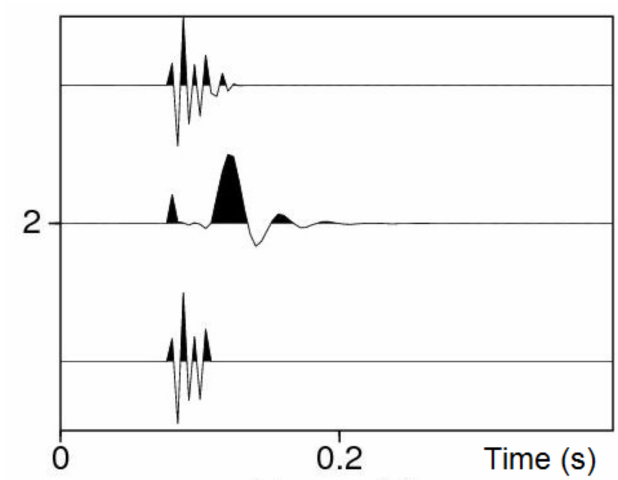

Figure 7 the order of top to bottom is: the minimum phase wavelet associated ,the component minimum phase filter of the extended Yule-Walker equations, the result of the convolution that is B^{α}.

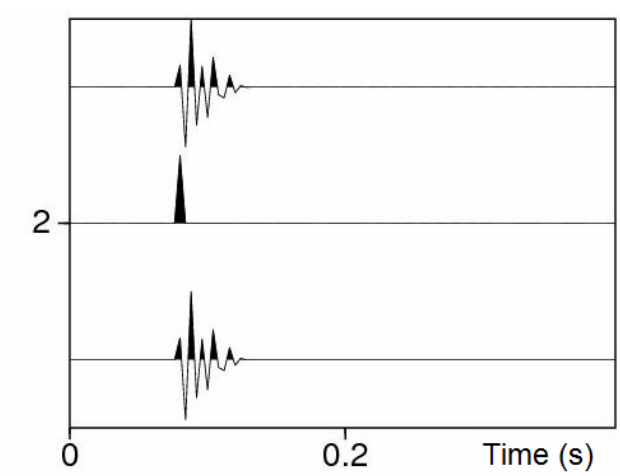

Figure 8 the order of top to bottom is: the minimum phase wavelet associated, the component minimum phase filter of the extended Yule-Walker equations, the result of the convolution that is B^{α}.

Mixed phase inverse filtering and wavelet estimation.

We tested the mixed phase inverse filter with the estimated wavelet (figure 11 and 12), with the seismic trace (convolution between the mixed phase wavelet and the random reflectivity) we estimated the ACF (Yilmaz OÈ. 1987) and the minimum phase wavelet associated, after all the analysis was with the minimum phase wavelet associated.

The figure 9 show the mixed phase wavelet estimation and the mixed phase inverse filter estimation, the result also a perfect spike, we move the B-parameter (equation 14 and 15) of the roots of B^{α} (equation 12) inside the unit circle, also B^{α}-roots can be described by a binary number, for each root the value 0 means outside of the unit circle, and the value 1 means inside of the unit circle, we tested several value and it change the phase of the mixed phase wavelet and the inversed filter, the figure 10 also show the change of the inverse filter and the mixed phase wavelet and the result of the deconvolution.

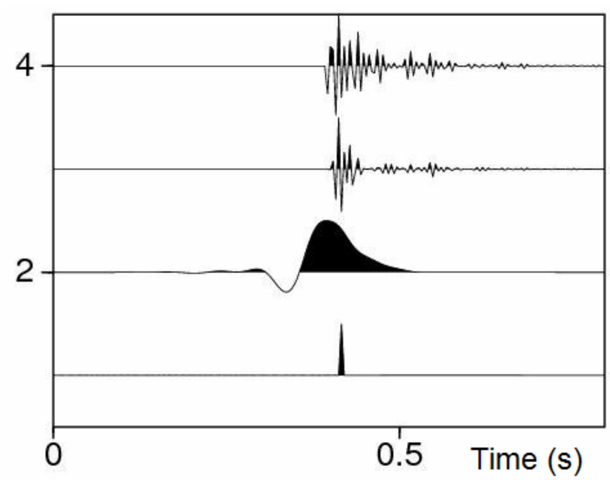

Figure 9 the order of top to bottom is: the minimum phase wavelet, the mixed phase wavelet, the mixed phase inversed filter, the pulse deconvolved with the mixed phase inverse filter.

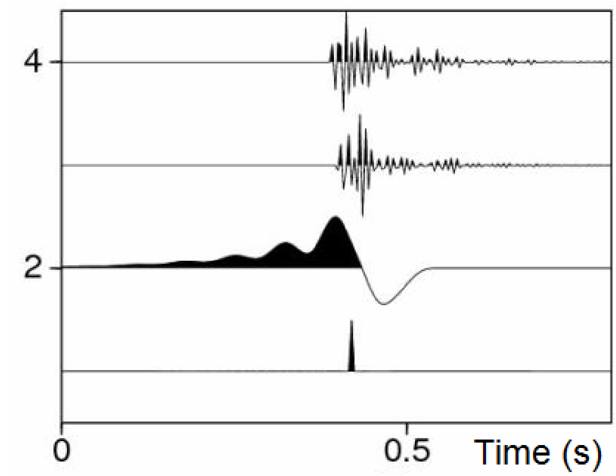

Figure 10 the order of top to bottom is: the minimum phase wavelet, the mixed phase wavelet, the mixed phase inversed filter, the pulse deconvolved with the mixed phase inverse filter.

Conclusions

A study of the algorithm for wavelet estimation and mixed phase filter for deconvolution was applied, and several filter also are generated with different B-parameter, the procedure is a generalization for extended deconvolution with the mixed phase inversed filter. First we applied in a synthetic wavelet and other in the synthetic seismic trace with the minimum phase wavelet associated, only tested the change of phase with the B-parameter and we generated several filter to changing the wavelet estimation and the mixed phase inversed filter, after for further work we estimated the optimal filter for the deconvolution.

Acknowledgments

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