Practical issues in the wave field extrapolation analysis from an observation surface

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Abstract

In this work it will be presented an analysis about the reliability of wave field extrapolation on the multiple reflection prediction based on the Kirchhoff-Helmholtz Integral approach. At a first moment, there is a brief review about the Kirchhoff-Helmholtz Integral theory. After that, it is performed a comparison between the direct wave field extrapolation and the Kirchhoff-Helmholtz approach in order to evaluate this methodology. The 2-D synthetic acoustic seismic modeling will be done by means of the Finite Difference Method and analyzing the possible issues of these processes for a homogeneous medium.

Introduction

The seismic method's essences are propagation and reflection of energy. The wave field extrapolation process was introduced by Huygens (1690) and this concept established the wave refraction and reflection laws (Robinson & Clark, 2007). Nevertheless, despite this process of predicting the wave front in a later time, it is deficient when treating the wave field direction and the amplitude quantification (Verschuur, 2006). This subject has remained unsolved until both Kirchhoff and Helmholtz made mathematical researches, in order to support the theory of the wave field extrapolation, based on integral representations from studies in a specific model (a surface ***S*** from a volume ***D***, see figure 2). However, this model was not physically suitable, since the earth surface is flat and extensive. So as to find a way to represent the wave field extrapolation to that case (flat and extensive surface), Rayleigh had conveyed the model and simplified the mathematical approach, which contributed, positively, to the validation of this theory to the seismic case.

The wave field extrapolation concept plays an important role in the imaging techniques and seismic processing. There are many ways to provide the wave field extrapolation: by the PSPI extrapolation operators, the Common Focus Point (CFP) technique, the Split-Step extrapolation operators, for example. The main purpose of this work is to present the wave field extrapolation concept by means of the Kirchhoff-Helmholtz Integral and analyze some characteristics and influences in the onsite application to predict and remove the ocean bottom multiple reflections. The evaluation of the reliability of the Kirchhoff-Helmholtz approach will be performed by the comparison between its theoretical extrapolation process and the forward modeling process (wave field directly registered in the desired position).

The approach and development of this subject have as main motivation to expand the knowledge about the extrapolation process in the context of how this field will behave when obtained in two different ways. This behavior can compromise the multiple reflections prediction, being an important analysis on the multiple reflection removal. This analysis will be performed by the 2-D synthetic acoustic seismic modeling, only for the homogenous case, using the Finite Difference Method (FDM) with fourth-order approximations for spatial derivatives and a second-order approximation for time derivative.

Fundamentals of Wave Field Extrapolation

According to the Huygens' Principle, all the points of a wave front may be considered as point sources of spherical secondary waves, which, after a certain period of time characterize the new wave front position being the tangent surface of these secondary waves (Robinson & Clark, 2007).

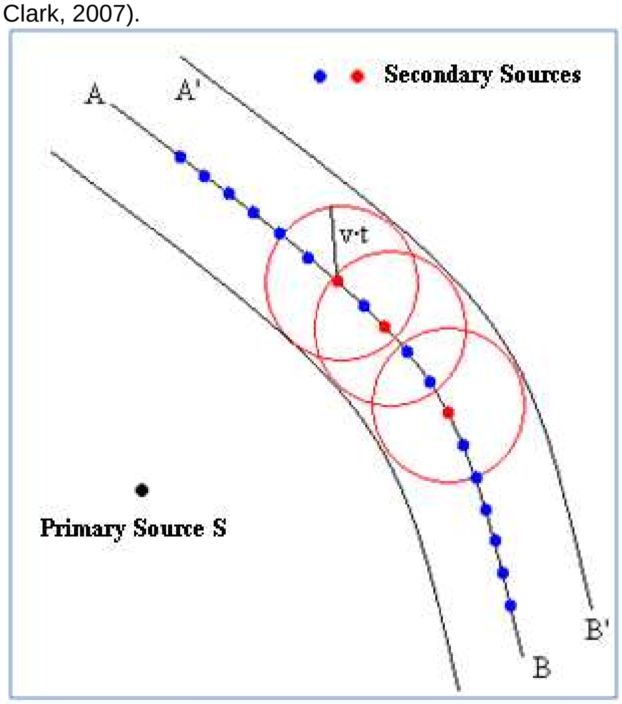


Figure 1: Illustration of Huygens' Principle: each point of the wave front ***AB*** – generated from a primary point source ***S*** – behaves as a secondary point source to the wave front ***A'B'***: a constructive interference between each secondary point source generates a new wave front, represented by the envelope of all secondary waves.

In a more specific way, from an initial wave front ***AB*** generated from a primary source ***S*** (black dot, figure 1), it is considered that all points of this wave front are composed by secondary point sources (red and blue dots, figure 1). Assuming that ***v*** is the propagation velocity for one point of this wave front, it is possible to determine the shape of the wave front ***A'B'*** at a time ***t*** by tracing circumferences centered in each secondary source with (***r=v.t***) radius, see figure 1. The new wave front (***A'B'***) is the envelope of the circumferences (if the medium is considered homogeneous and isotropic[1] the radius of these circumferences will be the same).

The wave front propagation process described by Huygens is fairly intuitive, although it neither quantifies the values of the amplitudes nor described the direction of the wave field propagation (Verschuur, 2006).

In 1883, Kirchhoff described, mathematically, the Huygens' Principle by solving the scalar wave equation, based on Helmholtz's researches (Bucci et al., 1994). In his researches, Kirchhoff has obtained, successfully, the mathematical quantification for the wave field prediction process by the Kirchhoff-Helmholtz Integral. This integral is also known as the Acoustic Representation Theorem, which allows calculating the acoustic pressure of a physic wave field at any point of a medium in terms of a volume and a closed surface integrals. This representation was built performing studies in a medium characterized as a volume ***D*** enclosed by a surface ***S*** (figure 2), being described by the expression, in the frequency domain (Wapenaar & Berkhout, 1989) as:

$$P(\vec{r}_A,\omega) = \frac{-1}{4\pi}\oint_S \left[P\,\frac{\partial G}{\partial n} - G\,\frac{\partial P}{\partial n}\right] dS \qquad (1)$$

where $\vec{r}_A$ is the position, inside the volume, of the point which is desired to know the pressure field (in this case the point ***A***), ***P*** is the pressure field on the surface ***S***, ***G*** is the Green's Function (which is represented as the impulse response of a source located at the point ***A*** recorded at the surface ***S***) and $\vec{\boldsymbol{n}}$ is an outward normal vector to ***S***.

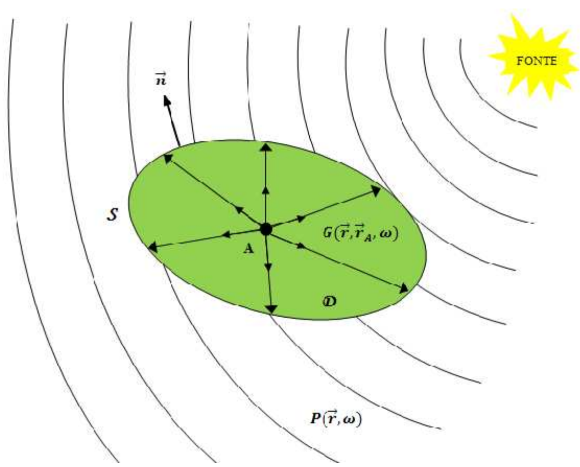


Figure 2: Geometry related to the Kirchhoff-Helmholtz Integral, which describes a pressure field at the point ***A*** based on measures on the surface (***S***) of a volume (***D***). The expression (1) allows calculating the wave field at any point inside the volume, by measures of the field on the surface of it.

The Green's Function calculation is a step that requires some care of interpretation. Its definition is described as an impulse response of a medium to an impulsive source applied at a certain position (figure 3). The source's function is an impulse in a time $\boldsymbol{t = t_0}$, located in a position $\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_{\boldsymbol{0}}$**.** The Green's Function is described by the source effect (i.e., the effect of this impulse) according to the shift in time and space. In the 2-D synthetic seismic modeling, the Green's wave field is that impulsive response convolved with the source function (in this work, the source function used is the second derivative of Gaussian's[2] function, introduced by Cunha (1997)). Therefore, the Green's Function presented in this work is a representative wave field.

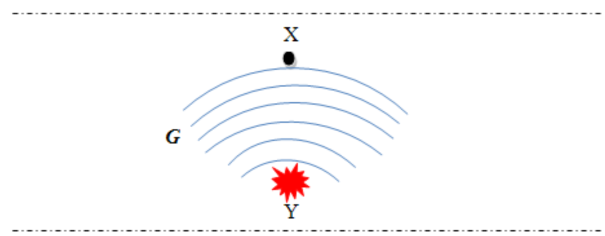


Figure 3: Illustration to the Green's Function (G) interpretation.

The expression (1), calculates the pressure field at a point inside a closed volume. Nevertheless, as the seismic configuration is characterized by an open, flat and extensive volume (figure 4), the application of this expression for this case would not be consistent. Moreover, the only information known is the pressure wave field on the earth's surface.

With this new configuration, the expression (1) can be simplified, since the terms of the integral become identical (in module), being possible added them. Moreover, the negative signal was also removed, since it was chosen the normal vector pointing downward (Wapenaar & Berkhout, 1989):

$$P(\vec{r}_A,\omega) = \frac{1}{2\pi}\int_S \left[P\,\frac{\partial G}{\partial z}\right] dS \qquad (2)$$

This expression is known as Rayleigh Integral II, which can be interpreted as a tool that reconstructs the wave field at a point ***A*** (position $\vec{r}_A$) by the sum of scaling (amplitude) and displacement (wave field) in time, throughout the surface ***S*** (Verschuur, 2006).

With this simplification, only the pressure field and the Green's Function (both of them are synthetic

[1] The same properties for all points in all directions of the medium.

[2] $f(t) = [1 - 2\pi(\pi.f_c.t)^2]e^{-\pi(\pi.f_c.t)^2}$, where f_c is the central frequency given by $f_c = \frac{f_{corte}}{3\sqrt{\pi}}$ (f_{corte} is the cut frequency) and *t* is the time.

seismograms) derivative are required (reducing the calculations). The Green's Function derivative is obtained from the central finite difference method[3].

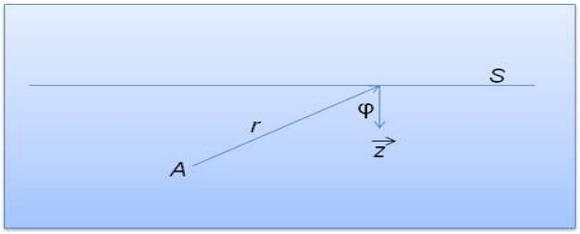


Figure 4: Simplification for the seismic case configuration: from pressure field measures on the surface S, it is possible to extrapolate the wave field to the point ***A***. In practice, the model is flat and infinite (not a closed volume). Considering this, it becomes possible to simplify the mathematical problem in order to apply the methodology.

It is possible to observe that, theoretically, these equations will provide the exactly extrapolated wave field whether all the medium properties are known – to the Green's Function calculation – (this situation does not occurs in real seismic survey) or the medium is homogeneous (if so, the medium where the physic wave field and Green's wave field are equals) (Martins, 2006).

Wave Field Extrapolation to a point – 2-D Synthetic Acoustic Seismic Modeling

From the Finite Difference Method (with fourth-order spatial and second-order time approximations for the scalar wave equation), the wave field extrapolation was modeled in a 2-D homogeneous medium with dimensions 3000mx3000m and velocity 1500m/s. The initial goal is to extrapolate the previously recorded wave field at 750m depth, to the point ***A*** = (750m, 1875m) of the model, using a convolution process between traces of the known field (known as P), recorded at 750m depth and the Green's wave field derivative at the same depth (750m). Subsequently, it will be performed the forward extrapolation, which the wave field will be recorded directly at the desired point. After that, it will be presented an analysis with these results, seeking for a match between them.

Initially, given a source applied at a position (1500m, 22m), the wave field is recorded (being identify as the field ***P*** of the Rayleigh Integral II (equation 2)) at 750m depth (figure 5).

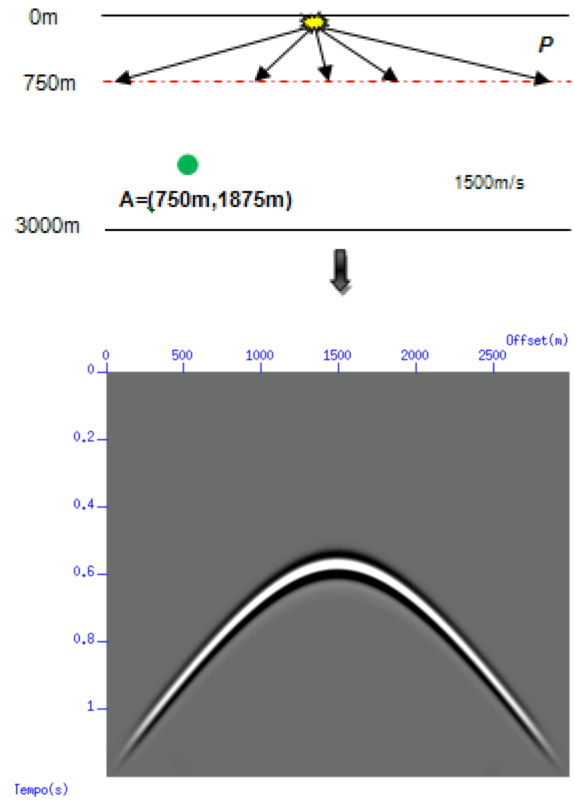
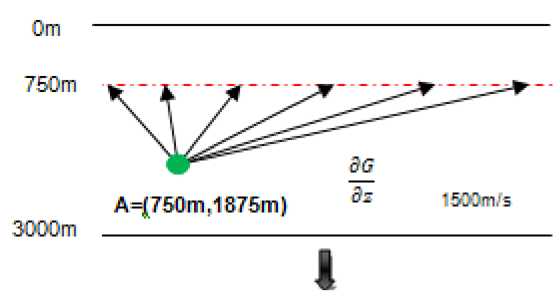


Figure 5: Illustration of the wave field acquisition at a certain depth followed by the equivalent synthetic seismogram: wave field ***P*** for a shot at (1500m, 22m) recorded at 750m depth.

After that, the Green's function derivative can be obtained using the central difference concept which represents the Green's wave field derivative, at 750m depth for a source applied at (750m, 1875m).



[3] $\left[\frac{dG}{dz}\right|_{z=i} = \frac{1}{2\Delta z}(G_{i+1} - G_{i-1})\Big]$

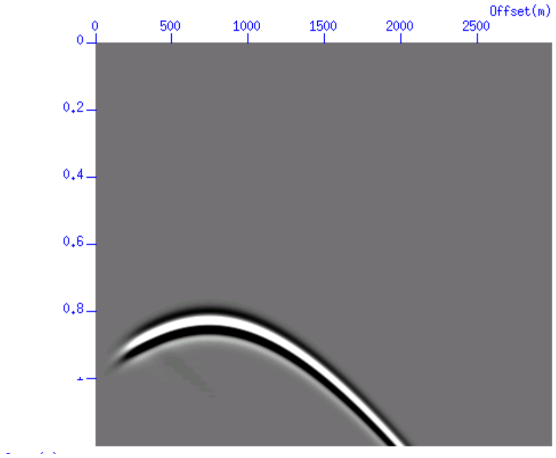


Figure 6: Illustration for the Green's wave field followed by the equivalent synthetic seismogram: Green's wave field derivative for a shot at (750m, 1875m) recorded at 750m depth.

Both wave fields (figure 5 and 6) are trace-by-trace time convolved, generating a single trace (figure 9) represented by the wave field of figure 8 – the extrapolated field to the desired point.

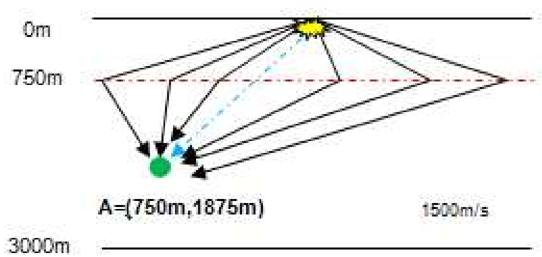


Figure 7: Illustration of the convolution process: all the traces of the pressure field ***P*** are time convolved with all Green's field derivative traces and then the result is summed, resulting in a single trace (blue arrow), which represents the summation of the extrapolated wave field to the point ***A***.

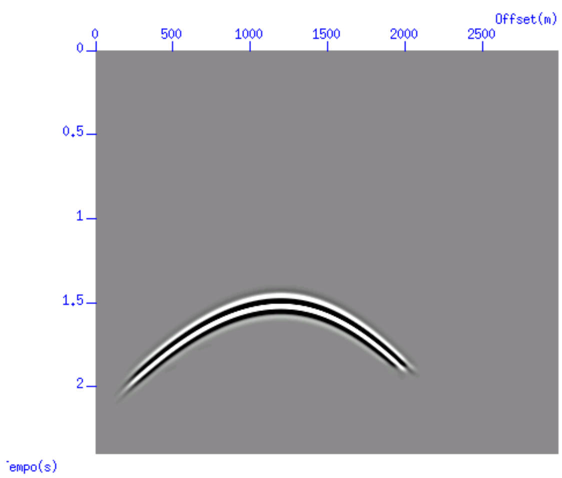


Figure 8: Wave field related to the convolution between the field P and the Green's wave field derivative.

The convolution traces (figure 8) are summed (figure 9), generating a single trace, which represents the extrapolated wave field to the point ***A*** = (750m, 1875m).

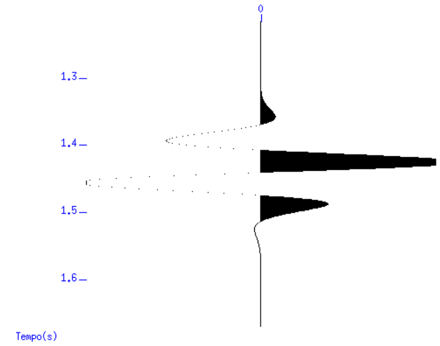


Figura 9: Trace related to the extrapolated wave field to the desired point: summation of all traces of the figure 8 wave field.

Looking at the figures 8 and 9 simultaneously (figure 10), it is possible to observe that the only contribution to the convolution summation was the wave field around the stationary point ***A*** (Verschuur, 2006). This fact is a consequence of the Fresnel Zones effects, which provide contributions related to both constructive and destructive interferences to the total field.

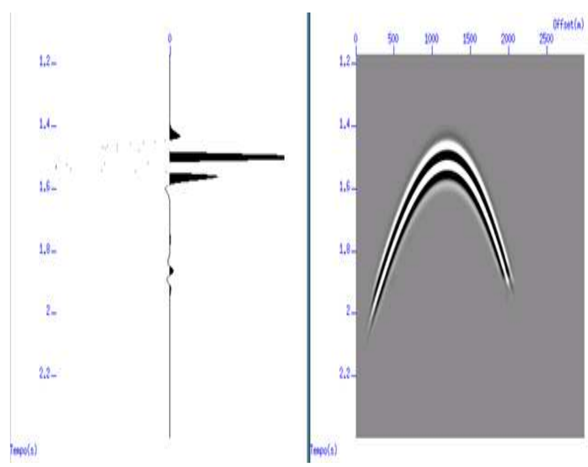


Figure 10: Illustration related to the contribution of the convolution traces to the wave field extrapolation for a point (the seismogram presented in this figure is the same of figure 8, with just a zoom view): just the traces around the stationary point (in this case, ***A*** = (750m, 1875m)) contributes in the summation.

In order to evaluate the extrapolation process, i.e., whether the theory is actually equivalent to the practical (seismic modeling) or not, another modeling was performed making the directly acquisition at the point where it is desired to know the wave field (i.e., ***A*** = (750m, 1875m)).

The source was applied at the same position as the previous example (1500m, 22m) and the wave field was recorded at (750m, 1875m). After that, this wave field was time convolved with the source function, obtaining the trace that represents the wave field directly extrapolated to the point ***A*** (figure 11).

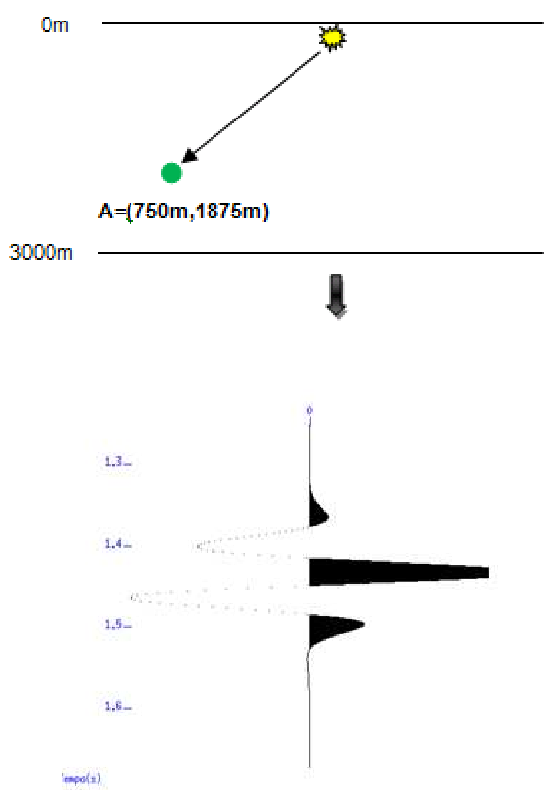


Figure 11: Trace related to the convolution between the wave field directly recorded at the point ***A*** = (750m, 1875m) and the source function.

Theoretically, these traces should be matching, in time and amplitude. Nevertheless, the computational modeling process is accomplished by means of approaches – the central finite difference method with fourth-order spatial and second-order time approximations – that may influence in some way, even small, on the generation of such data. In this modeling, this interference is characterized as a small discrepancy in time and amplitude of the traces (figure 12).

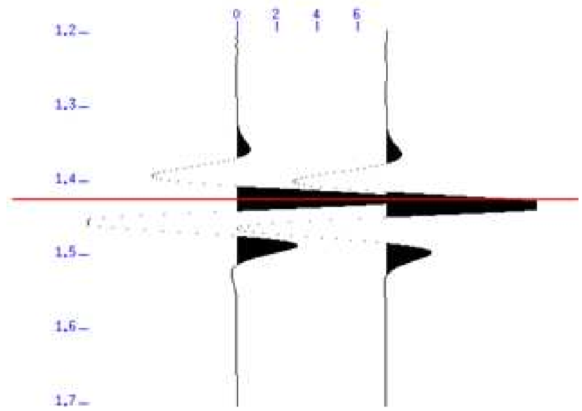


Figure 12: Comparison between the theoretical extrapolation (Kirchhoff-Helmholtz approach) (first trace of this figure) and the record of the desired point directly (second trace). The red line was designed as an auxiliary way to the data analysis.

Conclusions

The wave field extrapolation by means of Rayleigh Integral II has been a fairly useful tool in the prediction of the wave fields.

Although it was presented just the wave field extrapolation to a point, through this concept, it is possible to extrapolate the wave field to a surface just considering it consisted of several points. Performing the process for all these points, the extrapolated surface will be the result of the concatenation of each trace, as shown in the example.

The small discrepancy found between the extrapolation process by means of Rayleigh Integral II (Kirchhoff-Helmholtz Integral simplification) and the forward extrapolation arouses an investigation for some way to circumvent this problem. A more specific analysis about the source or characteristics in the Green's function modeling process may add important information to solve this problem, since to remove, successfully, the ocean bottom multiple reflections (in the sense of wave field extrapolation) is necessary to performed the correct multiple reflections prediction which depends on the reliable of this methodology.

Acknowledgements

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