Dynamic Effective Stress Coefficient: Results and Issues

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Abstract

Pore pressure prediction from seismic data is of paramount importance in oil exploration and production. During the exploration phase, seismic data may be used to access potential abnormal pore pressure regions, often known as geopressure zones. On the development phase, along the lifetime of a field, it is desirable to know the location of pressure barriers, in order to plan new injector wells. The estimation of pore pressure from seismic is usually based on the behavior of compressional-wave velocity or impedance with stress. This paper discusses the concept of effective stress, often misused on velocity to pressure transforms, and presents some experimental estimative of the effective stress coefficients. Some practical caveats on the measurement technique are also illustrated through the use of an idealized synthetic experiment.

Introduction

The knowledge of geopressure areas prior to well drilling is capital for petroleum exploration. The choice of drilling fluid and well casing may involve the largest expenses in a drilling program. An inadequate project can lead to unexpected costs, loss of time, fluid, well and even severe material and human life losses due to catastrophic events like blow-outs and rig fires. Some robust methods to abnormal pore pressure prediction, like that proposed by Eaton (1975), involve the seismic velocities and its behavior with depth or effective stress. In general, these studies assume that the effective stress, which governs the seismic velocities, is simply the difference between lithostatic stress and the pore pressure.

During the production history of an oil field it may be interesting to access the fluid pressure, in order to achieve a better injector wells location plan.

The majority of published data on pore pressure estimation from seismic data relies on the velocity dependence on stress, and assumes that the velocity is a function of the differential stress. This assumption agrees with the definition of effective stress, introduced by Terzaghi in 1923, as being the excess of the total stress over the neutral stress (pore pressure), that acts exclusively in the solid phase of soils. In fact Wyllie (1958) had shown that this is an excellent approximation for Berea Sandstone saturated with water. Nevertheless, several experimental and even theoretical studies suggest that, in some cases, it is not true.

Rather than depend on the differential stress, the velocities are functions of the effective stress which, in the lithostatic or isotropic case, can be defined as

$$P_e = P_C - n\,P_P \qquad (1)$$

The quantity n is the so-called effective stress coefficient or pore pressure coefficient that, in general, may be different to 1. P_C is the confining stress and P_P the pore pressure.

The effective stress concept may include any linear combination of confining and pore pressure that allows a reduction of independent variables. If we consider some physical property Q of a porous media that depends on the total stress (here confining pressure) and also on pore pressure, that acts hydrostatically over all the grain free surfaces, $Q = Q(P_C, P_P)$, the effective stress would be any linear combination as in Equation (1) such that $Q = Q(P_C, P_P) = Q(P_e)$.

On the theory of consolidation of porous media, Biot and Willis (1957) had introduced the coefficient

$$\alpha = 1 - \frac{K_{dry}}{K_{sol}} \qquad (2)$$

where K_{dry} and K_{sol} are the bulk moduli of the dry rock and the solid fraction, respectively. In fact, it was shown by Nur and Byerlee (1971) that, for the volumetric bulk compression of a porous media, the effective stress coefficient n is identical to the Biot-Willis coefficient α.

Todd and Simmons (1972) concluded that, for the compressional-wave velocity, the pore pressure coefficient or effective stress coefficient can be written as

$$n = 1 - \frac{\left(\dfrac{\partial V_P}{\partial P_P}\right)_{P_d}}{\left(\dfrac{\partial V_P}{\partial P_d}\right)_{P_P}} \qquad (3)$$

where $\left(\partial V_P / \partial P_P\right)_{P_d}$ and $\left(\partial V_P / \partial P_d\right)_{P_P}$ are the partial derivatives of the velocity with respect to the pore pressure, for constant differential stress, and to the differential stress, for a constant pore pressure, respectively. These derivatives may be obtained from special velocity measurement experiments on the laboratory, as illustrated in Figure 1, and *n* might be interpreted as an empirical pore pressure coefficient. This empirical *n* can be extended to any physical property *Q* of a porous media, by replacing the compressional-wave velocity on Equation (3) by this property *Q*.

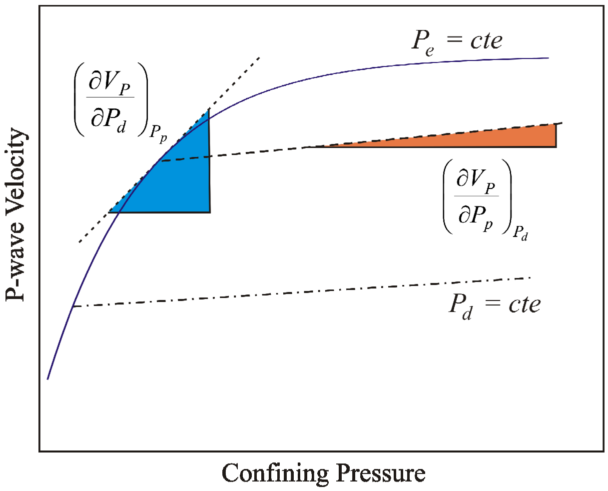

Figure 1 – *Schematic diagram illustrating how to obtain the partial derivatives in eq. (3) to compute the empirical effective stress coefficient as proposed by Todd and Simmons (1972).*

Note that, according to this graphical interpretation, a coefficient *n*=1 means the pore pressure compensates the effect of confining pressure, while *n*<1 means the effect of confining pressure is greater than that of pore pressure and *n*>1 means the pore pressure surpasses the confining pressure effect.

The few reported experimental results on *n* determination for natural rocks are not very conclusive. For instance, King (1966) found *n* values greater than 1 for the compressional and shear-wave velocities on Boise, Bandera, Berea and Torpedo sandstones. Christensen and Wang (1985) found *n*<1 for compressional-wave velocity and bulk modulus and *n*>1 for shear-wave velocity and Poisson's ratio of Berea sandstone. They suggest that the different behavior may be due to the presence of clay enveloping the grains. Prasad and Manghnani (1997) as well as Xu *et al*. (2006) found effective stress coefficients *n*<1 for Berea, Michigan and Lyons sandstones. Such a variety of results indicates that the behavior of *n* with stress and petrophysical properties needs further investigation.

Gurevich (2004) discussed the validity of the effective stress concept and showed that, for a rock with homogeneous and linearly elastic solid phase the coefficient *n* for the seismic velocities must be equal to 1. He claims that the violation of the homogeneity and linear elastic behavior may be the cause of the different experimental results. Berryman (1993) derived effective stress coefficients for some properties of rocks composed of different mineral constituents, showing that each physical quantity may be governed by a particular effective stress, associated with a given effective stress coefficient.

We present results on measurements of the effective stress coefficients on Brazilian reservoir rocks, as well on outcrop samples. A numerical experiment on an ideal rock with *n*=1 was made and had shown that there are some errors intrinsically associated to the estimation method.

Experimental Method

The effective stress coefficients were measured according to the scheme proposed by Todd and Simmons (1972). Elastic velocities were measured at ultrasonic frequencies on the laboratory, by the pulse transmission technique, on a set of poorly consolidated sandstone samples from an offshore Brazilian oil field, as well on two outcrop samples: Botucatu and Berea Sandstones.

From the experimental data, empirical pore pressure coefficients *n* were computed for compressional-wave and shear-wave velocities and also for the bulk and shear moduli. The empirical coefficients were derived using Equation (3). The velocity curves at constant pore pressure values were adjusted by a smooth function to obtain the partial derivatives while the velocity data for constant differential stress were fitted by straight lines. As reported by Avseth *et al*. (2005), the velocity may be expressed as a function of the differential stress as

$$V(P) = a_M - b_M \exp\left(-\frac{P_d}{c_M}\right) \quad (4)$$

Some of the samples were saturated with fresh water, while others were saturated with ethanol, in order to investigate the effect of different fluids on the effective stress coefficient *n* of similar rocks. The outcrop rock samples were saturated only with fresh water.

It is important to notice that the bulk modulus of the pore fluid varies with the pore pressure. This effect can be removed using Gassmann's (1951) equation, although this approach does not take into account frequency related effects on seismic velocities. The normalized bulk modulus K_N can be obtained from

$$\frac{K_N}{K_{ma} - K_N} = \frac{K_{sat}}{K_{ma} - K_{sat}} - \frac{1}{\phi}\left[\frac{K_f}{K_{ma} - K_f} - \frac{K_{fN}}{K_{ma} - K_{fN}}\right] \quad (5)$$

were K_{ma} is the bulk modulus of the solid material, K_{sat} is the rock bulk modulus measured on the experiment with a saturating fluid with bulk modulus K_f. K_{fN} is the fluid bulk modulus used for normalization and ϕ is the porosity of the rock sample. In this study the pore pressure 250 psi (1.72 MPa) was chosen as the normalization condition. The fluid properties were estimated with the models published by Batzle and Wang (1992).

Sample Description

The present study was conducted with a turbiditic unconsolidated sandstone from an offshore Brazilian oil field, aging from Late Oligocene to Early Miocene. Two outcrop rocks, relatively well consolidated, were also used for verification purposes: the Berea and Botucatu Sandstones.

The oil field reservoir sandstone samples comprises loose, clean sands with less than 2% clay, medium to fine and very fine grained, poorly sorted, with angular grains. The porosities were from 28 to 32 % and the permeability from 1500 to 3000 md.

The Berea Sandstone is an arcosic coastal sandstone, fine to medium grained, well sorted, with subrounded grains cemented by silica and, eventually, calcite. The particular sample used has 18% porosity and 213 md permeability. The Botucatu Sandstone is a quartzarenite originated on an eolic environment; it is reddish sand due to subaerial oxidation, well sorted, and fine to medium grained. The grains are well rounded; the porosity of the sample is 21.9 % and permeability 382 md.

These measurements on outcrop rocks aimed to verify the behavior of the effective stress coefficient of rocks usually chosen as “reference” on academic as well on oil industry reports. Although no natural rock behaves as a reference or standard, these rock samples has a relatively homogeneous pore space, when compared to real reservoir rocks.

The results obtained on these rock samples are compared with previously reported data by Vasquez *et al*. (2009) on other reservoir rocks from offshore Brazil, including a relatively unconsolidated limestone and a well consolidated sandstone. These limestone samples comprise algal biolitites and calcirudite to rodolites with a matrix that varies from micritic to calcarenitic, aging from Late Oligocene to Early Miocene. Some of the samples exhibit siliciclastic grains, composed mainly by quartz. The porosity of the rock samples ranges between 23 and 34 %, and its permeability from 2 to 1240 md. The consolidated sandstone samples are from Santonian age, and comprise arcosic sandstones with more than 50 % feldspar, fine to medium grained, poorly to moderately sorted and rich in chlorite. The porosity of the particular samples studied ranges from 13 to 19.4 % and permeability from 0.3 to 16 md.

Results

Figures 2 and 3 illustrate the typical velocity data collected on this study, for a loose sand sample saturated with water. On this example compressional and shear-wave velocity were measured for confining pressures up to 3500 psi (24.13 MPa) and pore pressure were increased from ambient up to 2500 psi (17.24 MPa). Velocity measurements were made at each 250 psi (1.72 MPa) confining stress step, for constant pore pressure values. It was verified that the velocity does not depend on the stress path, but only on the confining stress and pore pressure pair.

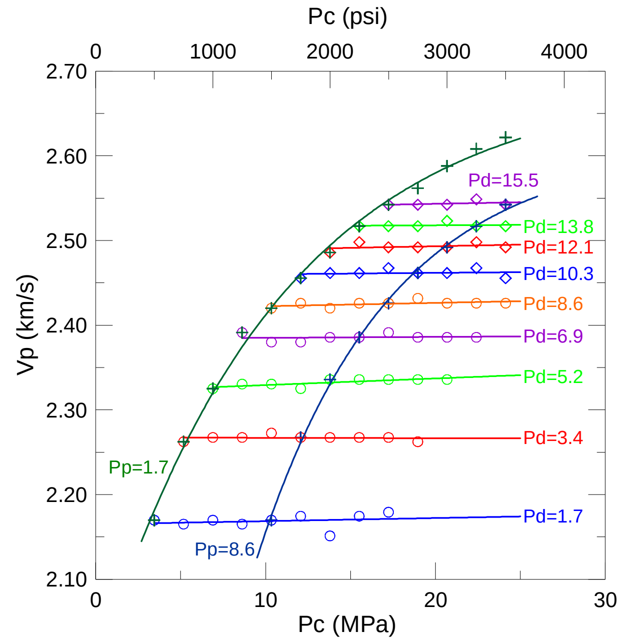

Figure 2 – *Compressional-wave velocity versus confining stress for loose sand sample 6155 for several differential stress values. Velocity curves for two specific pore pressure values are also shown.*

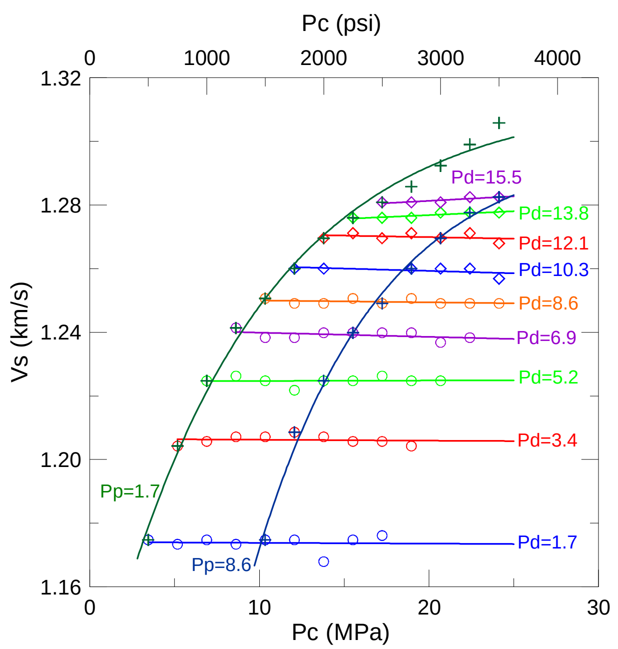

Figure 3 – *Shear-wave velocity versus confining stress for loose sand sample 6155 for several differential stress values. Velocity curves for two specific pore pressure values are also shown.*

Figures 4 and 5 are examples for the same sand sample with water, illustrating the bulk modulus before and after the normalization process with aid of Equation (5). It can be noticed that there is a change in the gradient of the data corresponding to constant differential stress, which implies in a change in the effective stress coefficient.

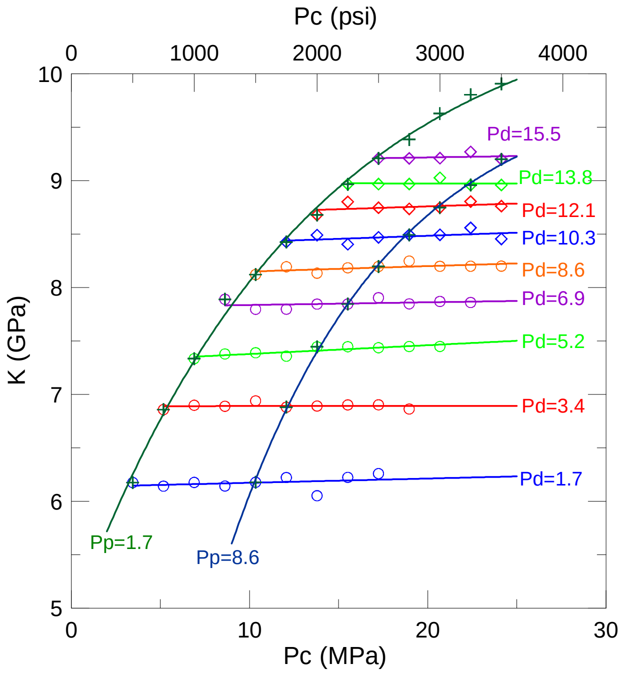

***Figure 4** – Bulk modulus computed for sample 6155 versus confining stress for several differential stress values. Curves for two specific pore pressure values are also shown.*

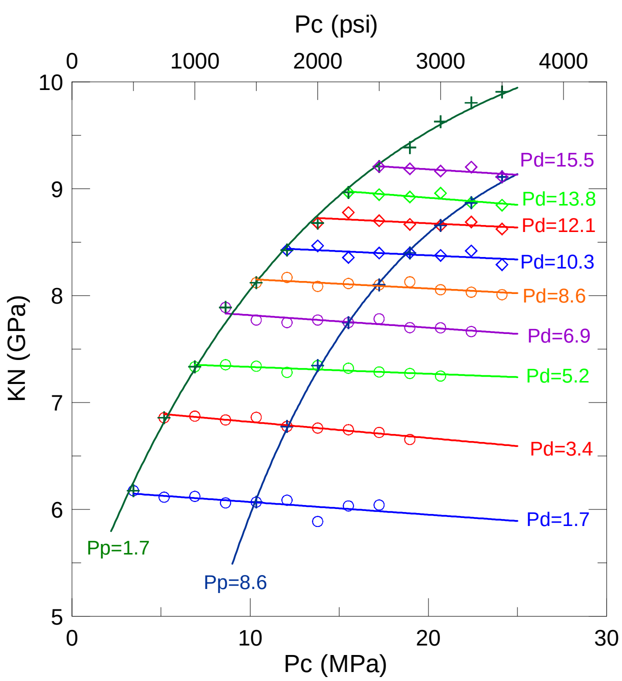

***Figure 5** – Bulk modulus of sample 6155 after the normalization with eq. (5).*

The effective stress coefficient for the dynamic bulk modulus (computed from the velocity data) can be compared to the Biot-Willis coefficient. In general, but not always, it was found that the dynamic empirical effective stress coefficient is greater than the Biot-Willis coefficient given by Equation (2). Figure 6 presents the comparison between these two estimative for the loose sands saturated with water at 250 psi pore pressure (1.72 MPa), as a function of differential stress.

From Figure 6 it can be seen that the effective stress coefficient for the dynamic bulk modulus may be even greater than 1, although the Biot-Willis coefficient cannot be smaller than zero nor greater than 1. It must be kept in mind that the effective stress coefficient is a measure of the influence of pore pressure and confining stress on the seismic behavior, and is not constrained by the dry rock and mineral modulus, as the Biot-Willis coefficient. For higher differential stress values, the estimated effective stress coefficients exhibit some fluctuations. It was observed on other rocks previously analyzed as well.

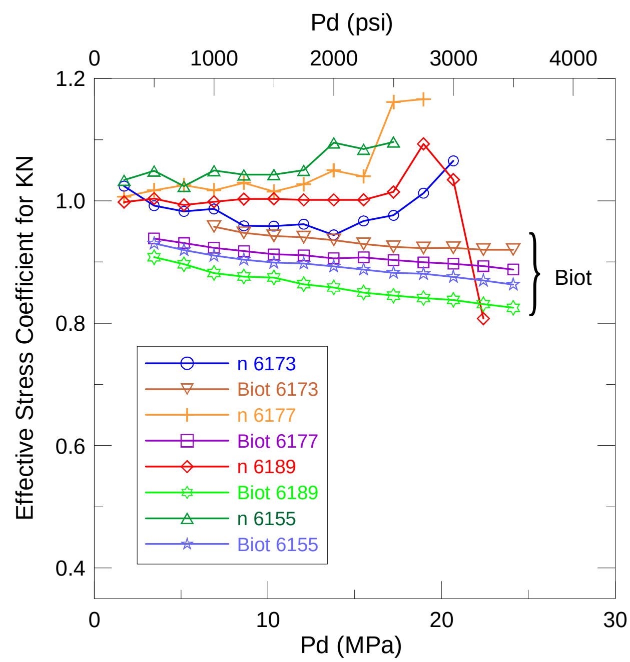

Figure 6 – *Effective stress coefficients for the loose sands with water along with the Biot-Willis coefficient as a function of differential stress for a 250 psi pore pressure.*

For the water saturated sands, *n* values from 0.95 up to 1.05 were obtained, while on the samples with ethanol these coefficients were in the range between 0.8 and 1.1. There is some evidence that the effective stress coefficient may depend on the fluid type, but no conclusive assertion can be done with the results from this study.

The effective stress coefficients for the outcrop samples are close to 1 as well, as illustrated in Figure 7 for a constant pore pressure of 1000 psi (6.89 MPa). It is worth noticing that these rocks are less porous than the reservoir rock samples. Note also that even in this case there is a peculiar behavior for high differential stresses.

In Figure 8 a comparison of the present results with those from previous studies involving reservoir rocks is illustrated. The effective stress coefficients for water saturated rocks are plotted as a function of rock porosity for a particular stress state (2500 psi confining stress and 500 psi pore pressure). There is a clear trend of increasing n with increasing porosity. Note that the rocks exhibit very distinct textures and compositions.

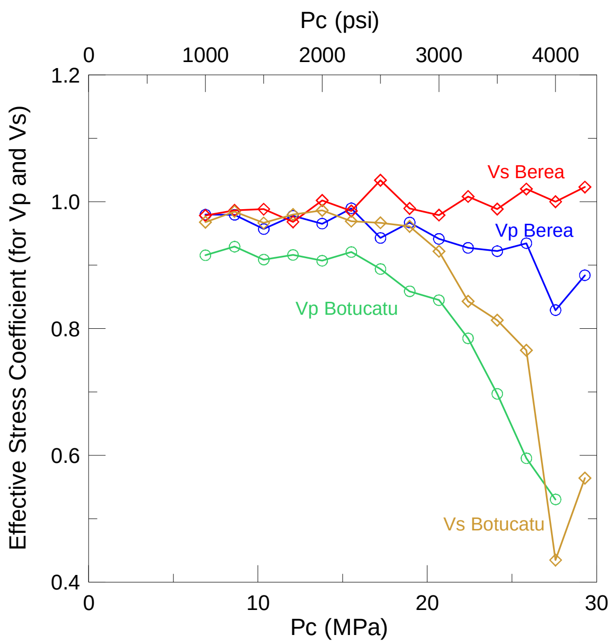

Figure 7 – *Effective stress coefficients for compressional and shear-wave velocities of the Berea and Botucatu sandstones.*

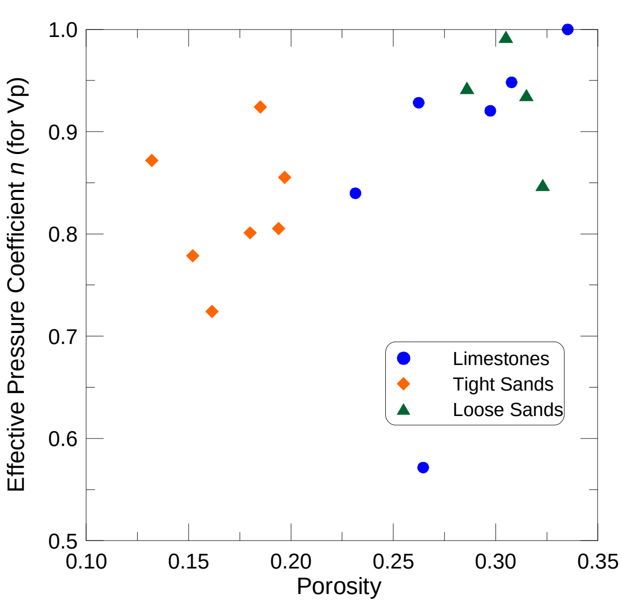

Figure 8 – *Effective stress coefficients for the compressional-wave velocity on the reservoir rock samples as a function of porosity, for a 2500 psi confining stress and 500 psi pore pressure (17.24 and 3.45 MPa, respectively). In spite of the different rock textures, there is a clear trend of increasing* n *with porosity.*

Numerical Experiment

In order to get an insight on the uncertainties involved on the effective stress coefficient estimation, a numerical experiment was made. First, synthetic velocity curves for an idealized rock sample with a n coefficient exactly equal to 1 were generated, according to:

$$V_P = 2.876 - 0.8686 \exp(-P_d / 12.26) \qquad (10)$$

where the velocity is in km/s and the differential pressure P_d is in MPa.

Then, a series of velocity data were generated for various pore pressure values, and these data were disturbed with an error of 0.0281 km/s. This "uncertainty" corresponds to a random error of 1 % of the median velocity value for zero pore pressure, and it was simulated by adding a normal distribution with zero mean and standard deviation 0.0562 km/s to the "ideal" or "exact" velocity data. This is a reasonable uncertainty in real measurements, although in practice there are some non-random components in the experiments.

Three data sets were generated. The first one simulated an experiment with the confining stress and pore pressure varying from ambient to 9000 psi (62.05 MPa), and the velocity were "measured" at each 250 psi (1.72 MPa) step. The second data set simulated the confining and pore pressure from atmosphere to 5000 psi, with velocity data at each 250 psi step (this is simply a truncation of the first experiment). The third experiment was just a sub sampling of the second one, simulating velocity measurements at each 500 psi step (3.45 MPa).

The resulting n values are plotted on Figure 9 as a function of confining pressure for a 500 psi pore pressure. It can be noticed that the n coefficients become unstable for differential stresses equal to half of the maximum confining stress used on the experiment. The sub sampling seems to have minor influence on the error introduced on the n values.

These observations made us concerned about how much confident we can be of effective stress coefficients measurements.

Discussion and Conclusions

Experimental results on the effective stress coefficient n of reservoir and outcrop rocks were presented. It was observed that n depends on rock porosity, but also has some influence of texture, composition and even fluid bulk modulus. However, synthetic numerical experiments indicate that experimental n values may be intrinsically contaminated by errors associated with the estimation method. That is an important observation that made us concern about the robustness of effective stress coefficient data obtained by this method.

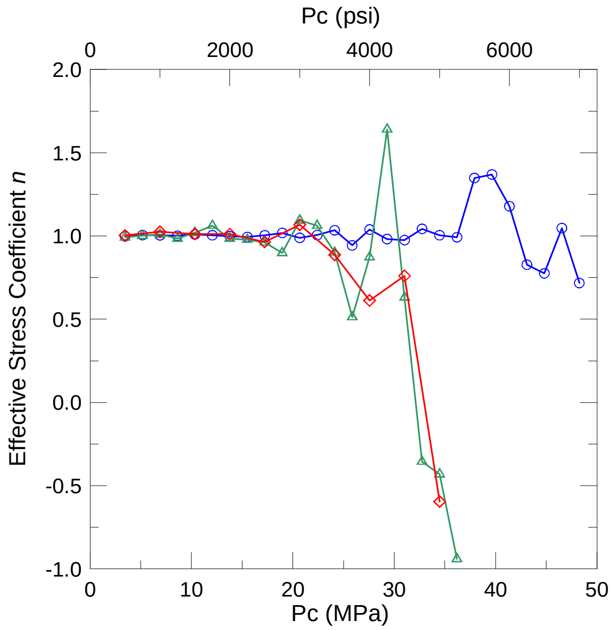

Figure 9 – *Effective stress coefficients estimated from a synthetic experiment for a 500 psi pore pressure (3.45 MPa). The blue circles represents the data generated for confining stresses up to 9000 psi, the green triangles corresponds to the data generated up to 5000 psi confining pressure and pore pressure steps of 250 psi, while for the red diamonds 500 psi "measurement" steps were used.*

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