Geophysical inversion using petrophysical constraints with application to lithology differentiation

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Abstract

Petrophysical information is sometimes available from well logs, laboratory measurements, etc. It has been shown that incorporation of such information into geophysical inversions, especially joint inversion of multiple geophysical data sets collected based on fundamentally different physics, can greatly improve the recovered models. However, when the known petrophysical distribution or relationship is not applicable throughout the entire region under investigation, the application of petrophysical constraints to geophysical inversions could seriously distort the resulting models. In this study, we propose a new inversion algorithm that is capable of building the petrophysical information into the inversion, while overcoming the aforementioned problem. The lithology differentiation is accomplished by applying fuzzy c-means (FCM) clustering algorithm to the models inverted from this new method. We illustrate this method with a synthetic crosshole seismic example and a gravity survey that measures both surface and borehole gravity response.

Introduction

Geophysical inversion has been widely used in mineral and petroleum explorations, as well as in large-scale geodynamic studies. Inversion provides a quantitative description of the distribution of physical properties of interest (e.g. density, velocity) from measured geophysical data, which serves as an important avenue to understanding the subsurface structure.

It has been realized that inversion of a single geophysical data type may sometimes fail to give a reasonable image of the subsurface structure. To deal with this situation, geophysicists have developed algorithms that constrain geophysical inversion with geologic information (Li and Oldenburg, 2000; Farquharson et al, 2008, Lelievre, 2009). The idea behind such constrained inversions is that geophysical models should be consistent with all available a priori geologic information. The geologic information that can be incorporated into geophysical inversions includes, but is not limited to the following: physical property measurements, structural orientations, geostatistical information about the physical property distribution of each rock type present, expected shape of a target, etc. In this study, we only consider the petrophysical information given by physical property measurements, and present a new algorithm that effectively builds this valuable information into inversion by means of fuzzy c-means (FCM) clustering technique.

One important question with inversion constrained by petrophysical information is the applicability of such information. When the available petrophysical information is valid throughout the model region, inversion algorithms that effectively make use of such a priori information usually produce better models than would otherwise be possible without petrophysical constraints. However, when the petrophysics only describes part of the model region, application of such petrophysical constraints to the whole model region could seriously deteriorate the inverted models (Moorkamp, 2011). We show that our proposed inversion algorithm can deal with this situation where only part of the subsurface petrophysical information is available.

An image of the subsurface rock physical property distribution can be readily obtained from geophysical inversion. These physical properties are macroscopic parameters closely related to lithology (Bosch, 1999), and their relationship has been extensively studied in the field of rock physics. Generally, different lithologies have distinct ranges of physical properties such as density and velocity. Inference about the subsurface lithology can then be made by grouping physical property values obtained from geophysical inversions into several clusters on the basis of their distances from each other. FCM clustering technique is a powerful tool to explore the similarity between data (e.g. physical property values in this study) and classify the data under consideration into clusters (i.e. lithologies in this study) rapidly and objectively (Bezdek, 1981; Duda et al, 2000). Therefore, the task of differentiating between different lithological regions is accomplished by applying FCM clustering algorithm to the reconstructed physical property models obtained from our proposed new inversion method.

In the following, we begin with a brief introduction of the formulation of the inverse problems and the FCM clustering algorithm. We then demonstrate how this new method works using two synthetic examples.

Methodology

Formulation of inverse problems

The area under investigation is divided into cells in Cartesian coordinates with the density and seismic velocity being constant within each cell and to be estimated by inversion. $\vec{m}$ is the model vector containing the values of some physical property (e.g. density or velocity in this study) in all of the cells.

Suppose we have the observed gravity data or synthetic crosshole seismic traveltimes denoted by $\vec{d}$. The inverse problem is then formulated as an optimization problem that minimizes an objective function (Tikhonov and Arsenin, 1977):

$$\phi(\vec{m}) = \phi_d + \beta\phi_m \qquad (1)$$

where ϕ_d is data misfit term and measures how well the observed data can be reproduced by the inverted model, and ϕ_m is a measure of the model structure and determines the characteristics of the inverted model. β is known as the regularization parameter that balances between those two terms.

The data misfit term ϕ_d is defined as:

$$\phi_d = \left\| W_d(\vec{d} - G\vec{m}) \right\|_2^2 \qquad (2)$$

where G represents the forward modeling operator that maps the density model and velocity model to the gravity data and the seismic traveltimes, respectively. W_d is the data weighting matrix that accounts for the uncertainty associated with each datum and correlation between data.

For the model structure term, we use the following function developed by Li and Oldenburg (1994):

$$\phi_m = \alpha_s \int_v \omega_s (\vec{m} - \vec{m}_{ref})^2 dv + \alpha_x \int_v \omega_x \left[\frac{\partial}{\partial x}(\vec{m} - \vec{m}_{ref}) \right]^2 dv + \alpha_y \int_v \omega_y \left[\frac{\partial}{\partial y}(\vec{m} - \vec{m}_{ref}) \right]^2 dv + \alpha_z \int_v \omega_z \left[\frac{\partial}{\partial z}(\vec{m} - \vec{m}_{ref}) \right]^2 dv \qquad (3)$$

where $\vec{m}_{ref}$ is the reference model constructed based on geology, previous studies in the same area, or any other information we may have. Constants ω_s, ω_x, ω_y and ω_z reflect our confidence in the reference model. α_s, α_x, α_y and α_z are constants that control the closeness of the reconstructed model to the reference model and the roughness of reconstructed model in each direction. In this study, we use a zero reference model, and consider synthetic 2D models that have only *x* and *z* dimensions.

Fuzzy c-means (FCM) clustering

FCM clustering is one of the unsupervised clustering methods that can be used to organize data into groups based on similarities between data entries. And it has been utilized in a wide variety of applications, such as image processing and pattern recognition (Bezdek, 1981; Duda et al, 2000). It is different from hard clustering methods in that each data entry can be assigned to multiple clusters with different membership values. Mathematically, FCM clustering algorithm can be expressed as the minimization of the following objective function:

$$\phi_{fcm} = \sum_{j=1}^{N} \sum_{k=1}^{C} u_{jk}^{q} \left\| m_j - v_k \right\|_2^2 \qquad (4)$$

where N is the number of model cells, C is the number of clusters, m_j is the physical parameter value at the j^{th} cell, and v_k is the center of the k^{th} cluster. u_{jk} is the membership function that measures the degree to which the model parameter at the j^{th} cell belongs to k^{th} cluster. The parameter *q*, also known as fuzzification parameter, controls the degree of 'fuzziness' of the resulting membership functions, and satisfies $q \geq 1.0$. In this study, we set $q = 2.0$, which is widely accepted as a good choice (Hathaway and Bezdek, 2001). But other choices of the value of q would also work. Throughout this paper, we assume that the total number of clusters, C, is known.

By taking the derivatives of the objective function (4) with respect to the cluster centers, v_k, and the membership functions, u_{jk}, and setting the resulting derivative equations to be zero, we can iteratively update the cluster centers and membership functions until a local minimum is found.

The FCM clustering technique is used to incorporate a priori petrophysical information to the separate inversion of gravity data and crosshole seismic traveltimes, and to differentiate between different rock units after inversions have been carried out.

Formulation of inverse problems with petrophysical constraints

Assume we know all the possible density and velocity values in the subsurface based on measurements on rock samples. And we would like to construct a density model or a velocity model that reproduces the observed geophysical data to a certain degree, that has minimum structure in a sense determined by the model structure term, and that is consistent with the a priori information about the physical parameter values. To accomplish this, we formulate the following constrained optimization problem:

Minimize: $\phi(\vec{m}) = \phi_d + \beta\phi_m$

Subject to: $\phi_{fcm} = \sum_{j=1}^{N} \sum_{k=1}^{C} u_{jk}^{q} \left\| m_j - v_k \right\|_2^2 < tolerance$

And $\sum_{k=1}^{C} \left\| v_k - t_k \right\|_2^2 < tolerance$

Here, v_k is the k^{th} cluster center for densities or velocities automatically updated by FCM algorithm, and t^k is the a priori physical property values determined from rock sample measurements. If the last condition is omitted, and we just let FCM algorithm to update the cluster centers, we find that the final cluster centers obtained are in general not close to those a priori values we have. Therefore, we add the last condition that penalizes the distance between cluster centers updated by FCM algorithm and target cluster centers determined from a priori petrophysics. One advantage of this strategy is that it does not compromise the well-behaved convergence of FCM algorithm, while at the same time it guides the search for cluster centers to the desired locations, based on a priori petrophysical information.

By applying the method of Lagrange multipliers, the above complicated constrained optimization problem can be transformed to an unconstrained optimization problem that minimizes the following objective function:

$$\phi(\vec{m}) = \phi_d + \beta\phi_m + \lambda(\sum_{j=1}^{N}\sum_{k=1}^{C} u_{jk}^q \left\| m_j - v_k \right\|_2^2 + \eta\sum_{k=1}^{C}\left\| v_k - t_k \right\|_2^2) \quad (5)$$

The values of the weighting coefficients λ and η are estimated by numerical experimentation. An iterative algorithm for minimizing (5) can be developed by differentiating $\phi(\vec{m})$ with respect to $\vec{m}$, as well as the cluster centers, v_k and membership functions, u_{jk}, and setting the resulting derivative equations to zero.

As mentioned in the Introduction, this new inversion algorithm can deal with the situation where we only have partial knowledge of the rock physics in the subsurface. For example, assume that there are three different rock units in the subsurface, and we have petrophysics information for two of them from measurements on rock samples. In this case, C = 3, and the last term in equation (5) would become:

$$\sum_{k=1}^{3}\left\| v_k - t_k \right\|_2^2 = \left\| v_1 - t_1 \right\|_2^2 + \left\| v_2 - t_2 \right\|_2^2 + 0\left\| v_3 - t_3 \right\|_2^2 \quad (6)$$

In other words, cluster centers, v_1 and v_2, for which we have petrophysical information available, are estimated by both FCM algorithm (i.e. $\sum_{j=1}^{N} u_{jk}^q \left\| m_j - v_k \right\|_2^2$, k = 1, 2) and the petrophysics constraints (i.e. $\left\| v_k - t_k \right\|_2^2$, k = 1, 2). However, the center for the third cluster, v_3, is updated only by FCM algorithm by turning off the third term, $\left\| v_3 - t_3 \right\|_2^2$, since no petrophysical information about the third rock type exists.

Synthetic Examples

<u>Inversion of crosshole seismic traveltimes with complete petrophysical information</u>

We present a synthetic crosshole seismic survey over a 2D area that extends 900*m* long and 600*m* deep. We position the transmitters evenly in one borehole and receivers in the other, as shown in Figure 1. The background velocity is 2000 *m/s*, which is equivalent to slowness of 0.5 *s/km*. We place two slowness anomalies within the model region with slowness 0.2 *s/km* above and below the background slowness. To simplify the problem, we calculate the traveltimes at each receiver using straight-ray tracing. We then contaminate the traveltime data with independent Gaussian noise. The model region is discretized into 864 25*m*x25*m* cells.

The histogram corresponding to this slowness model, as shown in Figure 2, clearly indicates that there are three rock types in the subsurface, and the slowness values these three rocks can take are 0.3, 0.5 and 0.7 *s/km*. Assume that this valuable petrophysical information is known a priori from measurements on rock samples, we now consider reconstructing the slowness model from the observed traveltimes and these petrophysical information.

Figure 3 shows the recovered slowness model by minimizing function (5) where t_k equals 0.3, 0.5 and 0.7 for k = 1, 2, 3 respectively. In other words, the a priori petrophysics information is built into inversion by letting t_k assume those possible slowness values determined from rock physics. The histogram associated with this slowness model is shown in Figure 4.

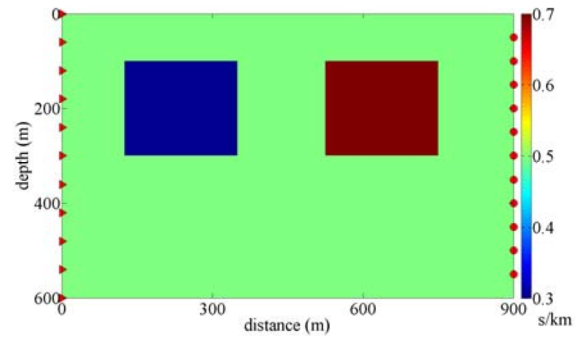

***Figure 1:** Slowness model and the geometry of the synthetic crosshole seismic experiment. The red triangles and circles mark the positions of the transmitters and receivers.*

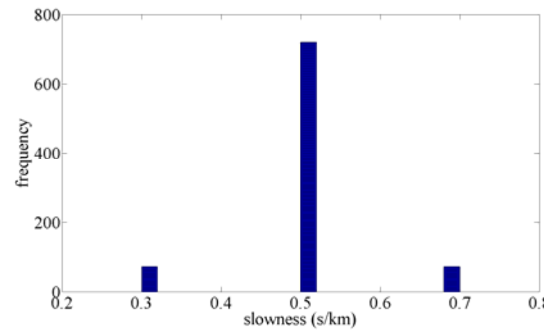

***Figure 2:** Histogram corresponding to the slowness model shown in Figure 1.*

For comparison, the slowness model recovered with only the traveltime data and without those petrophysical constraints is shown in Figure 5, and Figure 6 shows the corresponding histogram.

Comparing the slowness models shown in Figure 3 and Figure 5, we observe fewer artifacts in the slowness model recovered with both seismic traveltime data and petrophysical constraints, and therefore, the spatial extents of the slowness anomalies are better defined in Figure 3 than in Figure 5.

The histograms in Figures 4 and 6 show the distribution of the recovered slowness values. We compare the range of the recovered slowness values. The slowness values lie between 0.333 *s/km* and 0.694 *s/km* for the model recovered with petrophysical constraints, as compared to the range of [0.368, 0.630] for the model estimated using only traveltime information. It is clear that the former is closer to the true range, [0.300, 0.700]. Another observation is that the background slowness values enclosed by the red ellipse in Figure 4 are more spiky and tighter, and thus better resolved, but those in Figure 6 are more dispersed and less resolved. We then conclude that by incorporating petrophysics information into seismic traveltime inversion, we can produce a slowness model that not only honors the traveltime data, but also complies with the petrophysical information.

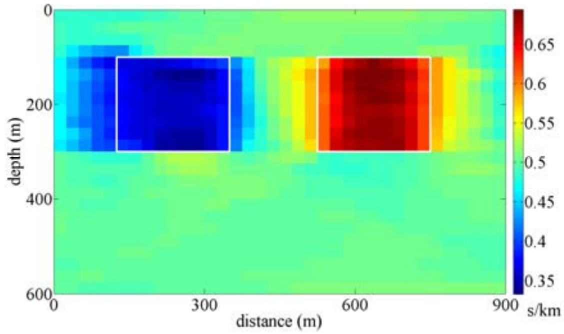

Figure 3: *Slowness model recovered from inverting crosshole seismic traveltime data with petrophysics constraints. The white boxes show the true locations of the slowness anomalies.*

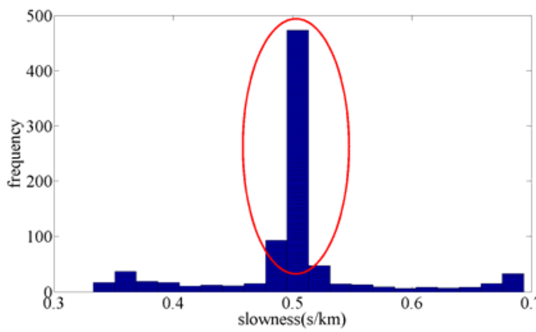

Figure 4: *Histogram corresponding to the slowness model in Figure 3.*

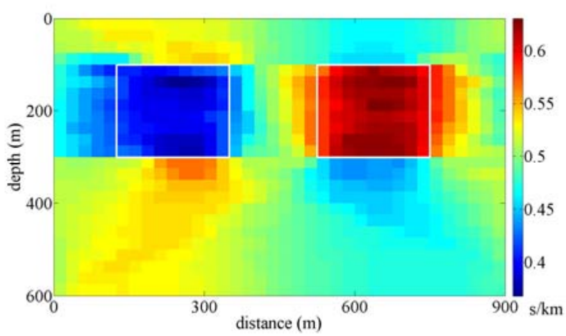

Figure 5: *Slowness model recovered without petrophysical constraints.*

This conclusion is further confirmed by the FCM clustering results which show the different rock types identified by this clustering technique. Figure 7 shows the lithology differentiation result based on the slowness model reconstructed using both seismic traveltime data and petrophysics. The three different colors, blue, green and red, indicate there are three different rock types in this region. Figure 8 shows the identified rock types by applying FCM algorithm to the slowness model using only seismic traveltime data.

<u>Inversion of crosshole seismic traveltimes with partial petrophysical information</u>

We next assume that we have only partial petrophysical knowledge in this region. The petrophysical information we have in this case is that there are three different rock units in the subsurface, and the possible slowness values are 0.3, 0.5 and one unknown value. We now use this partial information to constrain the seismic traveltime inversion. Figure 9 shows the recovered slowness model using seismic traveltime information and partial petrophysics information.

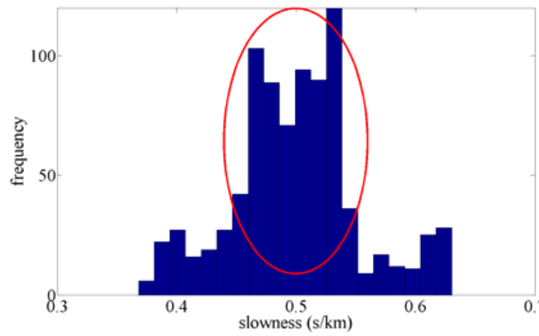

Figure 6: *Histogram that shows slowness value distribution recovered using only seismic traveltime data.*

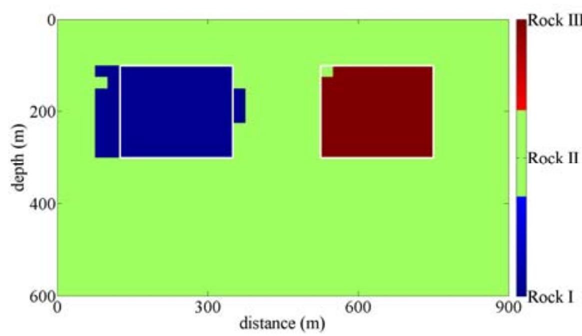

Figure 7: *FCM clustering result that shows the three different lithological units identified based on slowness model shown in Figure 3.*

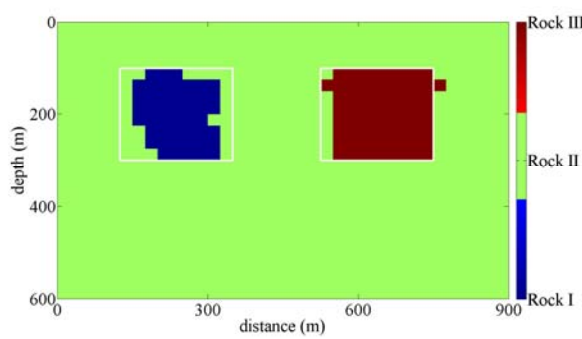

Figure 8: *FCM clustering result for the slowness model shown in Figure 5.*

If we compare the slowness model shown in Figure 9 with the model shown in Figure 5, it is clear that even with only part of the petrophysics information the improvement in the slowness model reconstruction is obvious.

The histogram shown in Figure 10 also clearly indicates the presence of three different lithology units. The FCM clustering result, shown in Figure 11, also outlines the locations of the two slowness anomalies reasonably. The cluster center for the Rock III, for which we have no petrophysical information, is 0.6564 *s/km*, which is very close to the cluster center estimated using complete petrophysics, i.e. 0.6647 *s/km*. The highest slowness that can recovered with partial petrophysical constraints is 0.6881 *s/km*, which is slightly lower than the value recovered with complete petrophysics, i.e. 0.6973 *s/km*, but is much higher than the values recovered without any petrophysical information, i.e. 0.6302 *s/km*.

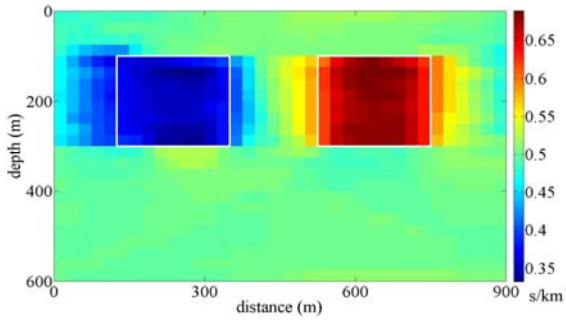

Figure 9: *Slowness model recovered by inverting crosshole seismic traveltimes with partial petrophysical constraints.*

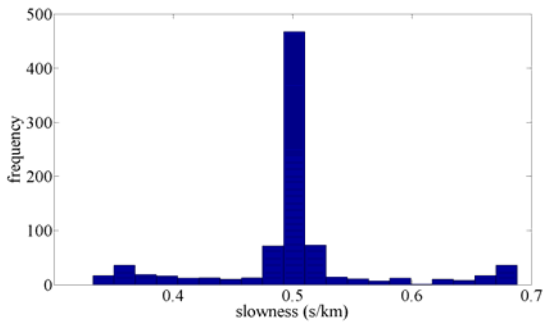

Figure 10: *Histogram that shows the distribution of the recovered slowness values as shown in Figure 9.*

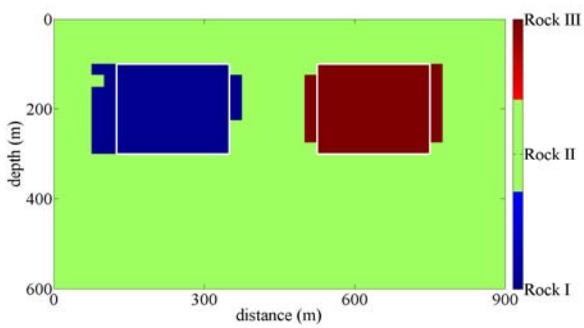

Figure 11: *Lithological differentiation results by applying FCM algorithm slowness model shown in Figure 9.*

It is interesting to notice that even if no information about the third rock type is incorporated into inversion, we can still get the location and the slowness value of that rock type recovered very well. This occurs because the incorporation of petrophysics information about the other two rock types improves the characterization of these two rock types, and consequently, we constrain the slowness value and the location of the third rock type indirectly.

<u>Inversion of surface and borehole gravity data</u>

In this synthetic example, we create a density model that is 800 meters deep by 1600 meters long, as shown in Figure 12. There are two density anomalies with density contrast of 0.4 *g/cm³*. We calculate the gravity response every 20 meters on the surface and every 40 meters in both of the two borehole located 50 meters off the ends of the model region. Independent Gaussian noise with zero mean and 1% standard deviation is added to simulate the noisy data collected in reality. The model is discretized into 2048 25*m*x25*m* cells.

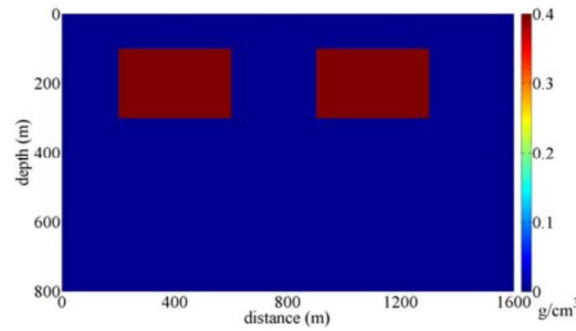

Figure 12: *Synthetic density model for gravity inversion.*

The histogram of the bulk density distribution is shown in Figure 13, and we can see that there are two different rock units with two distinct density values, 0 and 0.4 *g/cm³*. The frequency of each rock type is rarely available a priori in reality, and therefore, will not be considered as a priori petrophysics information that can be incorporated into inversion. We next implement the inversion of surface and borehole gravity data with the above mentioned petrophysics information as constraints.

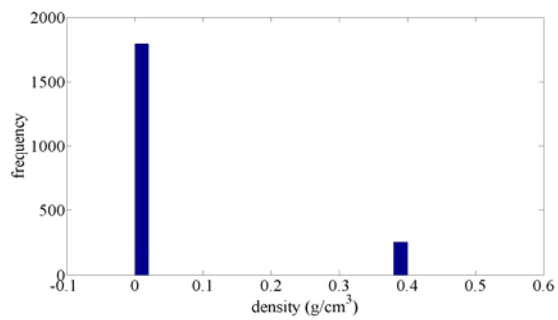

Figure 13: *Histogram that shows the distribution of bulk densities in this area.*

Figure 14 shows the resulting inverted model, and Figure 15 shows the corresponding histogram. It is observed in Figure 14 that the inverted density anomalies are placed at the correct locations by inversion, and the inverted density values at these two locations are mostly 0.4 g/cm³, which equals the true value. The same observation can be made by looking at the histogram, shown in Figure 15, which clearly shows the presence of two rock types with densities of 0 g/cm³ and 0.4 g/cm³. The histogram of the recovered densities is almost identical to the histogram of the true density model in Figure 13. In other words, we successfully recover a density model that can predict both the observed gravity data and the a priori petrophysical data. Figure 16 shows the lithology differentiation result by applying FCM clustering algorithm to the recovered density model. The two different lithologies are identified at correct positions and with reasonable shapes.

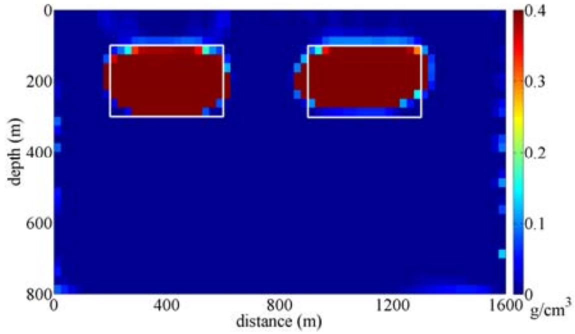

Figure 14: *Inverted density model with petrophysical constraints. The white boxes outline the boundaries of the true density anomalies.*

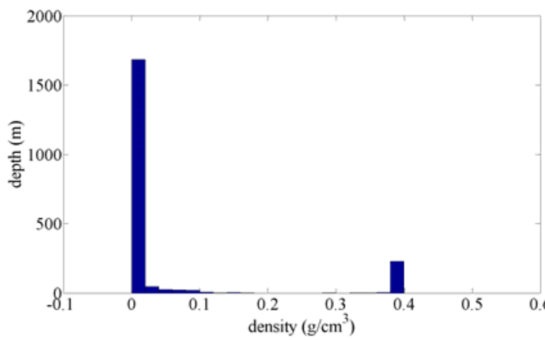

Figure 15: *Histogram that shows the distribution of the bulk densities inverted with petrophysical constraints.*

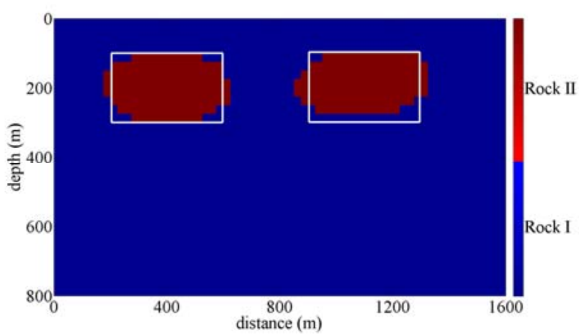

Figure 16: *Lithological differentiation result by applying FCM algorithm to the density model shown in Figure 14.*

Conclusions

Inversions of geophysical data have proven to be an effective way of obtaining quantitative information about the physical parameters of interest. Many efforts have been devoted to developing inversion algorithms that can incorporate geological structural information, physical property data and any other a priori information that may help better constrain the inverted model.

In this study, we consider the petrophysics information that is available from measurements on rock samples, surface mapping, or well log, and develop a new inversion algorithm that can effectively build this information into inversion by means of FCM clustering algorithm.

We demonstrate the effectiveness of this new inversion method using a synthetic crosshole seismic example and a gravity survey that measures both surface and borehole gravity response. Numerical results show that the final inverted model resulting from this new inversion algorithm honors both the observed geophysical data (e.g. seismic traveltimes, gravity) and the a priori petrophysics constraints. Therefore the subsurface geology is better represented than would otherwise if inversions were done only with geophysical data considered. These post-inversion models are further processed by FCM clustering algorithm to automatically determine the geometry and parameter characteristics of different lithology units present in the subsurface. In other words, a lithology map can be generated by inverting geophysical data with petrophysical constraints by means of FCM clustering technique.

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