Comparison of the Anisotropic-common-ray approximation of the Coupling ray theory for S waves with Fourier pseudo-spectral method in weakly anisotropic models

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Abstract

The standard anisotropic ray theory does not work properly or even fails when applied to S-wave propagation in inhomogeneous weakly anisotropic media or in the vicinity of shear-wave singularities, where the two shear waves propagate with similar phase velocities. The coupling ray theory was proposed to avoid this problem. In it, amplitudes of the two S waves are computed by solving two coupled, frequency-dependent differential equations along a common S-wave reference ray. In this paper, we test two recently developed algorithms of the coupling ray theory. As a reference, we use the Fourier pseudo-spectral method, which does not suffer from limitations of the ray method and yields very accurate results. We study the behavior of shear waves in weakly anisotropic media as well as in the vicinity of intersection or conical singularity. By comparing the coupling ray theory results with results of the Fourier pseudo-spectral method, we illustrate clearly applicability of the coupling ray theory in the mentioned regions.

Introduction

Shear waves propagating in inhomogeneous, “weakly anisotropic“ media in regions, in which the two S waves propagate with identical or close phase velocities, do not propagate independently. They are coupled. This effect is described by *coupling ray theory* (Kravtsov 1968; Coates and Chapman 1990; Pšenčík 1998; Bulant and Klimeš 2002; Farra and Pšenčík 2008, 2010), which is the generalization of both zero-order anisotropic and isotropic ray theories and provides continuous transition between them.

There is a broad variety of coupling-ray-theory algorithms differing by the used approximations, which simplify coding and increase performance of calculations, but diminish the accuracy of the coupling ray theory both with increasing frequency and increasing degree of anisotropy. Refer, e.g., to Bulant and Klimeš (2002, 2004) and Klimeš and Bulant (2004) for the description of different approximations and for their impact on synthetic seismograms. Most of these approximations can be avoided with minimal effort, with the exception of the common-ray approximation for S waves.

In the *common-ray approximation*, only one reference ray is traced for both anisotropic-ray-theory *S* waves, and both *S*-wave anisotropic ray-theory travel times in the coupling equations are approximated by perturbation expansion from the common reference ray. Whereas tracing the continuous system of anisotropic-ray-theory rays may be very difficult in the vicinity of an *S*-wave singularity at which the *S*-wave slowness surfaces coincide, this problem does not occur in the common-ray approximation. The common-ray approximation thus eliminates problems with ray tracing through *S*-wave singularities and also considerably simplifies coding of the coupling ray theory and numerical calculations, but may introduce errors in travel times due to the perturbation. These travel time errors can deteriorate the coupling-ray-theory solution. It is thus of principal importance in numerical applications to estimate the travel time errors due to the common-ray approximation, and then the related error of the wavefield.

In this paper, we follow the work of Pšenčík *et.al.* (2011), who compared their approximation of the coupling ray theory with the standard anisotropic ray method in several weakly anisotropic models in vicinity of shear-wave singularities. Using the results of the Fourier pseudo-spectral method (e.g., Kosloff and Baysal 1982) as the reference solution, they demonstrated clear superiority of the coupling ray theory method over the standard anisotropic ray theory, and showed the problems and dangers of using the anisotropic ray theory in such “weakly anisotropic” models. Since their method is based on the common S-wave rays obtained by the first-order ray tracing method (Farra and Pšenčík 2010), the approximations used during the first-order ray tracing of the reference ray may also deteriorate the coupling-ray theory solution. In this paper we thus compare the results of the coupling ray theory along the first-order rays by Farra and Pšenčík with the results obtained by coupling ray theory algorithm of Bulant and Klimeš (2002), where emphasis was put not on the performance of the method, but on the avoiding of unnecessary approximations of the coupling ray theory and of the common ray calculation. We again use the results calculated by Fourier pseudo-spectral method as a reference solution.

Method

In the coupling ray theory algorithm by Farra and Pšenčík (2011), the S-wave travel times in the coupling equations are approximated by the second-order perturbation expansion (Farra and Pšenčík 2010) from the first-order common reference ray (Farra and Pšenčík 2008), we will thus use abbreviation FOCRT for this method.

In the numerical algorithm of coupling ray theory (CRT) by Bulant and Klimeš (2002), anisotropic common rays are traced in the anisotropic model using the averaged Hamiltonian of both anisotropic ray theory S waves according to the dynamic ray tracing algorithm proposed by Klimeš (2006). The S-wave travel times in the coupling equations are approximated by the first-order perturbation expansion from the anisotropic common reference ray, or, in models without caustics, by the second-order terms in the perturbation expansion (Bulant and Klimeš 2008).

For tests of accuracy of the above-mentioned coupling ray theories, we used a code based on the Fourier pseudo-spectral method (FM), see, e.g., Kosloff and Baysal (1982). The time integration is done by a 4th order Taylor expansion of the evolution operator. The highly accurate computations of spatial derivatives in connection with a higher order time integration technique result in negligible numerical dispersion. The method has similarities to the one described by Carcione et al. (1992), the major difference being the type of time integration. The FM does not suffer from limitations of the ray method. It is applicable to any type and strength of anisotropy. It works equally well in regular as well as in singular regions of the ray method. The accuracy of the FM makes it thus an almost ideal tool for testing the coupling ray theory.

Numerical examples

In this paper, we follow the work of Pšenčík et.al. (2011) and we compare the coupling ray theory seismograms with FM in different weakly anisotropic models called QI, SC1_I, and ORT. In each model, the matrices of the density-normalized elastic moduli measured in (km/s)^2 are specified at two depths. Between these depths, each element of the matrices is linearly interpolated generating a medium with constant vertical gradient of elastic moduli. The density r in all models is considered to be constant, r = 1 g/cm^3. The S-wave anisotropy in the studied profiles, defined as $2(c_{max}-c_{min})/(c_{max}+c_{min})$x100% for each wave, is maximum 10% (SC1_I model). The S-wave separation in the studied profiles, defined as $2|c_{S1}-c_{S2}|/|c_{S1}+c_{S2}|$ x 100%, is maximum 13% (model QI4). For detailed description of the models refer to Pšenčík et.al. (2011).

The synthetic seismograms, corresponding to vertical force F = $(0,0,100)^T$ at position $(0,0,0)^T$, are calculated at receivers located in a vertical well in a distance 1 km from the source, see Fig. 1. The source time function is the Gabor signal $\cos(2p f t) \exp[-(2p f t / 4)^2]$ with reference frequency f = 50 Hz, bandpass filtered by a cosine filter given by frequencies 0, 5, 60 and 100 Hz. The numbers and depths of the receivers differ for individual models, and are shown on the figures with seismograms. The receivers record the vertical (positive downwards), transverse and radial (along the line connecting the source and the top of the borehole; positive away from the source) components of the wave field. The recording system is right-handed. All calculated seismograms are shown with no differential scaling between components and traces, so that true relative amplitudes are shown. See Figures 2 to 4 for the synthetic seismograms calculated in the individual models.

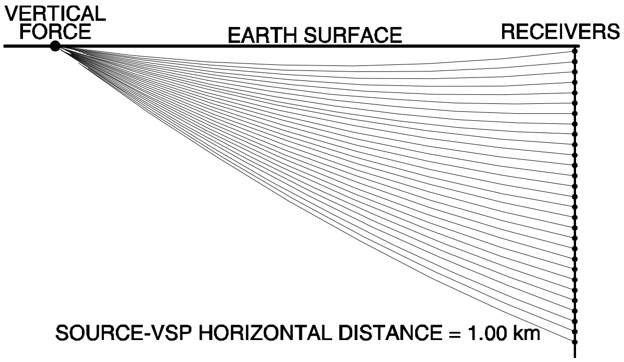

Figure 1: Source-receiver configuration for the calculations of synthetic seismograms. The numbers and depths of the receivers differ for individual models, and are shown on the figures with seismograms.

Results

First we consider model QI proposed by Bulant and Klimeš (2008) for illustration of coupling effects. The model is vertically inhomogeneous, transversely isotropic with the horizontal axis of symmetry rotated counterclockwise in the horizontal plane by 45^o from the x-axis. The medium in model SC1_I is transversely isotropic. Phase velocity surfaces of S waves at the top of this model intersect each other, giving rise to an intersection singularity. ORT is a model of vertically inhomogeneous weakly orthorhombic medium. It has four conical (point) singularities, and the waves propagating from the source to the profile of receivers pass close to one of the singularities. In Figures 2 to 4 we can see an overall good fit of the seismograms of both coupling ray theories with the FM seismograms. The results are nearly identical despite the existence of the above-mentioned singularities.

Conclusions

The tests described in this paper clearly show that the coupling ray theory, where applicable, yields results very close to those generated by the FM, which we consider as a very accurate reference.

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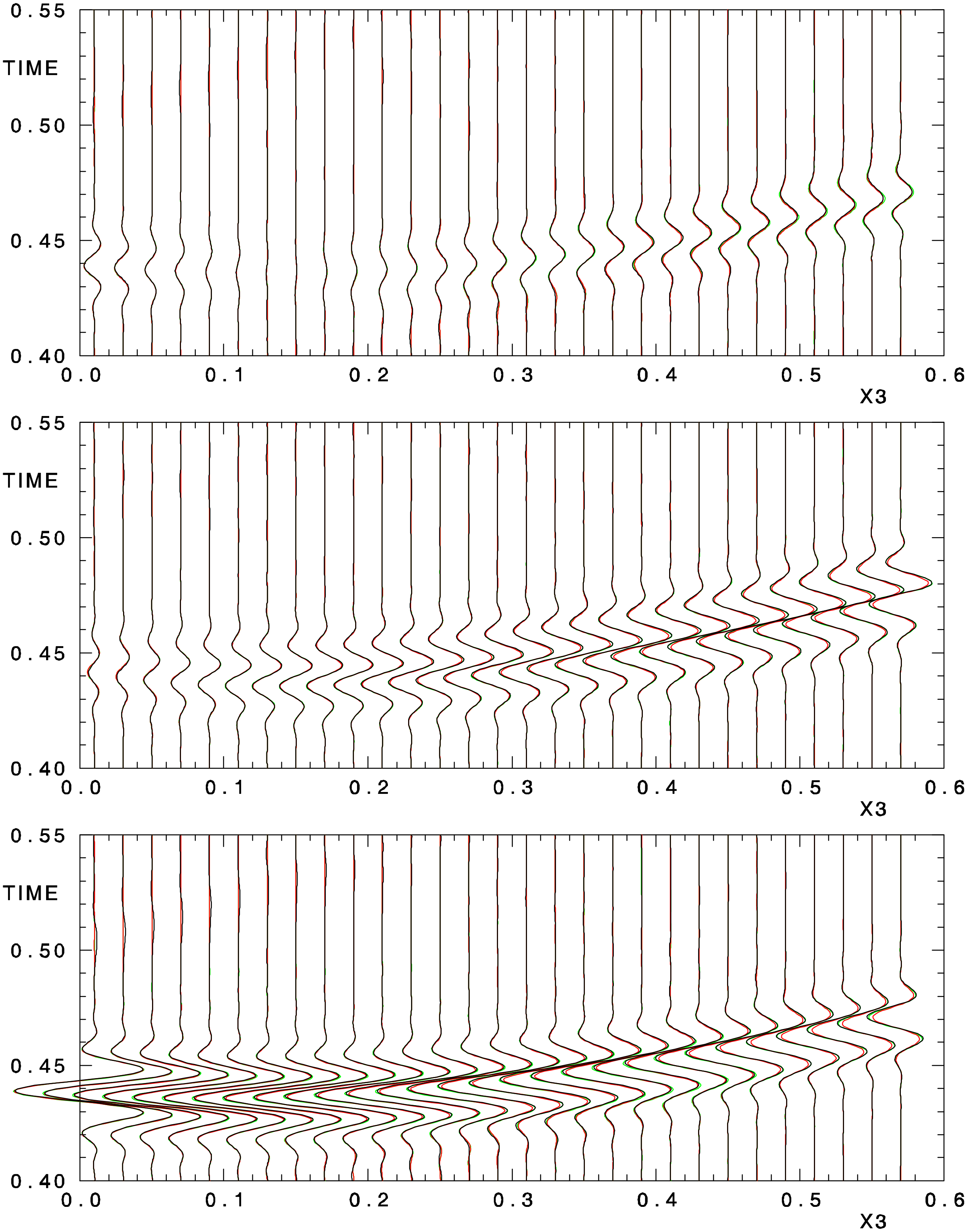

Figure 2: Seismograms calculated in model QI. Vertical axis shows travel time in seconds, horizontal axis shows receiver depth in kilometers. All calculated seismograms are shown with no differential scaling between components and traces, so that true relative amplitudes are shown. Black color is used for the Fourier pseudo-spectral method (FM) seismograms, red color for coupling ray theory of Farra and Pšenčík (FOCRT), green for coupling ray theory by Bulant and Klimeš (CRT). All the seismograms are in a good agreement.

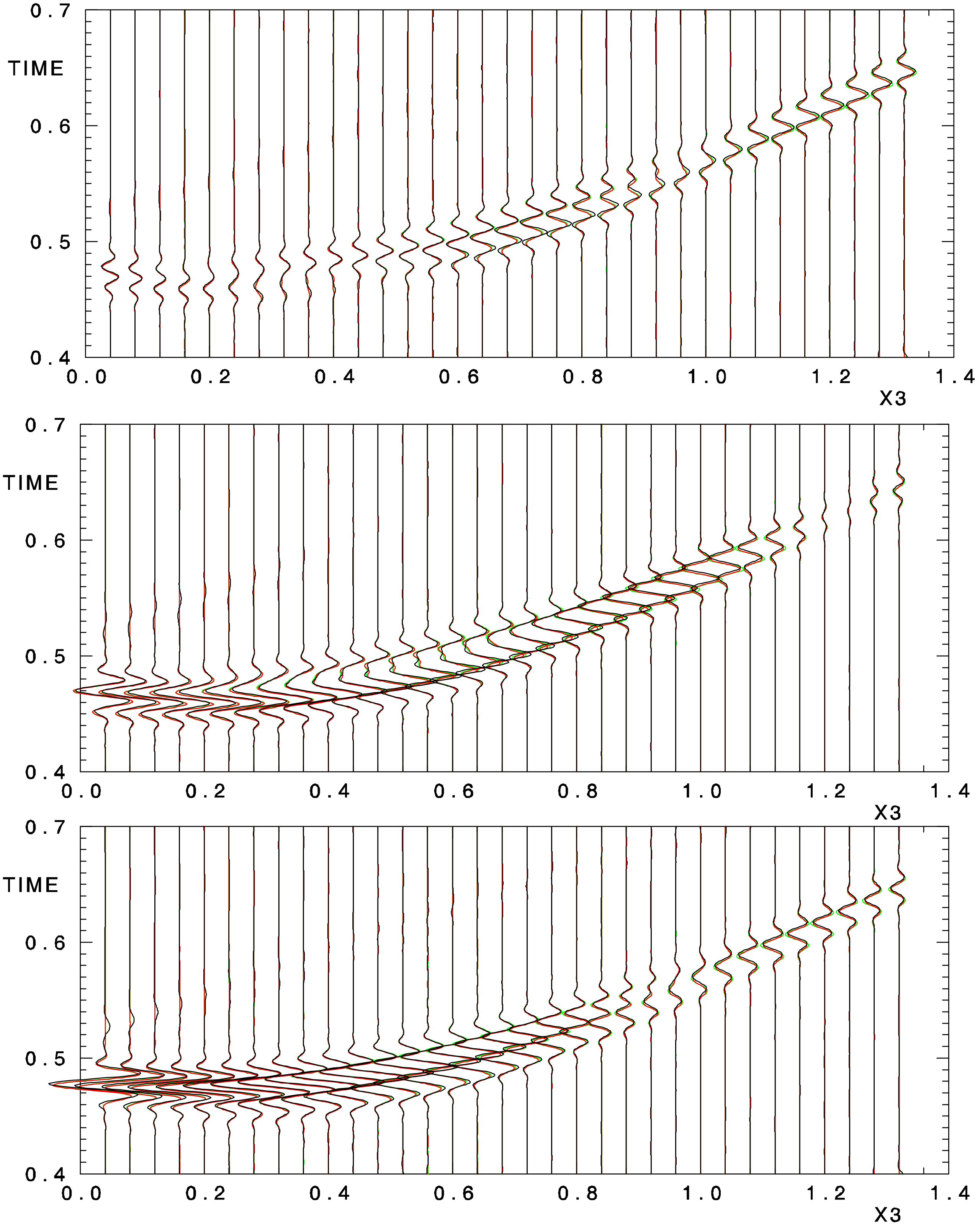

Figure 3: Seismograms calculated in model SC1_I. Vertical axis shows travel time in seconds, horizontal axis shows receiver depth in kilometers. All calculated seismograms are shown with no differential scaling between components and traces, so that true relative amplitudes are shown. Black color is used for the Fourier pseudo-spectral method (FM) seismograms, red color for coupling ray theory of Farra and Pšenčík (FOCRT), green for coupling ray theory by Bulant and Klimeš (CRT). All the seismograms are in a good agreement.

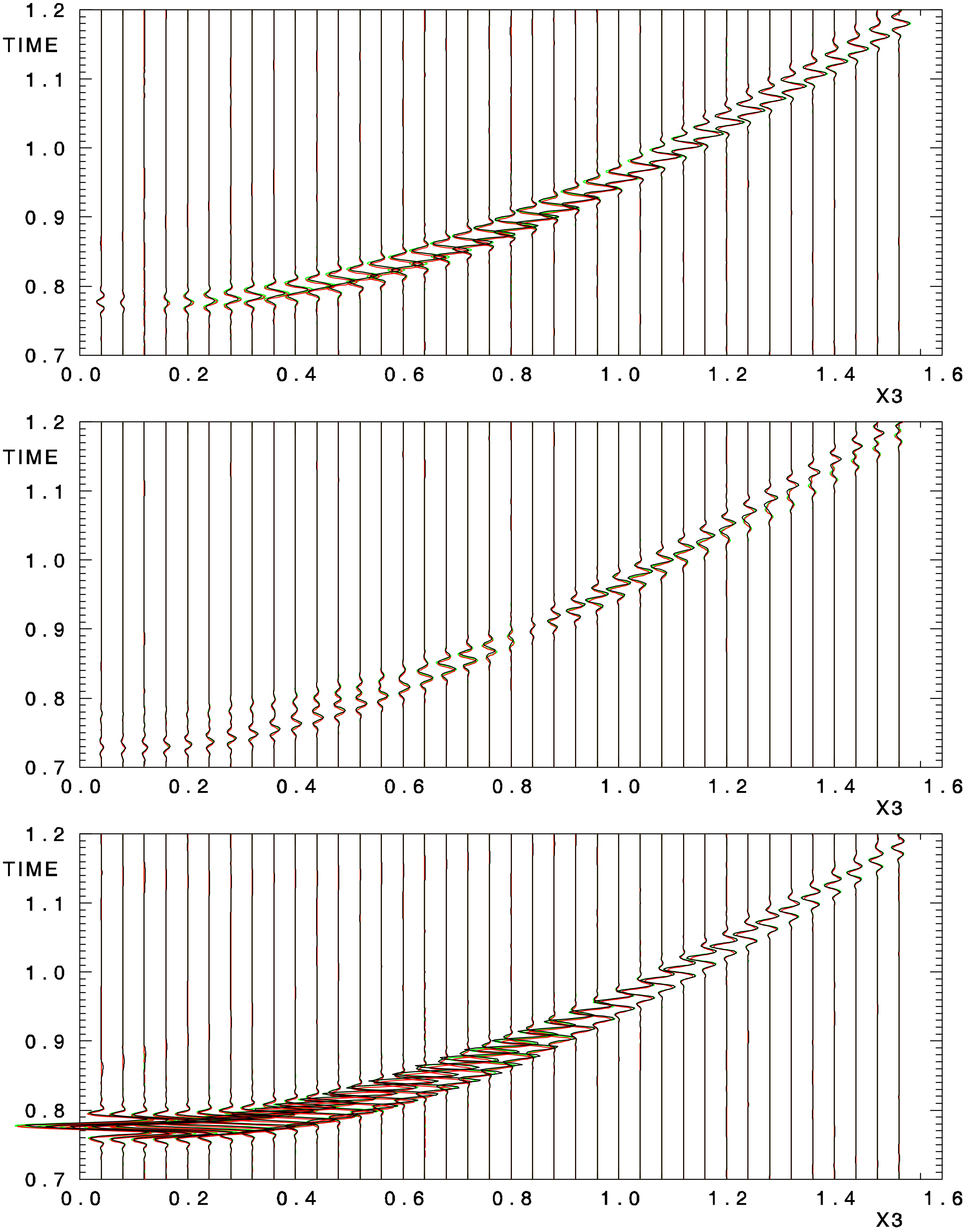

Figure 4: Seismograms calculated in model ORT. Vertical axis shows travel time in seconds, horizontal axis shows receiver depth in kilometers. All calculated seismograms are shown with no differential scaling between components and traces, so that true relative amplitudes are shown. Black color is used for the Fourier pseudo-spectral method (FM) seismograms, red color for coupling ray theory of Farra and Pšenčík (FOCRT), green for coupling ray theory by Bulant and Klimeš (CRT). All the seismograms are in a good agreement.