A new method to estimate the crustal and lithosphere thickness using elevation, geoid anomaly and heat flow

Alexandrino C. H. Observatório Nacional, Rio de Janeiro, E-mail chalex@on.br.

Valiya M. Hamza Observatório Nacional, Rio de Janeiro.

Abstract

We present a novel approach for joint inversion of heat flow, elevation and geoid anomalies which allows simultaneous determination of the deep thermal field as well as crustal and lithospheric structures. The technique is based on computationally stable iteration schemes and provide simultaneous checks for compatibility of the inversion results with the observational data on surface heat flow, radiogenic heat production, elevation and geoid anomalies. The results are found to be far more robust and realistic than those obtained in conventional thermal models. Unlike the previous attempts, the new approach incorporates surface heat flow variation as an independent constraining parameter and at the same time allow for the non-linear effects of thermal conductivity variation with temperature and in the crustal layers. The method was used in determining deep thermal structures of crustal blocks composing the São Francisco craton. The results obtained point to substantial variations in the crustal thickness; with mean values falling in the interval of 32 to 47 km. Such variations in crustal thickness are accompanied also by substantial intra-cratonic variations in the lithospheric thickness; with mean values extending over the the range of 90 – 210 km. These results are in reasonable agreement with values obtained in studies of seismic tomography.

Introduction

Knowledge of isostasy is often considered crucial in assessment of processes tectonic processes, in studies of evolution of continental margins and in understanding the history of sedimentary basins. Thermal isostasy is the process in which changes in density induced by changes in the subsurface thermal field contribute to vertical movements of the crustal and lithosphere. Since the pioneering work of Lachenbruch and Morgan (1970) a number of attempts have been made in understanding relation between geoid height and heat flow within the framework of thermal isostasy. Recently, Fullea et al. (2007) presented a method for automatic modeling of crustal and lithospheric structure using elevation and geoid data. This approach also incorporates information on Moho temperatures. However, the approach adopted by Fullea et al (2007) does not make use of measured heat flow and thermal conductivity data as input parameters and in addition does not allow for vertical variations in thermophysical properties. Consequently, the results obtained provide only limited understanding of the deep thermal structure of the lithosphere.

In this work, we present a refinement of the method of Fullea et al (2007) which admits surface heat flow also as an input parameter and in addition allow for the effects of vertical variations in thermal conductivity. The technique employed is based on computationally stable iteration schemes and provide simultaneous checks for compatibility of the inversion results with the observational data on surface heat flow, radiogenic heat production, elevation and geoid anomalies. The model was applied to examine the crustal and lithospheric structure of the São Francisco craton.

Method

The assumption of isostasy allows us to develop the relations between elevation and density of lithosphere and asthenosphere. Buoyancy considerations may be employed in developing the relations between elevation of the lithosphere with respect to the sea level and the level of asthenosphere, illustrated in the schematic diagram of Figure (1).

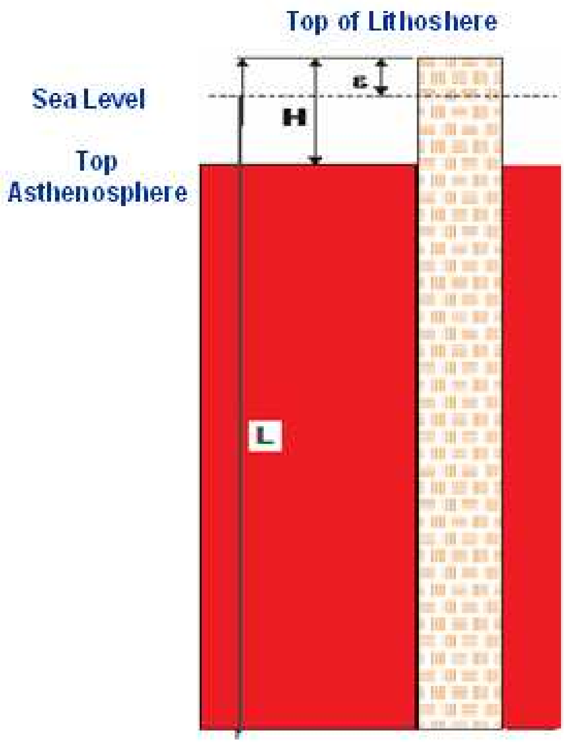

Figure (1) Isostatic balance between lithosphere and asthenosphere (Lachenbruch and Morgan, 1990).

In the above diagram ε is the elevation of lithosphere above sea level, L the lithospheric thickness, H the difference in height and H_0 the elevation of asthenosphere. We introduce now the concepts of geope of dry asthenosphere and hydro-geope of the asthenosphere. The relevant relations for a lithosphere subjected to tectonic processes, such as stretching, have been derived by Lachenbruch and Morgan (1990). Under such conditions the geoide height is proportional to the dipole moment of the vertical distribution of density (Ockendon e Turcotte, 1977; Turcotte and Oxburgh, 1982):

$$N = -\frac{2\pi G}{g}\int_{LC} z.\rho(z)\,dz + N_0 \qquad (1)$$

where z is depth, G is the universal gravitational constant (m^3 s^{-2} kg^{-1}), Δρ the density contrast and g is the terrestrial gravitational acceleration over the Earth's surface (m s^{-2}). The integration constant N_0 plays the role of a reference level, and is needed to adjust the zero level of the geoid anomalies.

Following Fullea et al (2007) we also consider a four-layered lithospheric model composed of crust, lithospheric mantle, sea water and the asthenosphere. The depth of the base of crust and lithosphere are related with elevation under local isostasy (Fullea et al., 2007):

$$z_c = \frac{\rho_a L_0 + E(\rho_c - \rho_w) + z_L(\rho_m - \rho_a)}{(\rho_m - \rho_c)} \qquad (2)$$

where z_c is the depth of the crust-mantle boundary (Moho), z_L is the depth of the Lithosphere–Asthenosphere Boundary (LAB), ρ_c and ρ_m are the average densities of the crust and lithospheric mantle. All depths are referred to the mean sea level. For the lithospheric mantle density, ρm, we consider a linear dependence on the temperature:

$$\rho_m(z) = \rho_a\left(1 + \alpha\left[T_a - T_m(z)\right]\right) \qquad (3)$$

where α is the linear coefficient of thermal expansion (K^{-1}), Ta is the temperature at the LAB and Tm(z) is the temperature at depth z in the lithosphere. The average value of the lithospheric mantle density, $\overline{\rho}_m$, can be determined by integrating Eq. (5) between z_c and z_L:

$$\overline{\rho}_m(z) = \frac{1}{z_L - z_c}\int_{z_c}^{z_L}\rho_m(z)dz = \rho_a\left(1 + \frac{\alpha}{2}\left[T_a - T_m(z)\right]\right) \qquad (4)$$

The temperature distribution for a one-dimensional medium with heat sources the steady-state heat conduction equation may be written as:

$$\frac{d}{dz}\left[\lambda(T)\frac{dT}{dz}\right] = -A_0\exp(-z/D) \qquad (5)$$

where z is the depth, *T* is temperature, *λ(T)* the thermal conductivity, A_o is the heat production in near surface layers and *D* is logarithmic decrease of heat production with depth. The following boundary conditions apply:

$$T(z=0) = T_0 \qquad (6a)$$

$$\lambda(T)\left.\frac{\partial T}{\partial z}\right|_{z=0} = q_0 \qquad (6b)$$

where T_0 is the temperature of the surface (z = 0), q_0 is heat flow observed in surface. The variation of thermal conductivity with temperature at depth is assumed to follow a relation of the type:

$$\lambda(T) = \frac{\lambda_{25°C}}{1 + BT} + C(273.15 + T)^3 \qquad (7)$$

In the above relation B and C are material constants (*B* = 10^{-4} ℃ $^{-1}$ and *C* = 10^{-10} ℃ $^{-1}$), determined by experimental studies on rock samples. The solution of (5) that satisfies the above boundary conditions is (Alexandrino and Hamza, 2008):

$$\frac{1}{B}\ln\{1 + B[T(z) - T_0]\} + \frac{C}{4\lambda_0}\{273.15 + [T(z) - T_0]\}^4 - U(z) = 0 \qquad (8)$$

where U is the variable of the Kirchoff transformation. FORTRAN subroutines available in IMSL Lbrary (1989) were used in numerical computation of the transforms.

Temperature (T_c) and thermal conductivity (λ_c) at the crust-mantle boundary are calculated by introducing $z=z_c$ and $z=z_L$ in Eq. (7) and (8). The general equation for isostasy may be expressed as a quadratic relation for the thickness of the lithosphere:

$$\gamma_1 z_L^2 + \gamma_2 z_L + \gamma_3 = 0 \qquad (9)$$

where

$$\gamma_1 = [(T_a\lambda_c) - \theta]$$

$$\gamma_2 = \left(z_c\left(T_a(\lambda_m - 2\lambda_c) + 2\theta\right) - \delta + T_a E\lambda_m - \frac{2\lambda_c}{\rho_a\alpha}[(\rho_a - \rho_c)z_c + \eta]\right)$$

$$\gamma_3 = \left(z_c[\delta - T_a(z_c\Delta\lambda + E\lambda_m) - z_c\theta] - \frac{2}{\rho_a\alpha}[(z_c\Delta\lambda + E\lambda_m)(\eta + (\rho_a - \rho_c)z_c)]\right)$$

$$\theta = \lambda_c T_0 + A_0 D^2\left\{1 - \exp\left[-\left(\frac{E + z_c}{D}\right)\right]\right\}$$

$$\delta = \lambda_m T_a(z_c + E)$$

$$\Delta\lambda = \lambda_m - \lambda_c$$

$$\eta = -[\rho_a L_0 + E(\rho_c - \rho_w)]$$

The geoid anomaly from Eq. (1) for a four-layered lithospheric model in which the density of the crust vary linearly with depth and the density of the lithospheric mantle is temperature dependent is:

$$N = -\frac{\pi G}{g}\left[\begin{array}{l}\rho_w E^2 + \frac{2\beta}{3}(z_c^3 - E^3) + (\beta E + \rho_c^T)(z_c^2 - E^2) \\ + (z_{max}^2 - z_c^2)\rho_a + \rho_a\alpha\frac{T_a - T_{mh}}{3}[(z_L - z_c)(z_L + 2z_c)]\end{array}\right] + N_0 \qquad (11)$$

where $\beta = \frac{\rho_c^B - \rho_c^T}{z_c + E}$ and z_{max} is the depth of the compensation level, and ρ_c^T and ρ_c^B are the densities of the top and the bottom of the crustal layer. Since we use absolute densities, it is necessary to determine the integration constant N_0 to tie the zero level of the geoid anomalies. The joint solution of the above equations allows us to calculate the average lithospheric mantle density, $\overline{\rho}_m$, as well as the depth of the Moho z_c, and the thickness of the lithosphere z_L, which simultaneously fit the elevation and the geoid anomaly under local isostasy.

Iterative computational schemes are necessary because of the non-linearity of the equations. The computational procedure employed in the present work is similar to that used by Fullea et al (2007) but include additional steps because of the complexity of the relations used for heat flow and thermal conductivity. These are:

1. Estimate the initial values for z_c and z_L, assuming constant density for crust and mantle;
2. Use the initial value of z_c for calculating the depth to the base of the lithosphere, which couples isostasy top the thermal field;
3. Calculate the temperature at the base of the crust (T_c) and lithosphere (T_a) using values of z_c and z_L of step 2;
4. Calculate the thermal conductivity the crust (λ_c) and lithosphere (λ_m) using values of T_c and T_a of step 3;
5. Calculate the geoid height using z_c, z_L T_c, T_a, λ_c, and λ_m obtained in steps 3 and 4;
6. Determine the residual anomaly (calculated – observed);
7. Change the value of z_c and repeat the process until the residual anomaly is minimized.

A key parameter in the joint inversion of elevation and geoid data is the reference level for geoid anomalies N_0, which depends on the particular configuration of the lithospheric column considered as reference. The range of meaningful solutions in the elevation – geoid height domain varies depending on the configuration of the reference lithospheric column considered. This model was employed for studying the elevation – geoid height – heat flow domain of the São Francisco province.

Data Base for the São Francisco Province

We discuss briefly the characteristics of the data base for heat flow, elevation, radiogenic heat production and geoid anomaly in the São Francisco Province. This information provides important constraints in application of the model proposed in the present work.

Heat Flow: A summary of temperature gradient, thermal conductivity and heat flow values calculated for the cratonic areas and surrounding fold belts of the São Francisco structural province is presented in Table (1).

Table (1) Mean values of temperature gradient (Γ), thermal conductivity (λ) and heat flow (q) for the main tectonic provinces in the study area.

Geologic Province	Γ (^{0}C/km)	λ (W/m°C)	q (mW/m^2)
Salvador Cráton	14.8 ± 4.3	2.9 ± 1.1	43 ± 9
Tocantins / Brasília Belts	16.0 ± 4.2	3.1 ± 0.5	50 ± 12
Araçuaí / Mantiqueira Belts	17.9 ± 7.2	2.8 ± 0.3	51 ± 10
São Francisco Cráton	16.6 ± 3.7	3.1 ± 0.3	51 ± 14
Northeast of Paraná Basin	28.7 ± 6.3	2.6 ± 0.6	73 ± 14
São Francisco Basin	29.4 ± 7.8	2.6 ± 0.2	76 ± 20

According to data in Table (1) the mean heat flow values are 43 +/- 9mW/m^2 for the Salvador cráton in the north, 51 +/- 10mW/m^2 for the Araçuaí and Mantiqueira fold belts in the east, 50 +/- 12mW/m^2 for the Brasilia and Tocantins fold belts in the west and 50 +/- 14mW/m^2. for the São Francisco cráton in south. Such heat flow values are typical of stable continental regions. On the other hand, the data indicates mean heat flow of 76 +/- 20mW/m^2 for the São Francisco basin. The distribution regional heat flow in region of Province Structural São Francisco is presented in the map of figure (2).

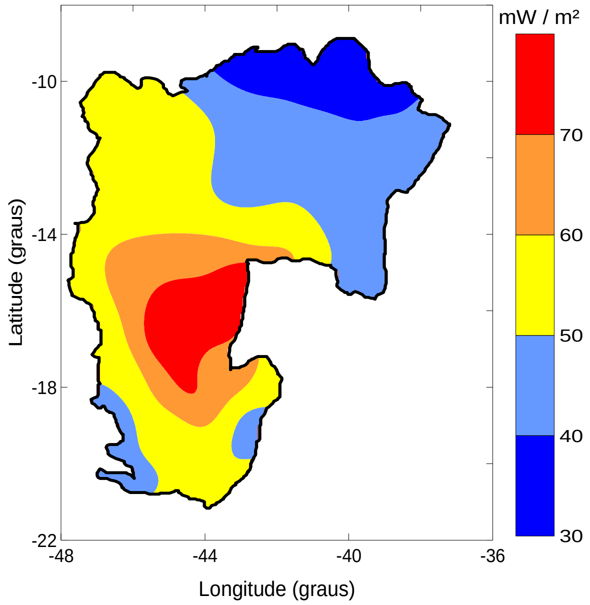

Figure 2 – Distribution heat flow in the São Francisco Structural Province.

Radiogenic Heat Production: Determinations of the abundances of natural radioactive elements (Uranium, Thorium and Potassium) have been carried out, over the last few decades, for a number of localities in Eastern Brazil. The experimental techniques employed include both gamma ray spectrometry as well as isotope mass spectrometry. The available data have been compiled in the present work for estimating representative near-surface heat production values. The results obtained are presented in Table (2).

Table (2) Values of heat production (A_0) and the depth parameter (D) for the tectonic provinces.

Geologic Province	A_0 (μW/m^3)	D (Km)
Salvador Cráton	1.3 ± 0.7	14.2 ± 3.1
Tocantins / Brasília Belts	1.2 ± 0.6	11.1 ± 2.1
Araçuaí / Mantiqueira Belts	1.2 ± 0.7	11.9 ± 2.1
São Francisco Cráton	1.3 ± 0.9	12.2 ± 2.2
Northeast Paraná Basin	2.9 ± 1.0	10.1 ± 1.8
São Francisco Basin	1.7 ± 1.0	12.8± 2.5

According to data in Table (2) the cratonic areas and metamorphic fold belts are characterized by relatively lower heat production values, in the range of 1.0 to 1.3μW/m^3. The sedimentary sequences of the intra-cratonic São Francisco basin are found to have the high values of heat production, with mean values in the range of 1.7 to1.7 ± 1.0μW/m^3. Also provided in Table (2) are the mean values of the parameter for logarithmic decrease of heat production with depth, for the main tectonic provinces.

Elevation and geoid data: The elevation data are derived from a public domain data base (available at the website www.heavens-above.com) for the entire world, which collect data from such organizations as US Geological Survey and The National Imaging and Mapping Agency. This program provides information on geographical location and elevation for every city, village, region, state, county in the World. Geoid anomaly data were taken from the EGM2008 global geopotential model. The official Earth Gravitational Model EGM2008 has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159.

Results of Model Studies

The crustal structure for the São Francisco Province, calculated from elevation, geoid anomaly and thermal field, is shown in figure (3). The lower value for depth to the crust-mantle boundary, z_c is in part of central São Francisco Basin, the Moho is rather flat (35–39) km. The Salvador craton is characterized by zc values relatively high in range of 41–47 km.

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Conclusions

The procedure proposed by Fullea et al (2007) for allows joint inversion of elevation and geoid data but ignores the crucial role of surface heat flow in inversion. As a result the model loads to overestimation of crust and lithospheric thickness and underestimation of moho temperature.

The modified method proposed in present work allows simultaneous inversion of heat flow, elevation and geoid height. It takes into consideration vertical variations in thermal properties and provides information on the thermal structure of lithosphere in region Southeast Brazil.

The correlation between geoid height and heat flow is a useful tool for detailed mapping crustal temperature field in geothermal areas. The results obtained are in agreement with values by studies of seismic wave tomography for region (Assumpção et al 2004).

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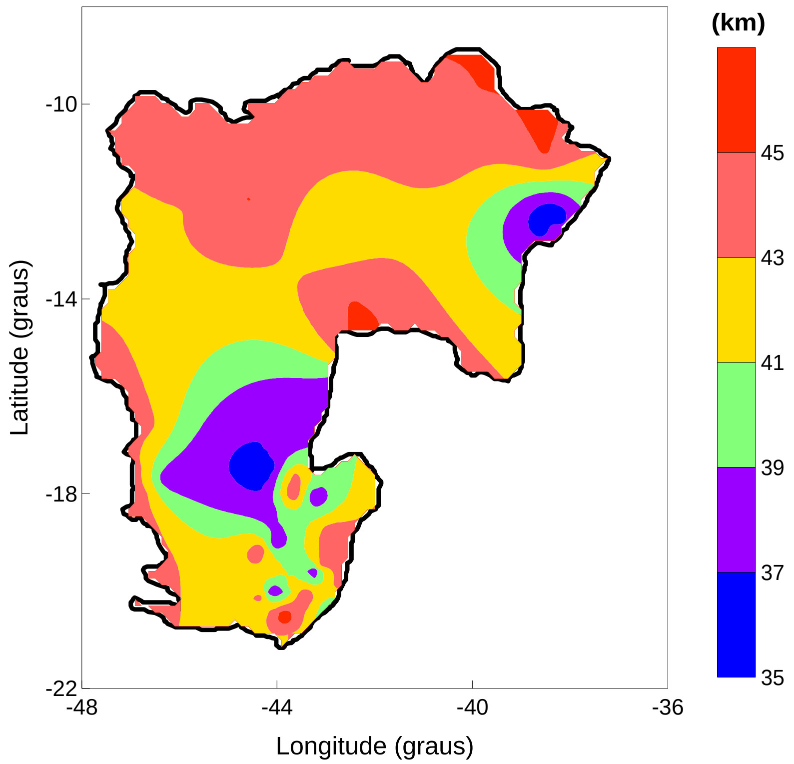

Figure (3) Map of Moho depths in the São Francisco Province derived from elevation, geoid height and heat flow data.

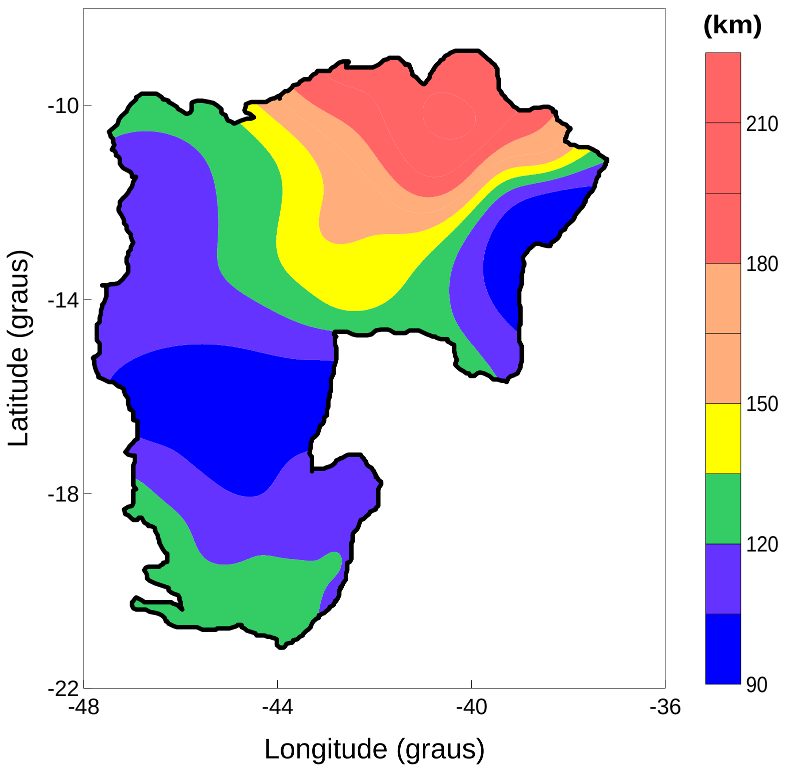

Figure (4) Map of lithosphere thickness in São Francisco Province derived from elevation, geoid anomaly and heat flow data.