Correction of primary amplitudes for plane-wave transmission loss through an acoustic or absorptive overburden with the inverse scattering series internal multiple attenuation algorithm: an initial study and 1D numerical examples

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Abstract

The objective of extracting the spatial location of a reflector, and its local angle-dependent reflection coefficient, from seismic data, depends on the ability to identify and to remove the effect on primary amplitudes of propagation down to and back from the reflector. All conventional methods that seek to correct for such transmission loss require estimates of the properties of the overburden. In this paper we propose a fundamentally new approach that will in principle permit correction of primaries for such transmission loss without requiring overburden properties as input. The approach is based on the amplitude of the first term of the inverse scattering series internal multiple attenuation algorithm, which predicts the correct phase and approximate amplitude of first order internal multiples. The amplitude is estimated to within a factor determined by plane wave transmission loss down to and across the reflector producing the event's shallowest downward reflection. Hence, the amplitude difference between a given predicted and actual multiple, both of which are directly available from the data and the algorithm output, in principle contain all necessary information to correct specific primary reflections for their overburden transmission losses. We identify absorptive overburdens/media as requiring particular focus, so as a first step, previous amplitude analysis of the internal multiple attenuation algorithm is here extended to include stratified absorptive media. Using this newly derived relationship between predicted and actual internal multiples, and existing results for acoustic/elastic media, correction operators, to be applied to specific, isolated primaries in both types of media, are then computed using combinations of multiples and their respective predictions. We illustrate the approach on synthetic data for the absorptive case with three Earth models with different Q profiles. Further research into the amplitudes of the plane wave internal multiple predictions in 2D and 3D media as a likely pre-requisite to field data application of this concept-level algorithm.

Introduction

A primary is a recorded seismic event whose history can be roughly subdivided into: propagation down from the source through the overburden, reflection at a target, and propagation back through the overburden up to the receiver:

$$\text{Primary} \quad = \quad [\text{TDown}] \times [\text{Reflection}] \times [\text{TUp}], \qquad (1)$$

where TDown and TUp stand for *transmission down* and *up*, respectively. In exploration seismology primaries are the main seismic source of subsurface information, and are used for structural mapping, parameter estimation, and, ultimately, petroleum delineation at the target. Techniques of *migration-inversion* (Weglein and Stolt, 1999) accomplish these goals by first generating maps of seismic reflectors at depth, typically positioning at these reflectors reflection coefficients as functions of angle, and, second, by using this behavior to determine local contrasts in medium properties. Therefore, an important part of migration-inversion is the processing of primary amplitudes, which are themselves essentially described by equation (1), to remove the effects of transmission down to and back from the point of reflection, "laying bare" the reflection coefficient information so that it may be used in parameter estimation. This removal as it is conventionally accomplished requires an accurate estimate of all medium properties above the target. In this study we describe an approach for the correction of primary amplitudes for transmission through various types of overburden, that avoids the requirement for prior characterization of overburden properties, thus aiding otherwise conventional migration-inversion methods. We seek a corrective operator, COp, derivable directly from the data, of the form

$$\text{COp} = ([\text{TDown}] \times [\text{TUp}])^{-1}, \qquad (2)$$

which, when applied to a particular primary as modeled by equation (1), provides the reflection coefficient information, CorP, required by the inversion component of migration-inversion:

$$\text{CorP} = \text{Primary} \times \text{C.Operator} = [\text{Reflection}]. \qquad (3)$$

Our approach derives from the inverse scattering series internal multiple attenuation algorithm (Araujo et al., 1994; Araújo, 1994; Weglein et al., 1997, 2003). The order of an internal multiple refers to the number of downward reflections experienced by the event anywhere in the subsurface (Weglein et al., 2003); e.g., first order internal multiples have one downward reflection, etc. The inverse scattering series has the ability to eliminate all multiples without *a priori* subsurface information (Weglein et al., 2003). The inverse series algorithm for free-surface multiples (Carvalho, 1992) eliminates a single order of free-surface multiples with a single algorithm term (of the same order). In contrast, each order of internal multiples requires a series for its removal. For instance, the internal multiple attenuation algorithm is a series, whose first term predicts the correct time and approximate amplitude of all first order internal multiples, and prepares the higher order multiples for attenuation by higher order terms in the algorithm. Research has additionally progressed towards an elimination algorithm. Ramirez and Weglein (2005a); Ramirez (2007) have provided a closed-form *elimination* algorithm for a subset of first-order internal multiples, which eliminates internal multiples generated at the shallowest reflector in the earth and improves the attenuation of internal multiples generated at deeper reflectors. Further aspects of the internal multiple attenuation algorithm have been reported in the literature by Carvalho et al. (1991); Matson (1997); Weglein et al. (1997); Weglein and Matson (1998); Kaplan et al. (2005); Nita and Weglein (2005); Weglein and Dragoset (2005). Our proposed primary correction approach derives from the properties of the first term of the internal multiple attenuation algorithm. The precise difference between the actual amplitude of an internal multiple and the amplitude predicted by the first term of the algorithm, for plane wave data in an acoustic medium (Weglein et al., 2003; Nita and Weglein, 2005), is a direct expression of plane wave transmission losses down to and across the reflector where the multiple's shallowest downward reflection has taken place (Weglein and Matson, 1998; Weglein et al., 2003; Ramirez and Weglein, 2005b,a). This means that the amplitude difference between a given multiple and its prediction, both of which are directly available from the data and the algorithm output, in principle contains all the information necessary to correct specific primary reflections for their overburden transmission losses[1]. The main goal of this paper is to use this information to construct a corrective operator essentially of the form described in equation (2). In doing this early-stage research, we assume that wavelet estimation and deconvolution, instrument response analysis, and de-ghosting have already been carried out, and that the requisite data events have been identified and can be separately studied. There are additional potential benefits associated with this idea: first, the information is a byproduct of an existing part of the *wave-theoretic* processing flow (the de-multiple phase) and comes at no additional cost. Second, this information becomes available at a convenient point during processing, just prior to its likely use in primary processing/inversion. Third, it is consistent with wave-theoretic processing. Fourth, it is not restricted to a production setting, but is also applicable in reconnaissance and exploration settings. We have in particular found that the design of the operator depends on whether or not the overburden is absorptive.

Amplitudes predicted by the multiple attenuation algorithm

The first term in the internal multiple attenuation algorithm acts non-linearly on reflection seismic data to calculate the exact phase and approximate amplitude of all orders of internal multiples:

$$b_{3IM}(k_g,k_s,q_g+q_s)=\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}dk_1e^{-iq_1(z_g-z_s)}$$
$$\int_{-\infty}^{\infty}dk_2e^{iq_2(z_g-z_s)}\left[\int_{-\infty}^{\infty}dz_1'b_1(k_g,k_1,z_1')e^{i(q_g+q_1)z_1'}\right.$$
$$\times\int_{-\infty}^{z_1'-\epsilon}dz_2'b_1(k_1,k_2,z_2')e^{-i(q_1+q_2)z_2'}$$
$$\left.\times\int_{z_2'+\epsilon}^{\infty}dz_3'b_1(k_2,k_s,z_3')e^{i(q_2+q_s)z_3'}\right], \tag{4}$$

where $q_g = \text{sgn}(\omega)\sqrt{(\frac{\omega}{c_0})^2-(k_g)^2}$, $q_s = \text{sgn}(\omega)\sqrt{(\frac{\omega}{c_0})^2-(k_s)^2}$, k_g and k_s are the horizontal wavenumbers conjugate to receiver and source coordinates (x_g,x_s), respectivelly, and ϵ is a small positive quantity. The input for the internal multiple attenuation algorithm is b_1, which is created from the pre-stack reflection seismic data. It is constructed as follows: the surface recorded data, deghosted and without free surface multiples, $D(x_g,x_s,t)$, is Fourier transformed over all variables, to produce $D(k_g,k_s,\omega)$. A change of variables is made, to $D(k_g,k_s,q_g+q_s)$, after which b_1 is defined as $b_1(k_g,k_s,q_g+q_s) = D(k_g,k_s,q_g+q_s)(2iq_s)$; b_1 is then inverse Fourier transformed over q_g+q_s to pseudo-depth. The result, $b_1(k_g,k_s,z)$, is used as input in equation (4), and the output, b_{3IM}, is the predicted internal multiple data set, produced without knowledge of Earth material properties or structure and it accommodating all Earth model types that satisfy the convolutional model (Ramirez and Weglein, 2005b).

The relationship between the predicted and the actual multiple amplitude

Being the first term in a series that removes first order internal multiples without subsurface information, the internal multiple attenuation algorithm provides the capability to predict the exact time of all first order internal multiples and it is the first term to predict the amplitudes of the first order internal multiples. Weglein and Matson (1998) and Ramirez and Weglein (2005b) examined the difference between the actual amplitudes of internal multiples and those of the internal multiple attenuation algorithm predictions. The latter authors called the difference the *amplitude factor*, and showed that it is related to the transmission coefficients down to and across the multiple generator interface (Weglein and Matson, 1998; Ramirez and Weglein, 2005b). The difference can be understood intuitively by considering the way the algorithm builds its prediction. Consider Figure 1. On the left panel we sketch an internal multiple and the three primaries that are used in the algorithm to predict it. The generator is interface 2. The multiple has the path $abcdijkl$. The algorithm predicts the multiple by multiplying the amplitudes of the three primaries, adding the phases of the deeper two, $abcdef$ and $ghijkl$, and subtracting the phase of the shallower, $ghef$. The phase of the actual multiple and the predicted multiple are therefore identical. However, the amplitude of the actual multiple,

$$T_{ab}T_{bc}R_{cd}(-R_{he})R_{ij}T_{jk}T_{kl},$$

[1]The use of the discrepancy for correcting for overburden effects was first suggested by Dennis Corrigan following discussions on the analytic example presented by A. Weglein at CWP and ARCO, later published by Weglein and Matson (1998).

and the multiplied amplitudes of the primaries in the prediction,

$$[T_{ab}T_{bc}R_{cd}T_{de}T_{ef}] \times [T_{gh}R_{he}T_{ef}] \times [T_{gh}T_{hi}R_{ij}T_{jk}T_{kl}],$$

clearly differ in that the actual multiple does not experience the transmission history of the shallower primary. That is, the terms T_{de}, T_{ef}, T_{gh}, T_{hi} in the prediction are extraneous. We note that this includes transmission across the generating interface. Let us next depart from schematics and consider the general accounting of this behavior for predicted multiples within an arbitrary stack of layers provided by Ramirez and Weglein (2005b). Figure 2 shows a 1-D acoustic model consisting of three reflectors, the lower two of which have associated reflection coefficients R_1 and R_2, and layer velocities c_0, c_1 and c_2. The transmission coefficient from layer i to layer j is T_{ij}. For example, a multiple generated at interface 1 in Figure 2 has an amplitude

$$M_1 = [T_{01}T_{12}R_2(-R_1)R_2T_{21}T_{10}]. \tag{5}$$

The predicted multiple amplitude is:

$$M_1^{\mathrm{PRED}} = [T_{01}T_{12}R_2(-R_1)R_2T_{21}T_{10}][(T_{01}T_{10})^2(T_{12}T_{21})]. \tag{6}$$

Comparing equations (5) and (6) we see that their ratio, as we now intuitively expect, carries information about the transmission coefficients down to and across the multiple generator interface. Ramirez and Weglein (2005b) refer to this ratio as the amplitude factor AF:

$$\mathrm{AF}_2 = \frac{M_1^{\mathrm{PRED}}}{M_1} = [T_{01}T_{10}]^2[T_{12}T_{21}]. \tag{7}$$

The index 2 anticipates our later use of this factor for corrective purposes, and signifies that the second interface is the generator. This terminology follows Ramirez and Weglein (2005b).

Extension of the amplitude analysis to absorptive media

Ramirez and Weglein (2005b) assume an acoustic medium, in which plane-wave transmission losses are local, occurring at the point at which the wave crosses a contrast in material properties. For an absorptive stack of layers, in which transmission loss occurs over the entire course of propagation, an extension of their results is required. In later sections we will see that this minor theoretical alteration leads to an important practical difference when the predicted-actual amplitude discrepancy is exploited. In order to study the transmission coefficients in an anelastic medium we select an intrinsic attenuation model to describe amplitude and phase alterations in a wave due to friction. These alterations are modeled by a generalization of the wavefield phase velocity to a complex, frequency-dependent quantity parameterized in terms of Q. A reasonably well-accepted Q model (Aki and Richards, 2002) alters the scalar propagation constant of the j'th layer, $k_j = \omega / c_j(z)$, to

$$k_j = \frac{\omega}{c_j(z)}\left[1 + \frac{F(\omega)}{Q_j(z)}\right], \tag{8}$$

where $F(\omega) = \frac{i}{2} - \frac{1}{\pi}\log(\omega/\omega_0)$. The reference frequency ω_0 may be considered a parameter to be estimated, or assumed to be the largest frequency available to a given experiment. The model divides propagation into three parts: a propagation component, an attenuation component, and a dispersion component. With this new definition of k_j, and assuming that in Figure 2 the two bottom layers are anelastic, we again construct the prediction. It is convenient to re-define the transmission coefficient of a given interface to incorporate absorptive amplitude loss within the layer *above* that interface. For instance, the coefficients T_{12} and T_{21} of the previous section derived using the k_j of equation 8, become:

$$\begin{aligned}
\mathcal{T}_{12} &= \left[\frac{2c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1}}{c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1} + c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1}}\right] \\
&\times \overbrace{e^{-\frac{\omega}{2Q_1c_1}(z_1-z_0)}}^{\text{attenuation component}} e^{\frac{i\omega}{\pi Q_1 c_1}\log\left(\frac{\omega}{\omega_0}\right)(z_1-z_0)} \qquad (9)\\
\mathcal{T}_{21} &= \left[\frac{2c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1}}{c_2\left(1+\frac{F(\omega)}{Q_2}\right)^{-1} + c_1\left(1+\frac{F(\omega)}{Q_1}\right)^{-1}}\right] \\
&\times \underbrace{e^{-\frac{\omega}{2Q_1c_1}(z_1-z_0)}}_{\text{attenuation component}} e^{\frac{i\omega}{\pi Q_1 c_1}\log\left(\frac{\omega}{\omega_0}\right)(z_1-z_0)}. \qquad (10)
\end{aligned}$$

We make particular note of the dependence (via the attenuation component) of this definition of transmission coefficients on the thickness of the layer *overlying* the interface in question. With this extension, we have essentially the same amplitude factor, for instance AF_2, in the anelastic case as we did in the elastic case. By analogy with equation (7):

$$\mathrm{AF}_2 = [T_{01}T_{10}]^2\mathcal{T}_{12}\mathcal{T}_{21}. \tag{11}$$

Correction of primary amplitudes using internal multiples

Let us make two comments about the amplitude error analysis above. First, we see that the discrepancy between the predicted and the actual multiple for a given generator is directly related to the transmission losses experienced by a primary associated with that generator. Second, we note that the discrepancy, characterized by the amplitude factor AF, is available directly from the data and the output of the internal multiple attenuation algorithm. In this section we use the information in the various AF factors as a direct means to correct the amplitude of the primary associated with the generator for transmission effects, in the sense we have put forward in the introduction. We define what will become the primary correction operator, PCO, to be built recursively from the data-determined AFs:

$$\mathrm{PCO}_n \equiv \frac{\mathrm{PCO}_{n-1}}{\mathrm{AF}_n}, \mathrm{PCO}_0 = 1. \tag{12}$$

Expanding this operator over several orders n clarifies that it will indeed act as a correction operator when applied to a primary whose upward reflection has occurred near the n'th interface. We find that the precise primary which should be corrected with the n'th operator depends on whether the medium is assumed to be absorptive or not.

Correction of primaries in acoustic/elastic media

Consider once again the multiple sketched in Figure 1, whose generator is interface 2. Setting $n = 2$, expanding equation (12), and employing the alphabetical indices we use in the figure, we have

$$\mathrm{PCO}_2 = \frac{1}{T_{gh}T_{hi}T_{de}T_{ef}}. \tag{13}$$

If the medium is acoustic/elastic, we note that for the primary depicted in the middle panel of Figure 1, the "last" overburden effect on the event before the reflection at interface 3 is the transmission through interface 2, and the "first" overburden effect on the event after the reflection is again transmission through interface 2. Consequently, PCO_2 is exactly appropriate as an operator to correct this (middle panel of Figure 1) primary. More generally, in the acoustic/elastic case, the operator PCO_n in equation (12) corrects the n'th primary, leaving the n'th reflection coefficient "bare" and suitable as input to other inverse procedures:

$$R_n = \mathrm{PCO}_n \times P_n. \tag{14}$$

Correction of primaries in absorptive media

Next, let us suppose that the medium in Figure 1 is absorptive, and again consider PCO_2. Recall that we may maintain the same form for the amplitude discrepancy between predicted and actual multiples in absorptive media and thereby this operator, PCO_2, provided we alter the transmission coefficients of a given interface to include absorptive propagation through the layer *above* that interface. With this arrangement PCO_2 is evidently no longer appropriate as an operator to correct primary 2, i.e., the primary depicted in the middle panel of Figure 1, because it does not account for absorptive propagation through the layer between the reflection and the multiple generator. To maintain the usefulness of the operator, we instead make an approximation. We assume that in an absorptive medium, the effect of the local transmission coefficient at a boundary on the amplitude of a primary is dwarfed by the effect of absorptive propagation. With that assumption we may simply change the primary being corrected by PCO_2 to the one depicted in the right panel of Figure 1. This statement is true to within the combined local transmission coefficient down and up across interface 2. More generally, in the absorptive case, the (now frequency-dependent) operator PCO_n in equation (12) corrects the $n-1$'th primary:

$$R_{n-1}(\omega) = \mathrm{PCO}_n(\omega) \times P_{n-1}(\omega). \tag{15}$$

Synthetic examples

In this section, we illustrate with simple synthetic examples the steps necessary to correct a primary for absorptive transmission losses. We generate zero-offset traces from plane waves normally incident on three horizontal layered models assuming the waves behave in accordance with the propagation constant in equation (8), and using the layer parameter values in Table 1. We include two primaries and a first order internal multiple. The traces are wavelet deconvolved, and bandlimited (3–50 Hz). Figure 3 shows the traces generated for each model, which differ in their Q values, ranging from relatively low attenuation to relatively high attenuation. The arrival times of the two primaries and the multiple are approximately 1.5s, 2.3s and 2.9s, respectively.

Depth (m)	c (m/s)	Q_1	Q_2	Q_3
000-500	1500	∞	∞	∞
500-1422	2200	200	100	50
1422-2422	2800	100	50	25
2422-∞	3300	50	25	10

Table 1. *Absorptive Earth models.*

The prescription for correcting the primary is:i) Each trace is used as input to the internal multiple attenuation algorithm, generating predictions of the internal multiples; ii)each internal multiple and its prediction are isolated and their spectra calculated; iii)the reciprocal of the ratio between the spectra of each internal multiple and its prediction is taken. By equation (12), this is the appropriate correction operator PCO and iv)the shallower primary is isolated, and the operator is applied to its spectrum. We compare the result to an equivalent primary which we model in the absence of all effects of transmission through the overburden.

Conclusions

We have presented a procedure for correcting a primary for transmission losses using internal multiples and the output of the inverse scattering series internal multiple attenuation algorithm. We have made particular mention and use of the distinction between situations involving significant absorption and situations that are largely acoustic or elastic. In spite of this broad categorization (that we have found to be practically important), one of the strengths of the approach is that it will act to correct transmission losses *whatever* their physical origin or mechanism, without requiring a precise model. In this sense the approach is truly data-driven – the events in the data, in comparison to one another, "decide" what the transmission loss must be. Our simple numerical results are encouraging and motivate examination of the approach in the presence of more complex media, both absorptive and otherwise. The main tool in this approach, the internal multiple algorithm, is immediately applicable in multiple dimensions, and since the amplitude error is in terms of plane wave transmission coefficients, a plane wave decomposition of 2D and/or 3D data will likely suffice to extend the method. Nevertheless, detailed extension of the approach stands as ongoing and future research. For these reasons in particular, we identify field data testing as a medium-term to long-term goal, contingent on the fundamental study of the internal multiple attenuation amplitudes in multiple dimensions.

Acknowledgments

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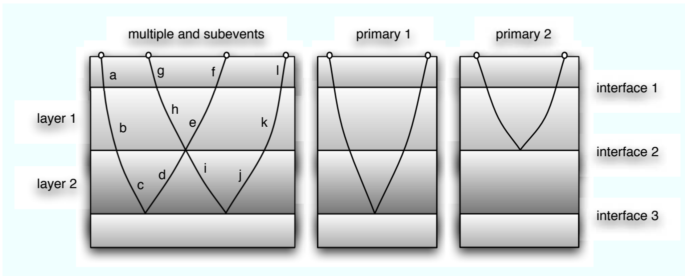

Figure 1: *Left panel: an internal multiple and the primary subevents used to predict it. Middle and right panels: associated primaries whose amplitudes may be corrected using the discrepancy between the amplitudes of the predicted and actual multiple on the right.*

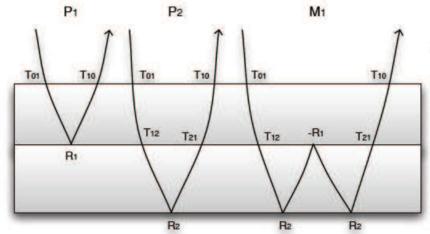

Figure 2: *Multilayered medium where the first layer,* 0*, is acoustic and the other two are anelastic. Three events are displayed and they were used to generate the data-set for testing the correction of the transmission losses of a primary in an absorptive medium. The parameters of the models are shown in table* 1*.*

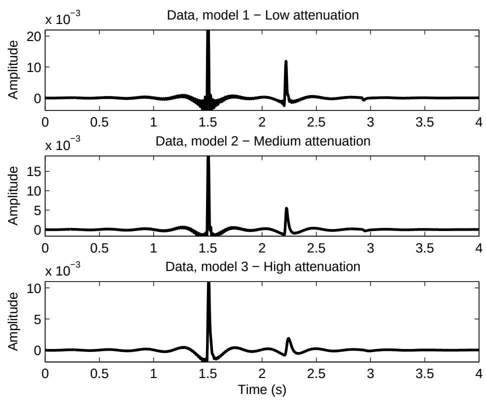

Figure 3: *Data generated for the numerical tests comprised of two primaries and one multiple for all models.*

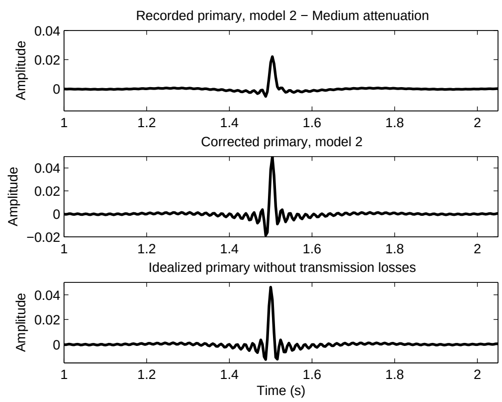

Figure 4: *A cartoon showing the actual primary in the top, the corrected primary in the middle and the idealized primary, that was generated by dropping the transmission coefficients in the modeling, in the bottom for model* 1*, the medium attenuation case.*