Nondeterministic Atomic Meshes

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Abstract

This article describes a new approach for the placement of the vertices of a finite element mesh for seismic images based in guide tables and gamma correction. A genetic algorithm moves mesh vertices considering the relative positioning between them and the underlying image. The meshes handled here are in the same category as the atomic meshes, that are induced by putting atoms, as if they were charged, assembling a triangle mesh for flow simulation. The density and position of these points tend to be aligned with image features.

Introduction

Finite element meshes (FEMs) created from image or volume features have many applications, such as flow simulation. A triangle or tetrahedron mesh may have its quality measured by the adequacy to the underlying features of interest and by the quality of the component elements.

Hale (6; 7) introduces the atomic meshes, a physical approach for the creation of FEMs, generated from the positioning of atoms that, indirectly, show a kind of segmentation of the image used.

Papers presented by Hale (6; 7) and by Esperança, Oliveira e Cavalcanti (4) show deterministic approaches to place the atoms that induce a mesh. Moreover, the way they move atoms in order to minimize a total potential energy made us question the need to move all the atoms.

Motivated by those observations, we propose a new nondeterministic method to put atoms on the image, suggested by a non-uniform discrete probability generator, modified to favour features of interest according to progressive gamma corrections. This new method is compared to the presented in (4). By observing the randomness of the atoms movement, we propose the usage of a genetic algorithm to achieve the same goal. Such algorithm has a simplified functioning of mutation and crossover, and considers energy gradients to minimize the total potential energy. This approach is numerically robust, and uses the modified energy formulation proposed in (4).

Related work

Hale (6) demonstrates a FEM generation technique based on charged particles (atoms). These atoms are arranged to produce a lattice using a triangulation process (like Delaunay), in which vertices density is adequate to density of features in the image. Moreover, a subset of the atoms forming the lattice is placed on the image features. Hale advocates that the process of mesh generation, replacing the image analysis process, follows the sequence: (i) process an image to enhance features of interest, (ii) fill space with a computational mesh aligned with image features and (iii) simulate some process on the space-filling mesh.

Hale's *atomic images* method can be computed in three steps:

1. Fill the space spanned by the image with a pseudo-regular lattice of atoms. The nominal distance between atoms varies smoothly, consistent with the density of features in the image;

2. Move atoms to minimize a total potential energy, defined to be a weighted sum of an atomic potential energy and an image potential energy;

3. Connect the atoms using Delaunay (or some other) triangulation to form a mesh.

Energy formulation

The force between two atoms is given by the function:

$$f(u) = \begin{cases} \frac{9}{8} - \frac{19}{8}u^2 + \frac{5}{4}u^3, & 0 \le u < \frac{3}{2} \\ 0, & \frac{3}{2} \le u, \end{cases} \qquad (1)$$

where u is the normalized distance between atoms, defined by:

$$u \equiv \frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{d}. \qquad (2)$$

The parameter d in Equation 2 refers to the cover distance of an atom, i.e., the distance beneath no other atom can be placed. The cover distance may be defined as a constant value or as a function of the underlying image pixel.

The resultant force incident to each atom is calculated using finite differences in a (scalar) potential field. In each image pixel, this field is calculated as:

$$\phi(u) = \begin{cases} \frac{153}{256} - \frac{9}{8}u + \frac{19}{24}u^3 - \frac{5}{16}u^4, & 0 \le u < \frac{3}{2} \\ 0, & \frac{3}{2} \le u. \end{cases} \qquad (3)$$

The scalar potential function $\phi(u)$ is used to define the atomic potential field given by:

$$a(\mathbf{x}_i) = \sum_{j=1}^{n} \phi \left[\frac{\|\mathbf{x}_i - \mathbf{x}_j\|}{d(\mathbf{x}_j)} \right]. \quad (4)$$

Figure 1 illustrates force and potential functions.

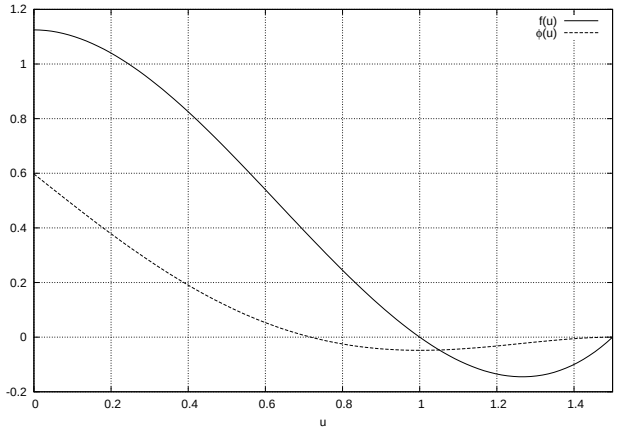

Figure 1: Force and potential functions.

Hence, the atomic potential energy is defined as:

$$A = A(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n) = \frac{1}{2} \sum_{i=1}^{n} a(\mathbf{x}_i). \quad (5)$$

In an analogous form, the image potential energy is defined as:

$$B = B(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n) = \sum_{i=1}^{n} b(\mathbf{x}_i), \quad (6)$$

where each element $b(\mathbf{x})$ is the image potential field at position $\mathbf{x}$. Hale (6; 7) suggests a previous processing of the image in order to enhance the interesting features and. Then, mapping from pixel values to image potential field values is straightforward.

Hence, the total potential energy is given by:

$$P = P(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_n) = (1-\beta)A + \beta B, \quad (7)$$

The initial position of atoms corresponds to positions of vertices of a regular crystal lattice. This initial lattice minimizes (locally) the atomic potential energy, since it is regular, and is consistent with the nominal distance function $d(x)$. The optimization step is, afterwards, in charge of moving atoms in such a way that atoms can be better positioned relative to image features, minimizing the image potential energy, and keep the lattice regularity.

Esperança, Oliveira and Cavalcanti (4) suggest a different approach for the initial lattice of atoms. They advocate that there is an intrinsic order in the positioning of atoms over the image. In their method, called *projection*, each atom is put as suggested by Hale in (6) with distance between atoms as $\frac{\sqrt{3}}{3} d(x)$. A point in which the image potential energy is maximal is searched in the spherical neighborhood of each atom of radius $d(x)$. Each pair (initial position, best neighbor) is put in a priority queue and the priority is the distance between the initial position and the best neighbor. Each popped pair is used to project the atom on the image, if possible. If the projection of an atom *a* is possible, then no other atom *b* can be put in the spherical region around *a*, with variable radius depending on the image potential field in that position. Initially, many atoms are put directly over the features of interest, and other are put elsewhere, respecting the density of features, diminishing the total potential energy comparatively to (7).

In (4), atom movement is governed by a different formulation of potential energy, relative to atomic potential energy, given by:

$$A = \kappa \left(\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \phi \left[\frac{|x_i - x_j|}{d(x_j)} \right] \right) + (1-\kappa) \sum_{i=1}^{n} \frac{\mathscr{L}(x_i)}{d(x_i)}, \quad (8)$$

where the factor $\mathscr{L}(x_i)$ corresponds to what they call "laplacian coordinates" of the point relative to its edge neighbors, determined by the Delaunay triangulation induced by the atoms. Their "laplacian coordinates" is formulated as:

$$\mathscr{L}(\mathbf{v}_i) = \mathbf{v}_i - \frac{1}{|\mathscr{N}(\mathbf{v}_i)|} \sum_{\mathbf{v}_j \in \mathscr{N}(\mathbf{v}_j)} \mathbf{v}_j. \quad (9)$$

The suggested value for the weight κ is 0.7. This atomic potential energy formulation tends to enhance the lattice regularity.

For a different propose, Machado (8) uses a nonuniform discrete probabilities generator to select candidate points of mesh that tends to be aligned with faults in a processed seismic datum. This approach, based on the "growing neural gas" algorithm, also suggests that there is an intrinsic order in selecting points in the image to put atoms onto.

Proposed method

Setting atoms initial placement

The method of guide tables was chosen to select a new location for an atom. Devroye (3) and Machado (8) show that the performance of this method for selecting a sample in a population, guided by its probability, is superior to other inversion methods.

Initially image or volume samples are transformed to probability values summing one. The positions of new atoms are chosen based on the probability of the underlying pixel. When a new atom is placed, it determines a region where no other atom can be placed inside. The radius of this region depends on the image potential field. There are many ways to deny the placement of new atoms near to existent atoms. One of them is to set zero to the probability in the spherical vicinity, but it would cause the reconstruction of the guide tables. So, we chose to store a circular structure for each atom. Hence, the placement of a new atom occurs if the selected position is in none of the stored circular regions.

If only one image is used, few atoms are placed over the features of interest, and then, the initial total potential

energy tends to be high. Also, some triangles tend to be thin. Therefore, the proposed method generates some derived "probabilities images" where the distribution of these probabilities becomes more uniform as illustrates the Figure 2. These images correspond to a normalization process applied after growing gamma corrections. In Figure 3 we used gamma with values 0.25, 0.5, 1.0, 2.0 e 4.0. The placement of atoms using the first and second images reinforce image features, and using the last two make atoms to be placed more uniformly.

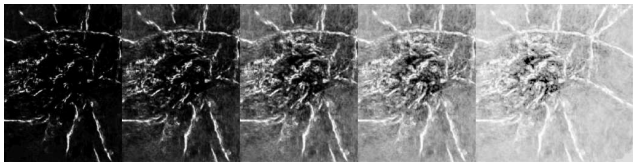

Figure 2: Normalization applied to images with gamma corrections of Figure 3.

We use one guide table per normalized image after adequate gamma correction.

Gamma correction is a technique that changes the intensity of component signals, in an image, for example. If the correction has a value greater than one, then the resulting image will have more luminance (lighter). If this value is less than one, then the original image will become darker. Gamma values are positive and different from zero. This correction can be given by the expression:

$$S_o = S_i^{1/\gamma},$$

where S_i and S_o are input and output signals, respectively. The Figure 3 shows the effect of different values of gamma correction applied to a gray scale image. Figure 4 shows the plots of gamma corrections for the sequence of images.

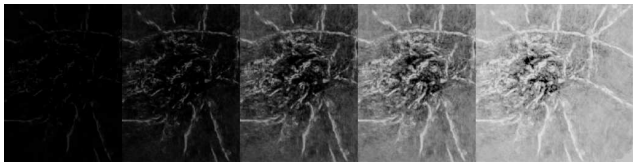

Figure 3: Gamma correction applied to image. Original is centered, adapted from Hale (6).

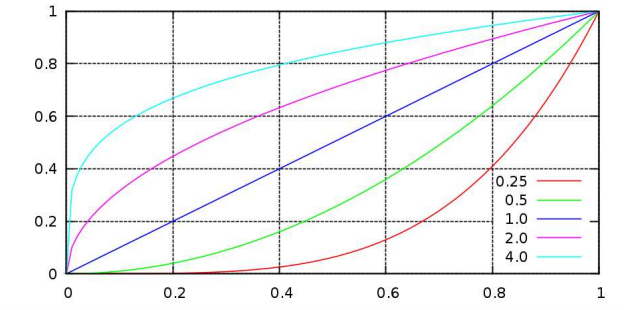

Figure 4: Plots of graphs of gamma corrections used in images of Figure 3

Input: A gray scale image I with enhanced features, the maximum quantity of atoms to be placed m and maximum number of successive failures placing an atom f.
Output: a set of atoms A
01. Generate, from the input image, a list of n guide tables T
02. Make A empty
03. $pos \leftarrow 1$
04. While $((pos \leq n)$ **and** $(|A| < m))$, do:
05. Let x be the corresponding position to the probability generated following $T[pos]$
06. If an atom can be placed at x on the image:
07. Make failure counter be zero
08. Insert the atom in A at position x reserving space around it
09. Else:
10. Increase failure counter by one
11. If failure counter is greater than f:
12. pos ← pos + 1
13. Return A

Figure 5: Algorithm for initial placement of atoms

Figure 5 resumes the proposed algorithm for the initial placement of atoms, detailed afterwords.

The generation of guide tables is as follows. The image is initially converted to an array of probabilities summing one. This array is then adjusted using gamma corrections, and the result is stored. The values of these corrections are increasing. After each correction, the values are normalized again making them sum one. For each array of probabilities created, a guide table is created as in (3). If this normalization were applied directly to the images of Figure 3, the result would be as shown in Figure 2.

When the limit of successive failures is reached, a guide table with greater value of gamma correction is then used to generate probabilities. By doing this procedure, we initially place atoms on the features of interest – increasing the image potential energy – and, after an amount of failures, we cover the whole image with atoms more regularly spaced – increasing the atomic potential energy and the regularity of the lattice.

Mesh optimization

In (6; 7) an optimizer is in charge of minimizing the total potential energy by moving atoms. Before executing the optimizer, all atoms forming the lattice are displaced up to 10% of the nominal interatomic distance in a random direction. The process of moving atoms and then optimizing the configuration is run until no significant enhancement is reached. The same process occurs in (4). In (1) steepest descent is used to optimize the lattice after the same random moves.

In this work, we propose the usage of a genetic algorithm to optimize the lattice. Each individual in a population of solutions is formed by the positions of all atoms generated by the earlier process of placement. A detailed description of genetic algorithms can be found in (5). Each time a new generation is created, the solution with best fitness,

that means with less total potential energy, is selected. The individuals of the next generation descend from the best solution. The evolution of a population (a generation) occurs when crossovers and mutations take place.

By crossover we mean the exchanging positions of (identified) atoms between two selected distinct solutions. The Figure 6 illustrates this process. A crossing position is selected (an index of the array of atoms). From that position to the end of the both solutions, the positions of the (identified) atoms are exchanged.

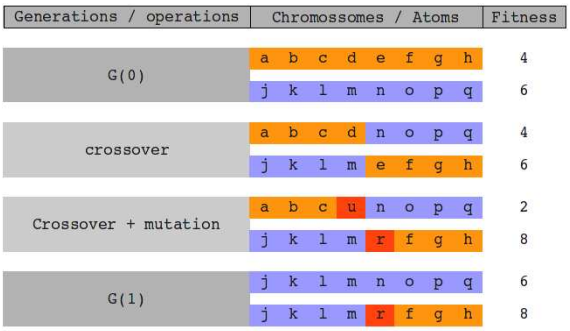

Figure 6: Illustration of crossover and mutation in population of size two. The fitness function is the number of consonants minus number of vowels.

After a crossover, an individual (or solution) can suffer mutation. In genetic algorithms terminology, mutation is the change of the value (or genetic contents) in a small part of an individual. In this work, mutation means atom movement, that is, if an atom suffers mutation, then its position will be slightly altered. The displacement of an atom follows the suggestion in (6; 7). The algorithm chooses the direction of this displacement. We used two possible directions with the same probability of being chosen: (i) random and (ii) following the descent gradient of total potential energy.

After an amount of crossovers between pairs of individuals in a population, and after some mutations, the result is a different set of solutions, meaning different lattices on the image. The algorithm returns the individual (or the solution) that has the best fitness value, i. e., that minimizes the total potential energy.

Results

Two seismic images, illustrated in Figure 7, were used to compare our nondeterministic method to the projection method proposed by Esperança, Oliveira and Cavalcanti (4). This comparison was done using a program developed in C++ that uses the CGAL (2) library to compute the triangulations and from which we used the data structures to calculate the quality of each generated mesh.

For both methods we initially put a set of atoms that could not move on the board of the image.

The Table 1 shows the parameters used in the tests.

For the 41 levels of gamma correction, we used values given by the formula:

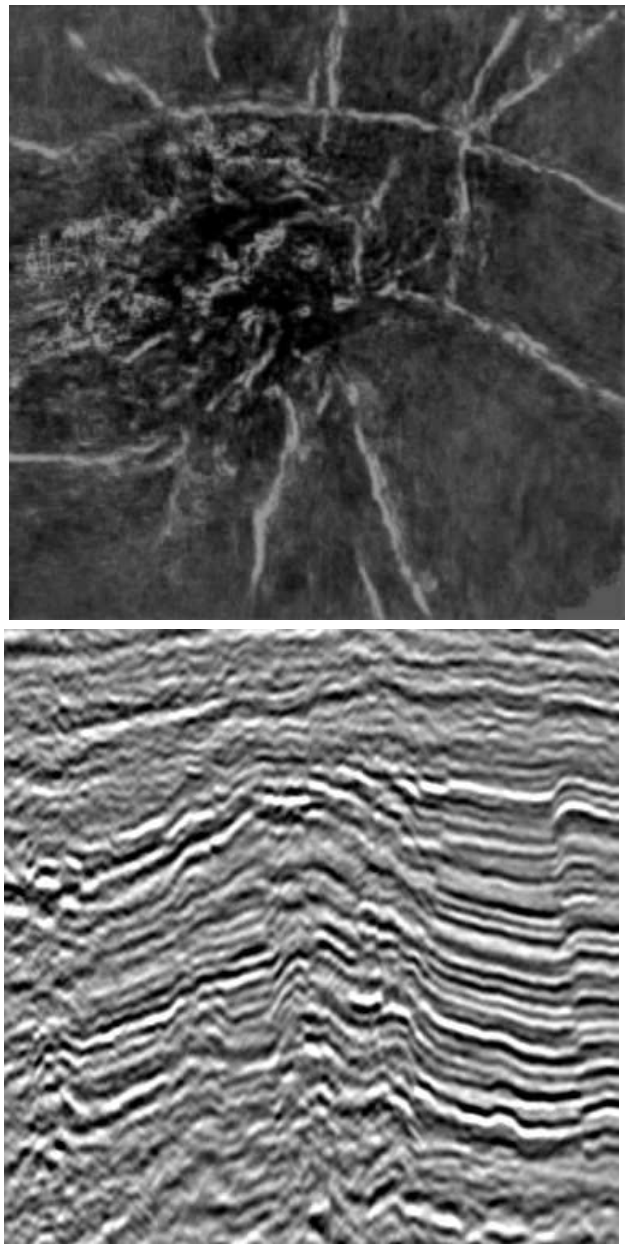

Figure 7: Images used in our test. Constant time section (adapted from Hale (6)) at the top and small interval of a vertical section of a 3D seismic acquisition.

Parameter	Value
Nominal interatomic distance	14
Maximum # of atoms	1024
Maximum # of failures per guide table	1024
# of guide tables	41

Table 1: Parameters used in the tests.

$$\gamma = \begin{cases} 1/i, & \text{if} \quad i > 1 \\ |i|+1, & \text{if} \quad i \leq 1 \end{cases},$$

with $i = [-20, ..., 20], i \in \mathbb{Z}$.

The comparison between the methods considers the

Guide table with gamma corrections					
Image	**NoA**	**QWT**	$\overline{Q}$	Δt	**TPE**
Time section	740	0.474	0.877	9.08	-256.95
Vertical section	627	0.531	0.877	5.08	-325.18
Projection					
Image	**NoA**	**QWT**	$\overline{Q}$	Δt	**TPE**
Time section	799	0.485	0.870	19.75	-300.65
Vertical section	662	0.555	0.872	7.01	-357.58

Table 2: Comparison between algorithms of initial placement.

number of atoms put on the image (NoA), the mesh quality, represented by the quality of the worst triangle in the mesh (QWT) and by the average quality of the mesh ($\overline{Q}$). We use the same quality metrics as in (4), having the formula

$$Q = \frac{4\sqrt{3}.A}{l_1^2 + l_2^2 + l_3^2},$$

where A is the area of the triangle and l_1, l_2 and l_3 are the lengths of the sides. We collected the execution time (Δt, in seconds) and the value of the total potential energy (TPE). Each algorithm of initial placement of atoms was executed ten times and we show the average of the observed data in Table 2. All tests were run in a computer with Intel Core 2 Duo @ 2.2GHz processor and 2Gb RAM.

For the mesh optimization step, our genetic algorithm was configured to have a population of ten individuals.

Figure 8 shows the initial placement of atoms following the proposed method. After some iterations using our genetic algorithm, the atoms were moved as shown in the Figure 9, generating the lattice as in the Figure 10.

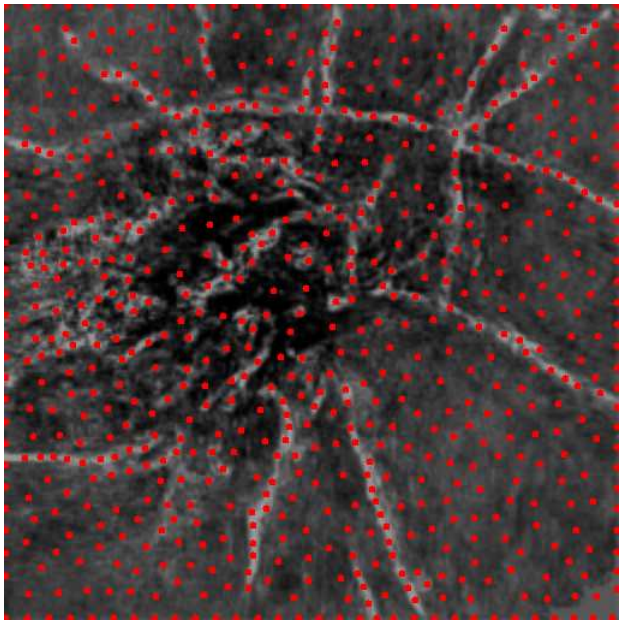

Figure 8: Initial placement of atoms using proposed method.

The Table 3 shows the evolution of the placement of the atoms in successive iterations of the genetic algorithm. We can observe that the total potential energy is decreasing and the average quality of the mesh is increasing.

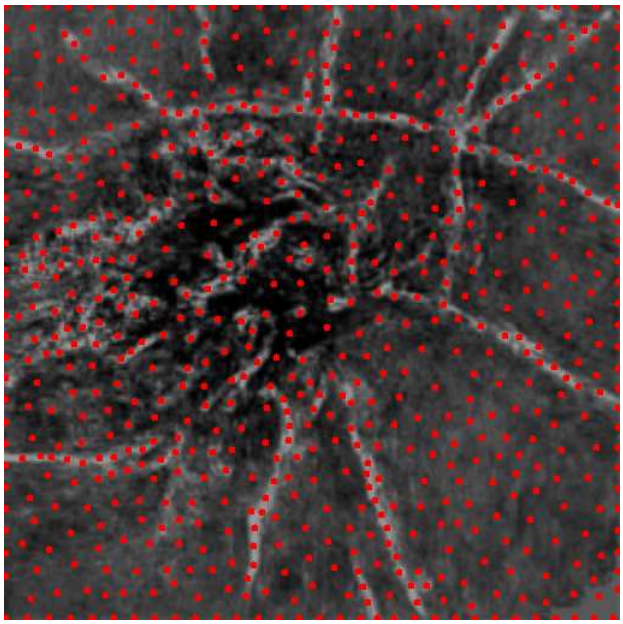

Figure 9: Final placement of atoms using the genetic algorithm.

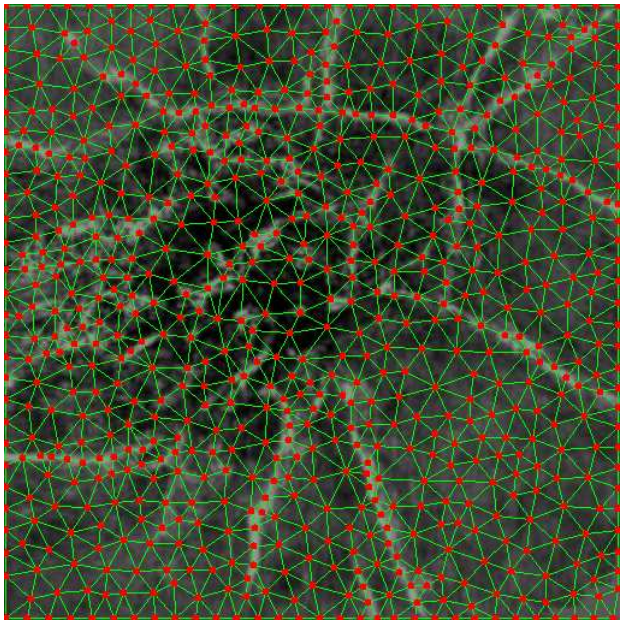

Figure 10: Mesh generated with Delaunay triangulation after optimization.

Conclusions

The placement of atoms using guide tables modified by gamma corrections has nice performance compared to the projection method. It is easy to use, and the quality of the initial mesh is as good or better than the one generated by the projection method.

The usage of genetic algorithm seems to be a valid option, despite its computational cost in relation to the steepest descent method.

Iteration	WQ	$\overline{Q}$	TPE
0	0.534	0.875	-258.725
1	0.555	0.887	-259.829
2	0.490	0.893	-259.896
3	0.513	0.897	-260.489
4	0.500	0.900	-260.372
5	0.479	0.902	-260.294
6	0.427	0.902	-260.007
7	0.473	0.905	-260.466

Table 3: Evolving mesh quality using the genetic algorithm. Initial placement differs from that in Table 2.

We believe that the finite element mesh aligned to image features can be used in geological interpretation procedures, such as adjusting the seismic data to interpreted horizons or to specific positions in wells. This is the target of the authors' efforts so far.

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