Non-hyperbolic term estimation for VTI inversion from walkaway VSP and surface seismic data

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Abstract

Walkaway VSP data provides information for interval anisotropic parameter estimates for transverse isotropy with vertical symmetry axis (VTI). For any depth tomographic/migration velocity analysis, we need an adequate initial depth velocity model. Dix-type approach provides us with explicit inversion formulas for velocity model estimation.

To estimate these interval parameters, using Dix-type inversion approach, we need to estimate non-hyperbolic term of NMO function. In walkaway VSP data, NMO function can be obtained from first breaks, while in surface seismic it is estimated through non-hyperbolic semblance analysis. In both cases, determination of non-hyperbolic term is the most unstable procedure in interval VTI inversion. Because exact explicit formula for reflected time in a layered (even isotropic) media is unknown, we have to use some approximation. Different approximations have different accuracy in different geology. Here we investigate accuracy of different approximations on four models with weak and strong anisotropy and modest and large vertical velocity changes.

Introduction

Inversion of reflection traveltimes can provide anisotropic parameters required for depth seismic processing and imaging. Non-hyperbolic P-wave moveout term for transverse isotropic model with a vertical axis (VTI) depends on the parameter η. To determine this parameter from the surface seismic data, three-term semblance velocity analysis is used Alkhalifah (1997). Knowing η and the vertical velocity from zero-offset VSP data, we can invert for the Thomsen parameters ε and δ, Grechka and Tsvankin (1998). Tsvankin and Thomsen (1995), discussed the feasibility of non-hyperbolic NMO inversion for η estimation. Blias (2009) suggested using walkaway VSP first breaks to obtain NMO function for virtual boundaries at each receiver depth, and then invert it into interval Thomsen parameters ε and δ. In industry, coefficient η and NMO velocity are estimated through semblance velocity analysis using a non-hyperbolic equation, suggested by Tsvankin and Thomsen (1995) and rewritten by Alkhalifah and Tsvankin (1995) in terms of coefficient η. This approximation works well for a homogeneous VTI layer, but for the subsurface with essential vertical velocity changes, it may provide large errors in η estimates. Blias (2009b, SEG) suggested using different NMO three-term approximations and testes the on a model data. However, there were not given any recommendation which approximation should be used in some cases. Here we consider four VTI layered models with different anisotropic and velocity properties, which allow us to come to some important conclusions about different approximations.

The main problem in VTI Dix-type inversion is estimation of non-hyperbolic term. Long-spread P-wave moveout can be estimated through a 2-D semblance scan using equation derived by Alkhalifah and Tsvankin (1995). For short spreadlength (about reflector depth) NMO function can be described with hyperbola, so to estimate non-hyperbolic term we need long offsets. It can be shown that standard deviation of non-hyperbolic term is proportional to $1/L^4$ where L is maximum offset. It implies that we need a long spreadlength, more than 1.5 – 1.8 of the reflector depth. Equation of Alkhalifah and Tsvankin is an approximation to exact NMO equation, and in some cases, using this equation leads to essentially biased estimates.

Non-hyperbolic parameter estimate from surface seismic dara is very sensitive to traveltime and amplitude noise. Extra to times, accuracy of semblance analysis depends on amplitude factors: the dominant frequency, regular noise (multiples, ground roll, super-critical reflections for large offsets, etc.), especially on long offsets, AVO effects. Even for the spreadlength is two times larger than the reflector depth, in most cases, surface seismic provides very low accuracy for non-hyperbolic term estimation. Walkaway VSP data provides longer offsets (up to three reflector depths) and first breaks are free f amplitude factor influence. Blias (2009a) suggested using walkaway first breaks for interval VTI inversion. In this paper, we will investigate influence of NMO approximation errors.

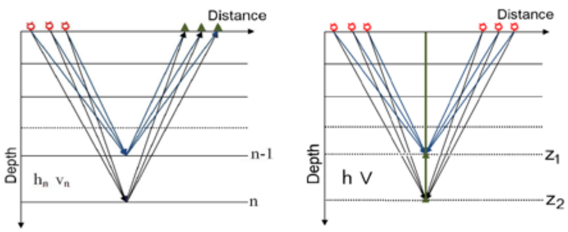
Method

We consider subsurface model composed of horizontal VTI layers. In VTI medium, velocity $V(\theta)$ depends on the angle θ between the ray and the vertical axis, fig. 1. Horizontal plane is the plane of anisotropy. VTI anisotropy is usually attributed to some combination of fine layering an inherent anisotropy of shales. To describe VTI anisotropy, we need five parameters: three for P waves and two for S. Here we consider only P-wave inversion, so we will use three parameters: vertical velocity V and two Thomsen parameters ***ε*** and ***δ***.

Fig.2. shows surface seismic geometry with the sources at the surface and the receivers in a well. Let t(x,z) be the first break time (time arrival for downgoing P wave) where x is the offset and z is a receiver depth. We can find the reflected traveltimes $t_1(x)$ and $t_2(x)$ for two virtual boundaries at the depth z_1 and z_2 of two receivers:

$$t_1(2x) = t(-x,z_1) + t(x,z_1)$$
$$t_2(2x) = t(-x,z_2) + t(x,z_2) \quad (1)$$

Figures 1a and 1b show that times $t_1(x)$ and $t_2(x)$ are the same as the times of the waves reflected from the top and the bottom of the layer number n. This implies that we can apply the same inversion scheme to the times $t_1(x)$ and $t_2(x)$ calculated from walkaway first breaks as if they were the reflected times from horizontal boundaries at the depths z_1 (boundary number "n-1") and z_2 (boundary number "n").



a b

Fig. 1 - Surface seismic ray scheme (a) and walkaway ray scheme (b).

Dix-type VTI inversion and NMO approximations

For time-offset function t(x) we can write:

$$t^2(x) = c_0 + c_1 x^2 + c_2 x^4 + c_3 x^6 + \ldots \quad (2)$$

Coefficients c_0, c_1 and c_2 are connected to interval vertical velocities and Thomsen parameters ε and δ (Blias, 2009a).

There are different three-term NMO approximations, but they all have the same Taylor coefficients c_0 c_1 and c_2. In other words, if $t(x) = F(a,b,c,x)$ is a three-term (three coefficients a, b and c) approximations then

$$F(a,b,c,x=0) = t_0 = \sqrt{c_0} \,,$$

$$\frac{\partial\left(F^2(x)\right)}{\partial\left(x^2\right)}(x=0) = c_1 = \frac{1}{V_{NMO}^2} \quad (3)$$

$$\frac{\partial^2\left(F^2(x)\right)}{\partial\left(x^2\right)^2}(x=0) = 2c_2$$

From VTI inversion theoretical point of view, all these approximaitons are equivalent: we need only time t and its two derivatives at zero offset. However, because all equations are approximate, we obtaine different estimates for the time derivatives at x = 0. It iplies that, in practice, estimate accuracy depends on approximate equation, which is used to estimate interval VTI anisotropic parameters.

Let us consider seven NMO approximations that can be used for non-hyperbolic NMO analysis. They all have the same zero offset time t_0, NMO velocity V_{NMO} and third parameter (S or η) connected to the time derivative at zero offset:

$$t(x) = \sqrt{\frac{t_0^2 + \left(\frac{1}{V_{NMO}^2} + c t_0^2\right) x^2}{1 + c x^2}} \quad \text{(A1)}$$

$$t(x) = t_0\left(1 - \frac{1}{S}\right) + \frac{1}{S}\sqrt{t_0^2 + S\frac{x^2}{V_{NMO}^2}} \quad \text{(A2)}$$

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{V_{NMO}^2} - \frac{2\eta_{eff} x^4}{V_{NMO}^2\left[t_0^2 V_{NMO}^2 + (1 + 2\eta_{eff})x^2\right]}} \quad \text{(A3)}$$

$$t(x) = \frac{1}{2}\sqrt{t_0^2 + \frac{1 - \sqrt{S-1}}{V_{NMO}^2}x^2} + \frac{1}{2}\sqrt{t_0^2 + \frac{1 + \sqrt{S-1}}{V_{NMO}^2}x^2} \quad \text{(A4)}$$

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{V_{NMO}^2\left(1 + 0.5 c x^2\right)^2}} \quad \text{(A5)}$$

$$t(x) = \sqrt{\frac{1}{2}t_0^2 + \frac{1}{V_{NMO}^2}\left(1 - \frac{1}{2}\sqrt{S-1}\right)x^2 + \frac{1}{2}t_0^2\sqrt{1 + \frac{2\sqrt{S-1}}{t_0^2 v_{NMO}^2}x^2}} \quad \text{(A6)}$$

$$t(x) = \sqrt{t_0^2 + \frac{x^2}{V_{NMO}^2} + \frac{1 - S}{4 t_0^2 v_{NMO}^4}x^4} \quad \text{(A7)}$$

Here

$$S = \frac{\left(\sum_{k=1}^{n}\frac{h_k}{v_k}\right)\left(\sum_{k=1}^{n} h_k v_k^3 (1 + 2\delta_k)^2 B_k\right)}{\left(\sum_{k=1}^{n} h_k v_k (1 + 2\delta_k)\right)^2} \qquad c = \frac{S - 1}{4 t_0^2 V_{NMO}^2}$$

where h_k is interval thickness, v_k is interval vertical velocity, ε_k and δ_k are Thomsen parameters and η_{eff} is coefficient introduced by Alkhalifah (1997); $S = 1 + 8\eta_{eff}$. Interval η is connected with ***ε*** and ***δ***:

$$\eta_k = \frac{\varepsilon_k - \delta_k}{1 + 2\delta_k}$$

Equations (A1) and (A4) were suggested by Blias (2009b). Shifted hyperbola (A2) for isotropic model was introduced by Malovichko (1978). Equation (A5) was suggested by Taner et al. (2005). Equation (A7) is Taylor series for t^2 truncated after third term; (A6) is a new equation. Formulas for explicit analytical NMO inversion in terms of coefficient S we derived by Blias (2009a), based by his results on joint P and S inversion (Blias, 1983, 1988); in terms of η were derived by Grechka and Tsvankin (1998).

Model study

To investigate accuracy of different approximations, we considered four models with eight VTI horizontal layers. All models have the same boundaries, but different vertical velocities and anisotropic parameters. Table 1 demonstrates parameters of first two models

Depth	V_{P1}	V_{s1}	ε_1	δ_1	V_{P2}	V_{s2}	ε_2	δ_1
0.25	1.80	0.50	0.12	0.05	1.80	0.50	0.06	0.04
0.55	2.20	0.80	0.20	0.08	2.20	0.80	0.08	0.06
0.85	2.40	0.90	0.15	0.12	2.40	0.90	0.09	0.05
1.20	2.60	1.15	0.24	0.10	2.60	1.15	0.11	0.06
1.55	2.80	1.25	020	0.16	4.80	1.25	0.08	0.04
1.80	4.20	1.80	0.16	0.04	5.20	1.80	0.11	0.06
2.15	4.40	2.1	0.20	0.08	5.40	2.1	0.07	0.05
2.50	4.80	2.4	0.32	0.04	5.80	2.4	0.11	0.06

Table 1. First and second model interval parameters

Depth	V_{P3}	V_{s3}	ε_3	δ_3	V_{P4}	V_{s4}	ε_4	δ_4
0.25	2.50	0.50	0.24	0.19	2.50	0.50	0.06	0.04
0.55	2.70	0.80	0.20	0.15	2.70	0.80	0.08	0.06
0.85	2.90	0.90	0.26	0.13	2.90	0.90	0.09	0.05
1.20	3.20	1.15	0.24	0.17	3.20	1.15	0.11	0.06
1.55	3.40	1.25	025	0.15	3.40	1.25	0.08	0.04
1.80	3.60	1.80	0.32	0.18	3.60	1.80	0.11	0.06
2.15	3.90	2.1	0.20	0.10	3.90	2.1	0.07	0.05
2.50	4.50	2.4	0.32	0.26	4.50	2.4	0.11	0.07

Table 2. Third and fourth model interval parameters

Coefficients with sub-index j correspond to the model "j", j=1, 2, 3, 4. First model has strong anisotropy, up to 30%. Its interval velocity increases from 1.8 till 4.8 km/s with jump of 1.4 km/s at the sixth layer. Second model has weak anisotropy (below 11%) and a large velocity jump in in fifth layer from 2.6 km/s to 4.8 km/s. Third model has strong anisotropy and essential gradual velocity increase with depth. Last model has weak anisotropy and modest vertical velocity changes.

For each model, time-offset functions were calculating through raytracing. Then a random time errrs were added to caculated time functions; these errors are between -3ms and +3ms. Each NMO approximation (7) - (13) was used to approximate the NMO curve using the least-squares method for three parameters. Let us analyze the modeling results for these models.

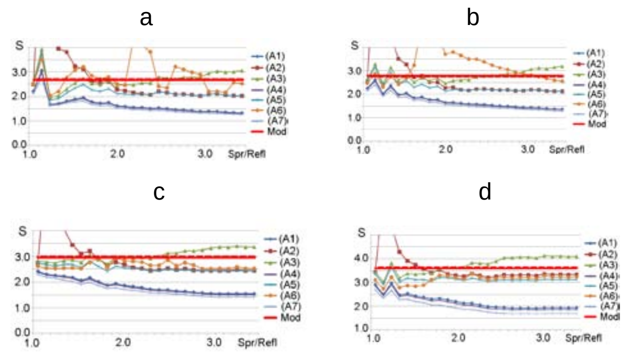


Fig. 2. Model 1 with strong anisotropy and essential vertical velocity changes. Coefficient S estimates using different approximations and different muting. Red line shows model value. Boundaries 4 (a), 5 (b), 7 (c) and 8 (d).

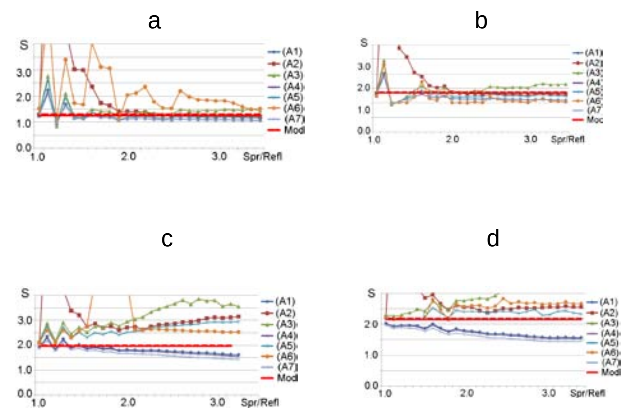


Fig. 3. Model 2 with weak anisotropy and large velocity increase in a layer 5. Coefficient S estimates using different approximations and different muting. Red line shows model value. Boundaries 3 (a), 4 (b), 5 (c) and 7 (d).

Figure 2 shows S-estimates for the first model with strong anisotropy and essential vertical velocity increase. Horizontal axis in this figure (and figures 5 - 7) is muting: We see that approximation (A3), used in the industry, provides accurate results for non-hyperbolic coefficient S estimates. It also shows that accurace strongly depends on muting, that is, on the ratio of the spreadlegth to reflector depth. For each equation, there is an optimium muting value that leads to the most accurate estimate.

Fig. 3 demonstrates S-estimates for the second model with weak anisotropy and large vertical velocity increase in fifth layer: from 2.6 km/s to 4.8 km/s. In fig. 4a and 4b (bundaries 3 and 4), most of approximations, incuding (A3) provides accurate results for non-hyperbolic term estimates. For upper bundaries 3 and 4, approximations (A2) and (A5) provide accurate estimates while equation (A3) leads to arger errors with spreadlength increase. For the boundaries 5 and 7, below large velocity increase (fig. 4c and 4d), equation (A3) lead to large errors. For boundary 5, the most accurate approximaitons are (A2) and (A5).

Fig. 4 shows S-estimates for the third model with strong anisotropy and modest vertical velocity changes, approximation (A3) provides the highest accuracy.

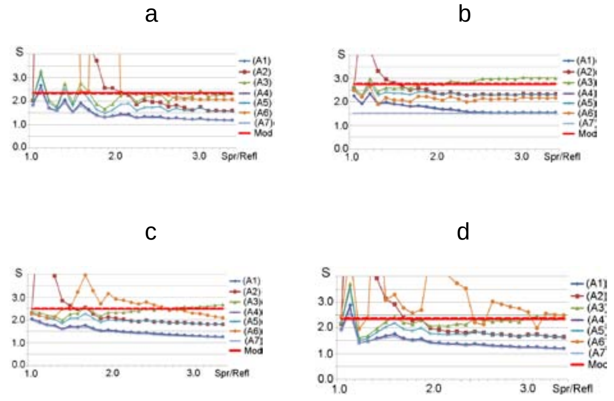


Fig. 4. Model 3 with strong anisotropy and modest vertical velocity increase. Coefficient S estimates using different approximations and different muting. Red line shows model value. Boundaries 4 (a), 5 (b), 7 (c) and 8 (d).

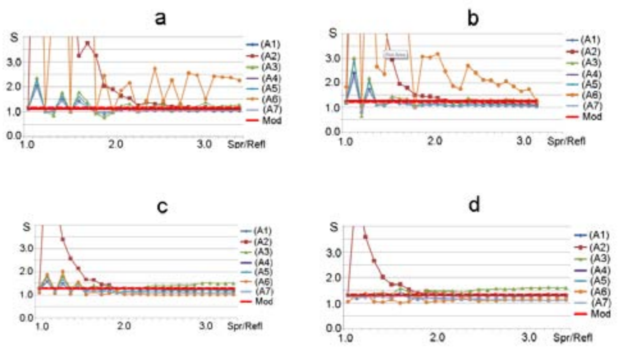


Fig. 5. Model 4 with weak anisotropy and modest vertical velocity increase. Coefficient S estimates using different approximations and different muting. Red line shows model value. Boundaries 4 (a), 5 (b), 7 (c) and 8 (d).

Fig. 5 shows S-estimates for the fourth model with weak anisotropy and modest velocity increase. Approximation (A3) provides quite accurate S-estimates for all boundaries except the deepest one. For deep reflectors (Fig. 5c and 5c) equations (A1) and (A5) have higher accuracy. For deep reflectors (Fig. 5c and 5d) equations (A1) and (A5) have higher accuracy. For deepest reflector, equation (A3) leads to large errors, while equations (A1), (A4) and (A7) provide accurate S-estimates.

Conclusions

Dix-type of NMO inversion provides initial anisotropic layered model from walkaway first breaks. The most challenging problem for VTI NMO inversion is estimation of non-hyperbolic coefficient. Different approximate equations can be used to estimate interval anisotropic parameters. For layered subsurface with gradual vertical velocity changes, equation (A3), suggested by Tsvankin and Alkhalifah, may provide accurate estimates of non-hyperbolic NMO coefficient. For this, optimum muting should be used. For most cases, the optimum spreadlength is 2.5 times of the reflector depth. For deep reflectors, other equations may provide better estimates.

At the same time, when we have large increase in interval velocity, equation (A3) leads to large errors in S-estimates. In this case, other approximations provide much more accurate estimates. This implies that we should use different equations to estimate effective coefficient η in different situations.

Effective η, obtained from surface seismic data, has much less accuracy than from walkaway VSP first breaks. It implies that walkaway VSP should be used to constraint surface seismic η estimates and anisotropic velocity model building.

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