Numerical Moveout Estimation for Migration Velocity Analysis in Super-Gathers

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Abstract

In migration velocity analysis (MVA), the residual moveout in the image gather is used to correct the velocity field. In this work we propose a numerical approach to describe the moveout in the image gather considering a dipping reflector. Through some numerical examples, we validate the strategy of fitting the moveout curves numerically. Furthermore, in our proposal many image gathers can be used altogether to obtain the parameters. This strategy provides more reliable values for the velocity correction factor.

Introduction

Migration velocity analysis (MVA) is a seismic processing technique that investigates the migration residual moveout to correct a priori velocity model. After migration, it is expect that in a common image gather (CIG) the events get flattened. However, if the velocity used in migration is lower than the correct one, events curve upward, whereas if the velocity is higher, events curve downward. This moveout can be used to correct the velocity.

As in the conventional normal moveout (NMO) processing, it is necessary to predicted such moveout. Al-Yahya (1989) proposed a formula based on a horizontal reflector embedded in a constant velocity overburden. Schleicher and Biloti (2007) proposed a generalization of Al-Yahya's formula considering dipping reflectors. During their derivations, the authors ended up with an equation that can not be solved analytically. To obtain a solution, Taylor's approximation was used. Besides, further approximations were made. Since the authors used Taylor's approximation, theoretically it is valid only for small dips.

Instead of making approximations, we adopt a numerical approach to describe the moveout in the image gather. In this way, we expect to get closer to the observed curve. Futhermore, in our proposal many image gathers are used altogether. This strategy provides more reliable values for the velocity correction factor.

The main purpose of this work is to validate the strategy of fitting the moveout curves numerically.

Migration velocity analysis

To a horizontal reflector in a homogeneous media, Al-Yahya (1989) proposed the following formula to describe the event in a CIG:

$$\tau^2 = t_0^2 + (\gamma^2 - 1) 4h^2 / v_m^2 \ , \tag{1}$$

where h is the half-offset, t_0 is the vertical time and $\gamma = v_m/v$ is the ratio between the velocity used in migration v_m and the correct velocity v.

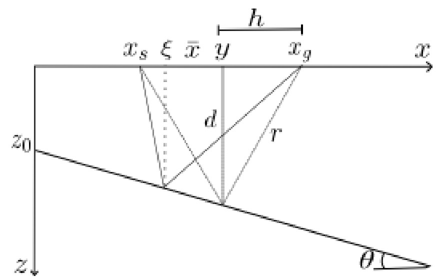

Figure 1: Dipping reflector geometry.

Considering also the dip of the reflector, Schleicher and Biloti (2007) generalized Al-Yahya's proposal. In the following, we repeat some theoretical derivations of their work, important to clarify our proposal. Lets start by the reflection traveltime, which is given by

$$t_{\text{ref}} = \frac{2}{v} \frac{r}{\sqrt{1+m^2}} = \frac{2}{v} r \cos\theta \ , \tag{2}$$

where z_0 is the depth of the reflector at reference zero coordinate, $d = my + z_0$ is the depth of the reflector vertically under the midpoint coordinate y, $r = \sqrt{d^2 + h^2}$ is the distance between this depth point to the source or receiver (see Figure 1).

The envelope of the family of isocrons

$$t(x,y,h) = \frac{2z}{v_m} = \frac{2b}{v_m} \sqrt{1 - \frac{(x-y)^2}{a^2}} \ , \tag{3}$$

where $a = v_m t_{\text{ref}}/2$ and $b = \sqrt{a^2 - h^2}$, describes the reflection event mapped in the image gather.

Replacing the reflection traveltime (2), equation (3) can be recasted as

$$t(x,y,h) = \frac{2}{v_m} \frac{1}{\gamma\sqrt{1+m^2}} \frac{pq}{r} \ , \tag{4}$$

where $p = \sqrt{\gamma^2 r^2 - (1+m^2)(x-y)^2}$ and $q = \sqrt{\gamma^2 r^2 - (1+m^2)h^2}$.

The envelope condition is

$$\frac{dt}{dy} = \frac{2}{v_m} \frac{1}{\gamma\sqrt{1+m^2}} \frac{1}{pqr^3} f(x,y,h) = 0 \ , \tag{5}$$

where

$$
\begin{aligned}
f(x,y,h) &= p^2(1+m^2)h^2m(my+z_0)+ \\
&+ q^2r^2\left[\gamma^2m(my+z_0)+(1+m^2)(x-y)\right] \quad . \quad (6)
\end{aligned}
$$

Since f is a polynomial function of degree five in y, $f=0$ can not be solved analytically for nonzero h. This motivated Schleicher and Biloti to use Taylor's expansion up to fourth order in m. By doing so, the condition $m \ll 1$ must be fulfilled, which means that their formula is valid for small dips. Moreover, they considered only one image gather at time. After y is obtained, the event location is calculated back substituing y into (4). The authors keep only the most important terms, consistently with the Taylor's expansion done before, to obtain their final expression for moveout curve.

In this work we propose to obtain y, solution of equation (5), numerically. Fixing $x=\xi$, we have to find a solution y for each h. In general, a reasonable initial guess for y is necessary. When $h=0$, we can solve (5) analytically, and the solution is given by

$$y_0 = \frac{\gamma^2mz_0+(1+m^2)\xi}{(1-\gamma^2)m^2+1} \quad . \quad (7)$$

Employing a continuation strategy, we are able to set reasonable the initial guess for each half-offset. After calculating the numerical solution y, it is replaced into (4) to obtain the moveout in the image gather.

The vertical time t_0 is calculated upon substitution of (7) into (4)

$$t_0 \equiv t(\xi,y_0,0) = \frac{2\gamma(m\xi+z_0)}{v_m}\frac{1}{\sqrt{|(1-\gamma^2)m^2+1|}} \quad . \quad (8)$$

From the above equation, we can obtain z_0 in terms of t_0. So that, instead of calculating the time in the image gather for each z_0, we can also use t_0.

To measure how good the numerical curve fits the event, semblance along this curve is calculated. The parameters which describe the best-fitted curve are obtained by maximizing the coherence measure. The most important parameter for the migration velocity analysis is the velocity correction parameter γ. Once γ is obtained, the velocity field is updated and the data has to be remigrated.

Numerical example

In this section, we apply our proposal to a synthetic data set. Lets consider three models with plane reflector with increasing dips of 10, 15 and 20 degree. In all models, the velocity above the reflector is 2.0 km/s. A noise of 40 signal-to-noise ratio has been added to data. Data has been corrected by the NMO/DMO effect and time migrated. The velocity used in the migration was 3.5 km/s as well as in NMO and DMO correction. Therefore, the theoretical value for the migration velocity correction parameter is $\gamma = 1.75$.

Using theoretical parameters values given in Table 1, we constructed the theoretical curves from Al-Yahya's formula, Schleicher and Biloti's formula and our numerical proposal, as shows Figure 2. Note that for dipping of 10°, Schleicher and Biloti's curve and ours are almost the same. For dipping of 15°, our proposal curve is better than Schleicher and Biloti, since it fits the event. For a dip of 20°, eventhought our curve is not passing through the event, it is the closest one. For all three dips, Al-Yahya's curve doesn't describe very well the event, specially when the dip increases, as expected.

Table 1: Theoretical parameters values.

Dip	m	z_0
10°	0.175	0.875
15°	0.275	1.375
20°	0.375	1.875

In both Al-Yahya's and Schleicher and Biloti's proposals, the parameters are obtained using the data of only one image gather. In our proposal, more than one image gather can be used to find the optimal parameters. In this way, the parameters are more reliable. We use image gathers nearby, on the left and on the right of a central image gather. In the following, we investigate the impact of using many image gathers.

To analyse the objective function behavior as we increase the number of image gathers used, the coherence measure is evaluated at a grid of parameters γ and m, for fixed z_0. For all γ and m, we construct our numerical curve and evaluate the semblance along that curve. Figure 3 shows the result for the model of dip 10°, for $z_0 = 0.875$. Analysing the semblance values, we note that as the number of image gathers increases, semblance get mores focused.

Once we have an idea of the objective function behavior, we apply an optimization method to find the parameters. For each z_0, first we construct an coarse grid in γ and m, evaluate the coherence value at grid points and find the highest value. The parameters γ and m associated with that highest value are the initial guesses for the optmization phase. In this work, we use a simplex method for unrestrict optmization, Nelder and Mead (1965). The results are shown in the following tables.

Table 2: Results for model of dip 10°.

# of i.g.	γ	m	z_0
1	1.749	0.225	0.815
3	1.743	0.165	0.855
5	1.755	0.173	0.870
7	1.753	0.175	0.870
9	1.753	0.175	0.875

Table 3: Results for model of dip 15°.

# of i.g.	γ	m	z_0
1	1.757	0.287	1.340
3	1.744	0.249	1.365
5	1.745	0.258	1.365
7	1.750	0.275	1.360
9	1.750	0.275	1.365

For the model of dip 10°, Table 2 shows that even for one image gather, the velocity update parameter γ found by applying an optmization method is very close to the theoretical one. The best result is when 9 image gathers

Table 4: Results for model of dip 20°.

# of i.g.	γ	m	z_0
1	1.733	0.477	1.615
3	1.730	0.377	1.935
5	1.728	0.334	1.955
7	1.727	0.334	1.965
9	1.724	0.328	1.990

are used at the same time to estimate the parameters. The parameters found in this case is pratically the same to the theoretical parameters. A similar behavior occurs for the model of dip 15°, as we can see in Table 3. In Table 4 are shown the results for the 20° dipping reflector. For the most important parameter, γ, even for dip 20°, our method using five image gathers found $\gamma = 1.728$, which is close to the correct parameter value 1.75. Indeed, using parameter value $\gamma = 1.728$, the updated migration velocity has an error of 1.26%.

Conclusions

In this work, we have validated our numerical curve and showed that it better fits the event than the Schleicher and Biloti's proposal. Besides, the principal extension we make upon their work is to use several image gathers at same time, instead of only one. By doing so, we are able to estimate more accurate parameters. Also from numerical results, we observe that as more image gathers are used, more the semblance value gets focused. This aspect is important to determine the optimal parameters values.

Acknowledgments

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References

Al-Yahya, K. M., 1989, Velocity analysis by iterative profile migration: Geophysics, **54**, 718–729.

Nelder, J., and Mead, R., 1965, A simplex method for function minimization: Computer Journal, **7**, 308–315.

Schleicher, J., and Biloti, R., 2007, Dip correction for coherence-based time migration velocity analysis: Geophysics, **72**, S41–S48.

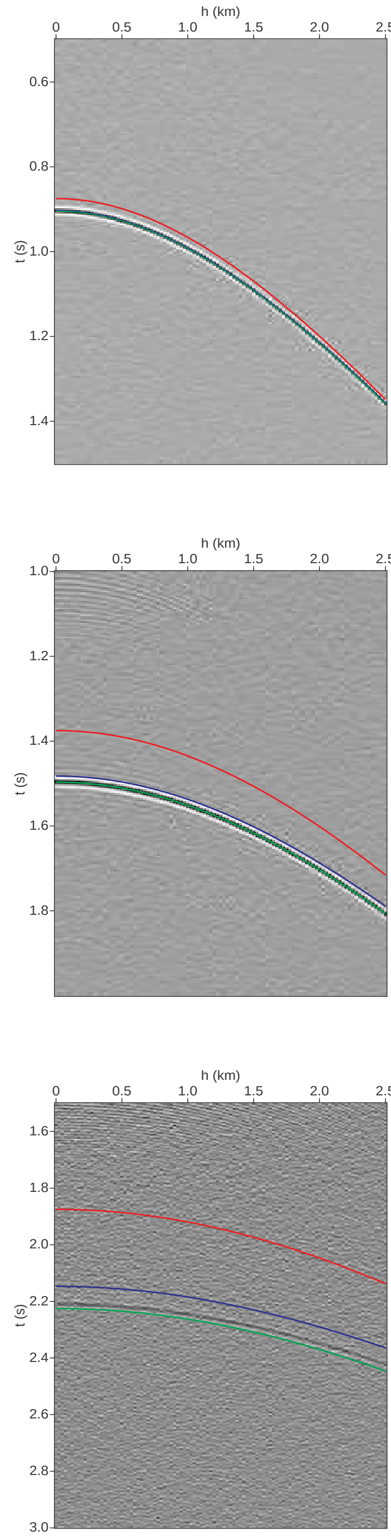

Figure 2: Image gather of the data migrated with an incorrect velocity for 10° (left), 15° (middle) and 20° (right) dipping reflector. The red line is from Al-Yahya's formula, blue is from Schleicher and Biloti and the green is our numerical proposal.

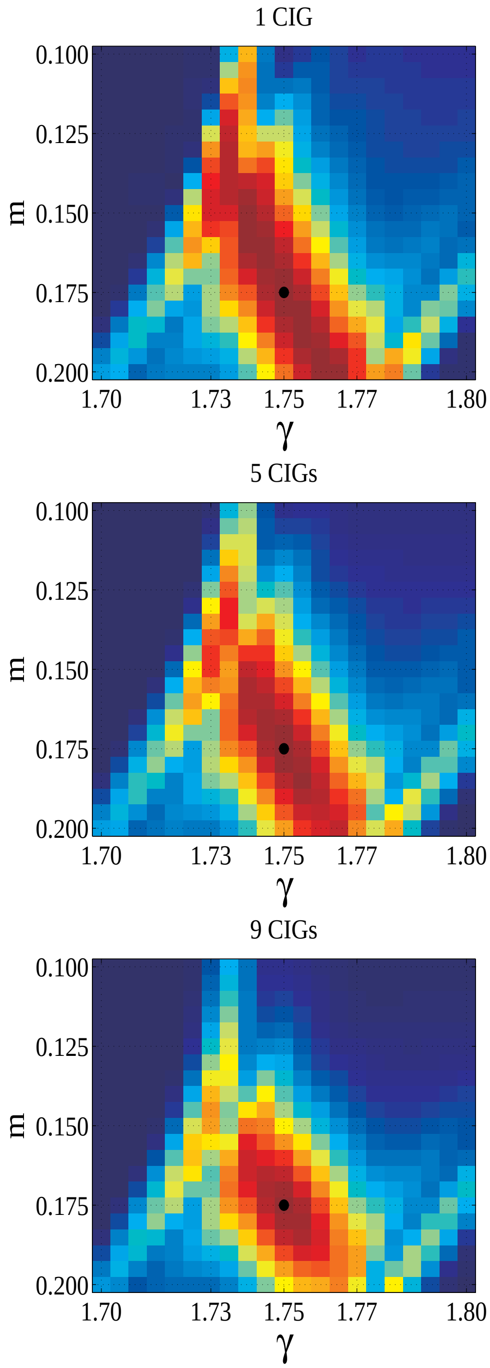

Figure 3: Semblance value for the model of dip 10° for 1 (above) , 5 (middle) and 9 (below) image gathers. The black dot marks the location of the theoretical parameters values (in these cases, theoretical parameters and parameters with highest semblance value are the same).