Velocity Analysis by OCO Rays

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Abstract

In general, time-imaging algorithms make use of RMS velocity field supposing the subsurface media is smooth enough to be represented by an equivalent media of constant velocity, in such a way that there is no big differences between the traveltimes in the equivalent media and the real media. Moreover, OCO rays are virtual rays used to map reflection events between seismic sections by offset continuation. In this manner, the main objective of this paper is providing a procedure to examine, by OCO rays, homogeneous and inhomogeneous velocity distributions aiming the definition of an equivalent media. In inhomogeneous case, the heterogeneity will be characterized by a velocity function and a certain group of parameters. Varying those parameters, it's possible to trace OCO rays for certain picked points in the seismic section in another sections, and, from this rays, to evaluate the tested parameters by a coherence criteria. This paper presents the basic concepts and necessary equations to the methodology, as well as the procedure for parameters evaluation. Numerical examples are also provided.

Introduction to OCO Rays

Let T be a point with coordinates (ξ_1, τ_1) picked in a reflection curve present in a 2D common-offset section with half-offset h_1 named here starting section, and suppose the depth point (x, z) that generated the event in point T also generates a reflection event in another common-offset section with half-offset h_2 named here target section. By an algorithm of offset continuation, it is possible to find the coordinates (ξ_2, τ_2) of the point T in the target section by means of a constant velocity transformation from the starting section to the target section. This procedure can be repeated for N different velocities, generating N points with coordinates (ξ_2, τ_2) in the target section. These points are designated image points and the line connecting such points is the OCO ray associated with point T in the target section. The OCO rays are presented originally by Filpo (2005). They are virtual rays representing the trajectory of a point in the seismic section when the imaging velocity is altered and they receive that designation because they are obtained by an offset continuation procedure. In this way, it's possible to infer that the intersection of the OCO ray with the reflection curve in the target section will provide the velocity, a RMS velocity kind, of the equivalent media for the picked point in the start section. This procedure is described in details by Pinheiro (2008) and can be used not only to obtain a constant velocity equivalent media, but also equivalent models with certain velocity distribution. In this last case, it is possible too, by OCO rays, to obtain equivalent models that take into account a heterogeneous distribution of velocity by the same criteria used for homogeneous media. In this paper, the OCO rays are used to test models with gradient of velocity and gradient of the square of slowness. Detailed studies about OCO rays and its applications, including the heterogeneous case, can be found in Pinheiro (2008).

General Procedure to Homogeneous Media

The process of velocity analysis using OCO rays can be interpreted as a cascaded application of analytical procedures of migration and demigration. The first step to apply these procedures is to interpret reflection-time curves of the same horizon in two common-offset seismic sections, defining one of them as the starting section and the other one as the target section. The next step is to choose the range of velocities to be tested. Once a reflection point with coordinates (ξ_1, τ_1) is chosen on the starting section, and supposing the dip ϕ_1 in that point is known, for each one of the N tested velocities it should calculate lateral $(\Delta\xi_1)$ and vertical (Δz) displacements from ξ_1 of the depth point where the reflection occurred. For homogeneous models this is analogous to solve the follow equations:

$$\phi_1 h_1^2 \Delta\xi_1^2 + \left(h_1^2 \tau_1 - \frac{V^2 \tau_1^3}{4}\right) \Delta\xi_1 - \frac{\phi_1 V^4 \tau_1^4}{16} = 0, \tag{1}$$

$$\Delta z = \sqrt{\left(\frac{V^2 \tau_1^2}{4} - h_1^2\right)\left(1 - \frac{4\Delta\xi_1^2}{V^2 \tau_1^2}\right)}. \tag{2}$$

The next step is to obtain the traveltime τ_2 where the reflected energy on the migrated point will be watched when a demigration using a half common-offset h_2 is carried out with same velocity V used in the migration. Now, the equation to be solved is

$$\frac{V^4 \tau_2^4}{16} - \frac{\left(2h_2^2 + \Delta z^2 + \theta^2 \Delta z^2\right) V^2 \tau_2^2}{4} + h_2^2 \left(h_2^2 + \Delta z^2\right) = 0. \tag{3}$$

where θ is the reflector's slope on migrated point, which is defined as

$$\theta = \frac{\Delta\xi_1}{\Delta z}\left(\frac{4h_1^2}{V^2\tau_1^2} - 1\right). \tag{4}$$

The lateral displacement $\Delta\xi_2$ from migrated point at time τ_2 is given by

$$\Delta\xi_2 = -\frac{V^2\tau_2^2\Delta z}{V^2\tau_2^2 - 4h_2^2}\,\theta. \tag{5}$$

The coordinate ξ_2 of the reflection event observed in τ_2 can be evaluate using the relation

$$\xi_2 = \xi_1 + \Delta\xi_1 - \Delta\xi_2. \tag{6}$$

All the analytical developments to obtain the equations from (1) to (6) is described by Pinheiro (2008) and will not be exposed here. These equations can be applied for a group of N velocities, producing N points with coordinates (ξ_2,τ_2) in the target section that, when connected by a line, generate the OCO ray associated with point (ξ_1,τ_1) selected on the starting section. The correct RMS velocity of equivalent media corresponds to the position where the OCO ray intercepts the reflection-time curve present in target section.

Obtaining Time Apparent Dip

Obtaining the time apparent dip ϕ_1 composes a fundamental stage to accomplish the analysis. The chosen form to obtain ϕ_1 in (ξ_1,τ_1) was to use coherence analysis, where several dips inside a range are tested, so that the correct dip is the one with maximum coherence value. This analysis is accomplished considering the group of samples located in the picked point's neighborhood. The dip that better aligns those data provides the largest coherence value and it will be the apparent dip considered in (ξ_1,τ_1). This procedure is described in details by Pinheiro (2008) and it is based on the covariance matrix formation presented by Gersztenkorn and Marfurt (1999).

Velocity Evaluation

A fundamental procedure to attain effectiveness and robustness in the method is the evaluation of each tested velocity. It is essential to evaluate automatically which velocity produced the better alignment of the image point (ξ_2,τ_2) with the reflection-time curve located in the target section. The scheme proposed to achieve this computation with the mentioned characteristics is measuring the similarity (coherence) between samples in the selected point's neighborhood (in the starting section) and the image point's neighborhood (in the target section) for each velocity. This coherence analysis is based on trace-to-trace semblance algorithm, that can be used to evaluate parameters of heterogeneous velocity models too. Detailed information about this stage are brought to light in the work of Pinheiro (2008).

Application to Heterogeneous Media

The velocity analysis by OCO rays to heterogeneous models follows, generally speaking, the same procedure previously indicated to homogeneous models. Pinheiro (2008) examines the models with gradient of velocity, where the velocity in a depth point (x,z) is given by

$$V = V_R + a_1(x - x_R) + a_3(z - z_R), \tag{7}$$

and those ones with gradient of the square of slowness, at which the velocity distribution along (x,z) is given by

$$\frac{1}{V^2} = \frac{1}{V_R^2} + a_1(x - x_R) + a_3(z - z_R). \tag{8}$$

In the equations (7) and (8), V_R is a reference velocity located in (x_R,z_R), a_1 is the lateral gradient and a_3 is the vertical gradient. In both models, the parameters characterizing the medium are (V_R,a_1,a_3).

In terms of methodology, the main difference involving heterogeneous models is that now the analysis has not only a scan over a range of velocities, but also a scan over a range of gradient values, in each chosen direction. Accordingly, the coherence analysis based on trace-to-trace semblance evaluates which parameters (V_R,a_1,a_3) produce the best estimative of heterogeneous equivalent medium for one picked point in the starting section.

Both mentioned heterogeneous models have analytic solutions obtained from ray tracing theory that the method accounts for. The main equations are found in Červený (2001), but complete analytical developments are presented by Pinheiro (2008), where are elaborated equations for migration, demigration, components of slowness, time and depth apparent dips, etc. All the expressions are obtained from a simple reflection event, considering a ray propagating from source to interface, and from this one to receiver. The model of equation (7) is probably the most popular case in seismology, while the most simple analytic description is obtained to the model of equation (8), since it have being put in terms of the parameter along the ray, what is defined by relation

$$\sigma = \int_0^\sigma V^2 d\tau. \tag{9}$$

Pinheiro (2008) also shows it is possible to calculate the parameter σ provided only the initial and final ray positions, what allows to evaluate the RMS velocity along the trajectory between these points as

$$V_{RMS}^2 = \frac{\int_0^\sigma V^2 d\tau}{\tau} = \frac{\sigma}{\tau}. \tag{10}$$

Application on Synthetic Data

The OCO rays are applied to obtain the RMS velocity from a horizon of 2D synthetic dataset generated by Kirchhoff modeling. Figure 1 shows the heterogeneous synthetic depth model. Initially, two common-offset sections are selected to perform the analysis. Figure 2 shows the seismic dataset referred as starting section, whose half-offset is $h_1 = 300$m, while figure 3 illustrates that one considered as target section, whose half-offset is $h_2 = 800$m. All points of the selected horizon are presented connected by a line on figure 2. The velocity for each picked point is varied from 1.4 km/s to 3.0 km/s. The increment of velocity is 0.01 km/s and the number of velocity steps is $N = 161$.

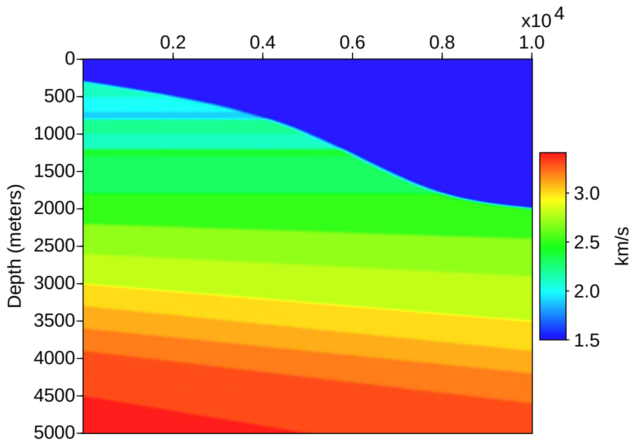

Figure 1: Synthetic model.

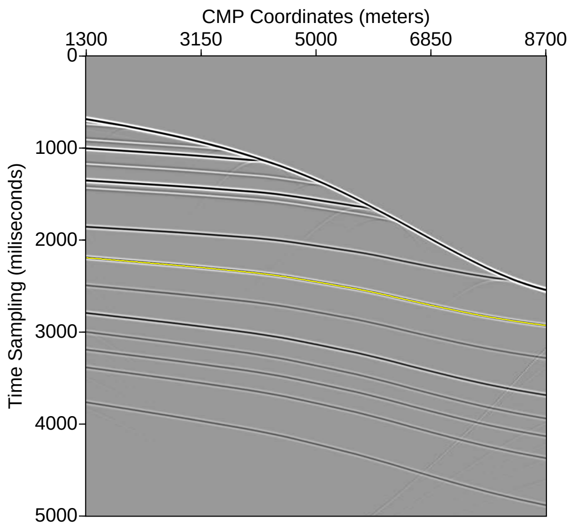

Figure 2: Synthetic data for starting section with half-offset $h = 300.0$ m. The line is marking the horizon's picked points.

The analysis starts with evaluation of dips on horizon's picked points by coherence algorithm. Consequently, varying velocity and using equations from (1) to (6) it is possible to obtain the coordinates (τ_2, ξ_2) of N image points for each picked point and to trace the respective OCO ray connecting the image points by a line. For the evaluation of RMS velocity by trace-to-trace semblance, a dataset around each picked point enclosing a relatively small subvolume with 11 traces and 31 samples was selected. Since the velocity is varied, these subvolume moves throughout the target section with each image point. The coherence is estimated between these subvolume and another subvolume around the image point, with same size and shape, but now enclosing dataset collected from target section. Figure 5 displays the maximum coherence value

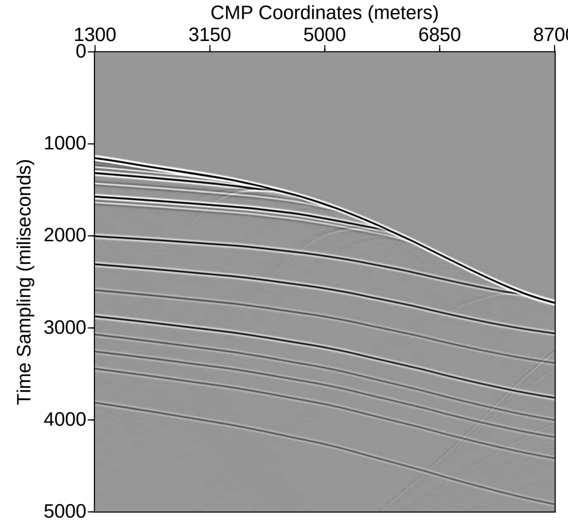

Figure 3: Synthetic data for target section with half-offset $h = 800$ m.

obtained for each point marked on figure 2. The respective RMS velocities associated with each maximum coherence measurement are showed on figure 4.

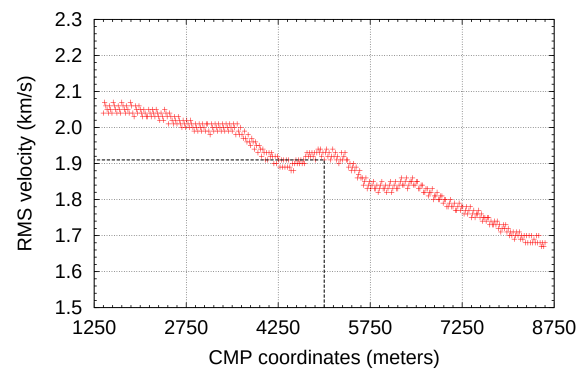

Figure 4: RMS velocities obtained for picked points on starting section.

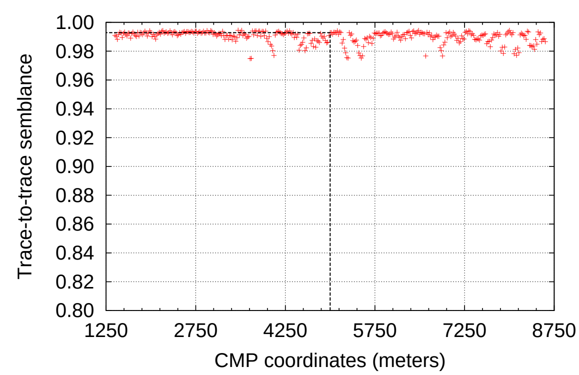

Figure 5: Maximum coherence values obtained for picked points on starting section.

Moreover, figures 4 and 5 also point out results for one picked point in starting section, located on coordinates $\xi_1 = 5000.0$m and $\tau_1 = 2452.0$ms. For this point, figure 6 illustrates the coherence values obtained for each tested velocity with the respective subvolume of dataset around it. From same figure, the maximum coherence is 0.9928 for $V_{RMS} = 1.91$km/s. This coherence value is pointed out on figure 5, and the respective RMS velocity is pointed out on figure 4.

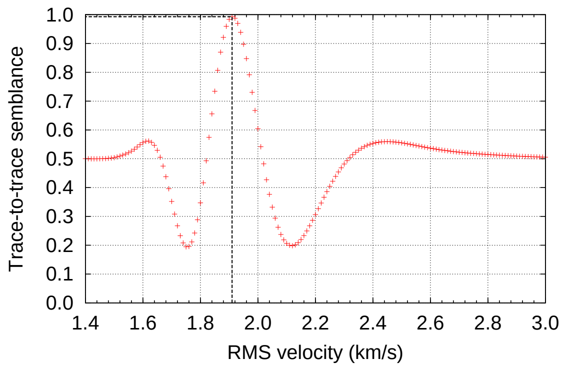

Figure 6: Coherence values obtained for OCO ray of picked point in $\xi_1 = 5000.0$m and $\tau_1 = 2452.0$ms.

The OCO ray associated with results of figure 6 is illustrated on figure 7, where is presented only the near area of the OCO ray and $V_{MIN} = 1.4$km/s and $V_{MAX} = 3.0$km/s. The maximum coherence shows up when the data around picked point are aligned with data from horizon located in target section.

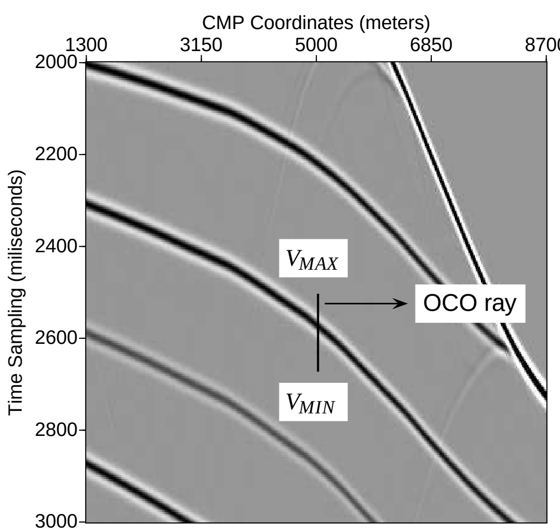

Figure 7: OCO ray associated with picked point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ in starting section. This is an ampliation of first target section $(h_2 = 800\,\text{m})$.

The previous velocity analysis considering homogeneous case also could be achieved considering the heterogeneous media of equations (7) and (8) using the analytic developments obtained by Pinheiro (2008). The main difference lies how the coherence results are presented. If, in addition to velocities, values of one gradient parameter are tested, the measures are presented in a velocity by gradient coherence plane. And in the case of two gradient parameters, the measures are presented in a coherence cube. Furthermore, increasing parameters to be tested increases the number of demanded target sections. A discussion about this characteristics can be found in Pinheiro (2008) too.

As an example of application to heterogeneous media, a velocity analysis is performed to the same point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ previously analyzed for homogeneous case. A medium with vertical gradient of the square of slowness is the chosen velocity distribution, which is equivalent to consider $a_1 = 0$ in equation (8). Accordingly, the parameters to be tested are reference velocity V_R and vertical gradient a_3. It is assumed that V_R is located in $(x_R, z_R) = (0, 0)$ and varied from 1.4km/s to 3.0km/s in steps of 0.02km/s. The vertical gradient is varied from $-2 \times 10^{-3}\,\text{s}^2/\text{km}^2/\text{km}$ to $-2 \times 10^{-1}\,\text{s}^2/\text{km}^2/\text{km}$ in steps of $2 \times 10^{-3}\,\text{s}^2/\text{km}^2/\text{km}$.

Figure 8 displays the coherence panel to the tested parameters using the target section of figure 3. However, the figure shows a lot of values with high coherence. To reduce the quantity of high coherence measurements, the analysis can be applied considering another target section, with different offset. Figure 9 displays the synthetic data for a target section with half-offset $h_2 = 1300.0$m.

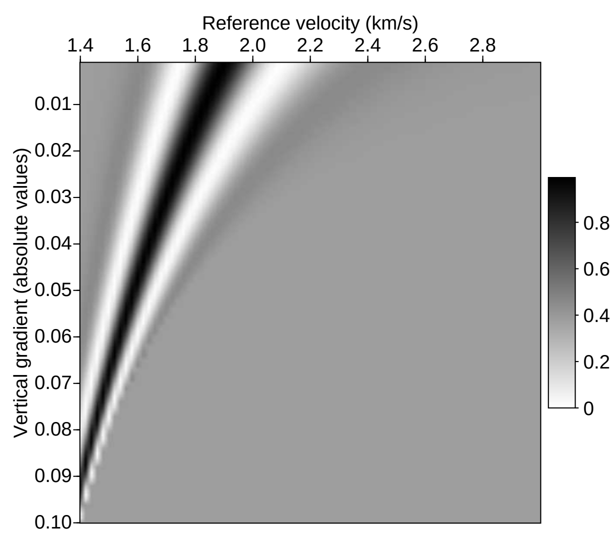

Figure 8: Coherence panel of the picked point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ considering now the variation of parameters V_R and a_3 of a medium with vertical gradient of the square of slowness. The vertical gradient in shown in absolute values, in $\text{s}^2/\text{km}^2/\text{km}$. The half-offset of the respective target section is $h_2 = 800$m.

Figure 10 exhibits the coherence panel to the tested parameters using the same target section of figure 9. The figure shows a lot of values with high coherence again, but located in a shorter group of values. The final coherence panel can be attained multiplying the panels of figures 8 and 10. The final result is illustrated by figure 11, where

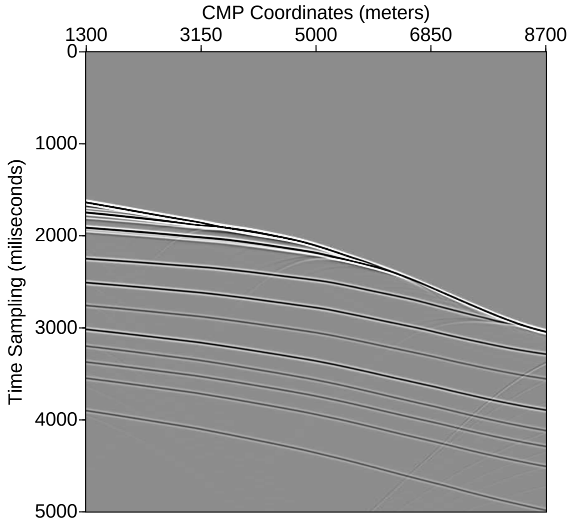

Figure 9: Synthetic data for target section with half-offset $h = 1300\,$m.

all coherence values smaller than 0.9 was clipped. It can be observed from figure 11 that just a fewer group of parameters remains with high coherence values. It is possible to descrease the number of parameters with high coherence values using another target section. The highest coherence obtained from panel of figure 11 was 0.9828 to $V_R = 1.52\,$km/s and $a_3 = -1.28 \times 10^{-1}$s^2/km^2/km.

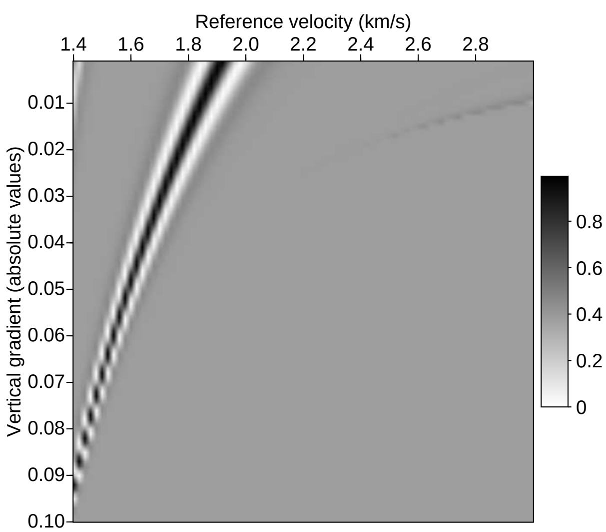

Figure 10: Coherence panel of the picked point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ considering now the variation of parameters V_R and a_3 of a medium with vertical gradient of the square of slowness. The vertical gradient in shown in absolute values, in s^2/km^2/km. The half-offset of the respective target section is $h_2 = 1300\,$m.

One OCO ray for $a_3 = -1.28 \times 10^{-1}$s^2/km^2/km is illustrated on figure 12, where is only presented, on target section of figure 9, the near area of the OCO ray and $V_{MIN} = 1.4\,$km/s and $V_{MAX} = 3.0\,$km/s. The maximum coherence shows up

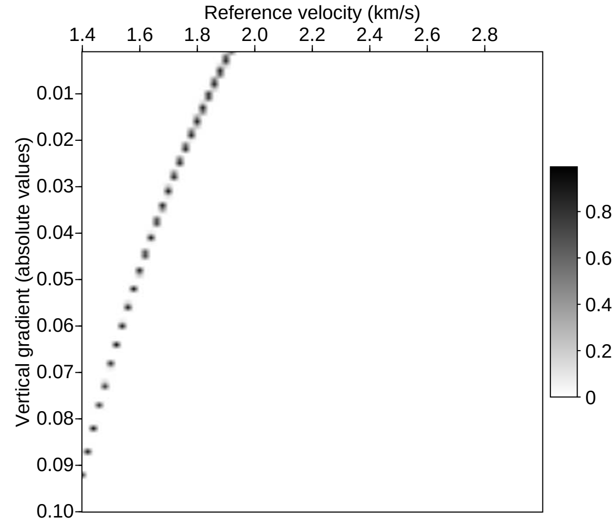

Figure 11: Final coherence panel of the picked point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ considering the variation of parameters V_R and a_3. The vertical gradient in shown in absolute values, in s^2/km^2/km. Only coherence values bigger than 0.9 are shown.

when the data around picked point are aligned with data from horizon located in target section, what happens when $V_R = 1.52\,$km/s.

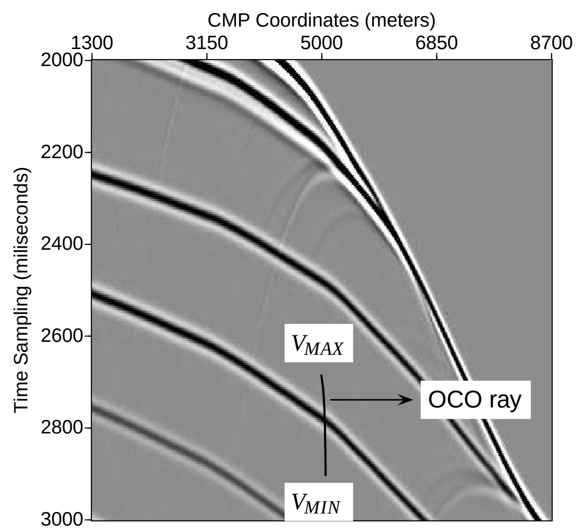

Figure 12: OCO ray associated with picked point $(\xi_1, \tau_1) = (5000.0, 2452.0)$ in starting section obtained for $a_3 = -1.28 \times 10^{-1}$s^2/km^2/km. This is an ampliation of second target section $(h_2 = 1300\,\mathrm{m})$.

Conclusions

This paper illustrates which stages are necessary to perform a velocity analysis using OCO rays, details the concepts originally presented by Filpo (2005), and incorporates coherence procedures in the original methodology.

The velocity analysis by OCO rays was successfully applied to 2-D synthetic seismic data and it can be used to obtain equivalent media of constant velocity with effectiveness and robustness. Besides, the methodology based on OCO rays also can be fully expanded to heterogeneous models with analytic distribution of velocity, as well as to 3-D seismic data, including the heterogeneous case. As the OCO rays are generated by means of analytic expressions, the full methodology can test a large amount of parameters fastly and accurately for each type of considered velocity distribution.

Furthermore, the method requires only the interpretation of horizons and $N_p + 1$ common-offset seismic data at least, where Np is the number of parameters to be tested. The evaluation of velocities and gradient parameters is performed automatically and does not require a large amount of seismic dataset. Consequently, it doesn't demand great computational resources. Also, the analysis by OCO rays can be refined by inclusion of more horizons, decreasing the increment of the parameters to be tested or considering additional seismic sections at any time.

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