# Separate P- and SV-wave equations for VTI media

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## Abstract

**In isotropic media we use the scalar acoustic wave equation to perform reverse time migration (RTM) of the recorded pressure wavefield data. In anisotropic media P- and SV-waves are coupled and the elastic wave equation should be used for RTM. However, an acoustic anisotropic wave equation is often used instead. This results in significant shear wave energy in both modeling and RTM. To avoid this undesired SV-wave energy, we propose a different approach to separate P- and SV-wave components for vertical transversely isotropic (VTI) media. We derive independent pseudo-differential wave equations for each mode. The derived equations for P- and SV-waves are stable and reduce to the isotropic case. The equations presented here can be effectively used to model and migrate seismic data in VTI media where $|\varepsilon - \delta|$ is small. The SV-wave equation we develop is now well-posed and triplications in the SV wavefront are removed resulting in stable wave propagation. We show modeling and RTM results using the derived pure P-wave mode in complex VTI media and use the rapid expansion method (REM) to propagate the wavefields in time.**

## Introduction

Reverse time migration (RTM) is becoming the standard tool for imaging areas where large velocity contrast and/or steep dips pose imaging challenges, e.g. around and below salt bodies. RTM propagates the source wavefield forward in time and the receiver wavefield backward in time to image the subsurface reflectors. By using the two-way acoustic wave equation, it naturally takes into account both down-going and up-going waves and thus enables imaging of the turning waves and prism waves that are able to enhance the image of steep salt flanks and other steeply dipping events associated with complex structures. In recent years RTM has gained popularity as computer power has increased enabling its routine application to prestack seismic data.

Seismic anisotropy is observed in many exploration areas. Conventional isotropic migration methods are insufficient in these areas. Thus, where required by analysis of the data, migration methods may be isotropic, or vertical transversely isotropic (VTI) or tilted transversely isotropic (TTI). While isotropic and VTI RTM are widely in use, TTI RTM remains challenging due to its complexity, numerical stability and computational cost. (Crawley et al., 2010).

Many researchers have implemented computationally efficient two-way wave equation modeling and reverse time migration in anisotropic media with the pseudo-acoustic approximation (Alkhalifah, 2000; Zhou et al., 2006; Du et al., 2008; Fowler et al., 2010a,b). Alkhalifah (2000) introduced a pseudo-acoustic approximation for VTI media by setting the vertical S-wave equal to zero in the dispersion relation. Although this dispersion relation for a scalar wavefield has kinematics close to those of the P-arrivals in the real elastic vector wavefield, it allows spurious events. This was initially categorized as a numerical artifact (Alkhalifah, 2000). Grechka et al. (2004) identified it as the SV-component, because simply setting $v_s = 0$ does not result in the vanishing of the shear wave phase velocity everywhere in an acoustic VTI medium (Liu et al., 2009). Methods have been proposed to suppress this artifact, e.g. It is well known that when the source point is located in an isotropic medium above the anisotropic medium, the artifact will disappear.

To avoid the undesired SV-wave energy, different approaches have recently been proposed to model the pure P-wave mode (Etgen and Brandsberg-Dahl, 2009; Liu et al., 2009). Recently, Liu et al. (2009) factorized the dispersion relation presented by Alkhalifah (2000) and obtained two pseudo-partial differential equations. The P-wave equation is well-posed for any value of the anisotropic parameters, but the SV-wave becomes well-posed only when $\varepsilon > \delta$ is satisfied.

Here we present a different approach to separate the P- and SV-wave components in VTI media and derive independent pseudo-differential wave equations for each mode. We show that the derived equations for P- and SV-waves are stable and reduce to the isotropic case. Rather than using the Alkhalifah (2000) dispersion relation, we start with the exact dispersion relation for VTI media as derived by Tsvankin (1996). Using a square root approximation we obtain approximations for the P- and SV-wave dispersion relations. The P-wave dispersion relation, under certain conditions, reduces to the expression used by Etgen and Brandsberg-Dahl (2009).

In this work we also solve the VTI equation system derived by Du et al. (2008) and the pure P-wave equation, using the rapid expansion method (REM), proposed by Pestana and Stoffa (2010), for explicit time extrapolation. REM has less numerical dispersion noise than other time extrapolation operators since it is a better approximation for the second time derivative.

We demonstrate the theory with some simple 2D synthetic examples and also illustrate the high-quality images obtained using REM RTM.

### Approximate scalar wave equations in VTI media

We start with the exact dispersion relations for VTI media as derived by Tsvankin (1996):

$$\frac{v^2(\theta)}{v_{po}^2} = 1+\varepsilon\sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\left[1+\frac{2\varepsilon\sin^2\theta}{f}\right]\left[1-\frac{2(\varepsilon-\delta)\sin^2 2\theta}{f(1+\frac{2\varepsilon\sin^2\theta}{f})^2}\right]^{1/2} \quad (1)$$

where $\theta$ is the phase angle measured from the symmetry axis. The plus sign corresponds to the P-wave and the minus sign corresponds to the SV-wave.

Here $f = 1-\left(\frac{v_{so}}{v_{po}}\right)^2$, $v_{po}$ and $v_{so}$ are the P- and S-wave velocities respectively, and $\varepsilon$ and $\delta$ are the Thomsen (1986) parameters. We expand the square root to first order ($\sqrt{1+X^2} = 1+\frac{1}{2}X$) and obtain the approximations

$$\frac{v^2(\theta)}{v_{po}^2} \approx 1+2\varepsilon\sin^2\theta - \frac{(\varepsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\varepsilon\sin^2\theta}{f})} \qquad \text{P-wave} \quad (2)$$

and

$$\frac{v^2(\theta)}{v_{po}^2} \approx 1-f+\frac{(\varepsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\varepsilon\sin^2\theta}{f})} \qquad \text{SV-wave} \quad (3)$$

In order to develop these equation further we introduce the horizontal P-wave velocity $v_h$ and the P-wave NMO-velocity $v_n$, as given by:

$$\begin{aligned} v_h^2 &= v_{po}^2(1+2\varepsilon) \\ v_n^2 &= v_{po}^2(1+2\delta). \end{aligned} \quad (4)$$

We also have $\sin(\theta) = \frac{v(\theta)k_r}{\omega}$ and $\cos(\theta) = \frac{v(\theta)k_z}{\omega}$ so that

$$v^2(\theta) = \frac{\omega^2}{k_r^2+k_z^2} \quad (5)$$

with $k_r^2 = k_x^2+k_y^2$. The results are the dispersion relations

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2 k_r^2 - \frac{(v_h^2-v_n^2)k_r^2 k_z^2}{k_z^2+F k_r^2} \quad (6)$$

for P-waves, and

$$\omega^2 = v_{so}^2(k_r^2+k_z^2) + \frac{(v_h^2-v_n^2)k_r^2 k_z^2}{k_z^2+F k_r^2} \quad (7)$$

for SV-waves.

Here

$$F = 1+\frac{2\varepsilon}{f} = \frac{v_h^2-v_{so}^2}{v_{po}^2-v_{so}^2} \quad (8)$$

The new equations 6 and 7 are good approximations for the P- and SV-wave dispersion relation if

$$\left|\frac{2(\varepsilon-\delta)\sin^2 2\theta}{f(1+\frac{2\varepsilon\sin^2\theta}{f})^2}\right| << 1 \quad (9)$$

When we set $v_{so} = 0$ (or $f = 1$) equations 6 and 7 reduce to the equations derived by Liu et al. (2009) using the dispersion relation presented by Alkhalifah (2000). If instead, we set $\varepsilon = 0$ in this expression, then $F = 1$, and equation 6 reduces to the dispersion relation used by Etgen and Brandsberg-Dahl (2009) and Crawley et al. (2010).

### Pure P-wave equation - Implementation

Based on the work of Zhang and Zhang (2009), the two-way wave equation can be transformed to a first order equation in time given by:

$$\left(\frac{\partial}{\partial t}+i\Phi\right)P(x,y,z,t) = 0 \quad (10)$$

where P is the complex pressure wavefield and $\Phi$ is a pseudo-differential operator in the space domain. In isotropic media, the operator is defined by $\Phi = v\sqrt{-\nabla^2}$ or by its symbol $\varphi = v(x,y,z)\sqrt{k_x^2+k_y^2+k_z^2}$ where $v$ is the velocity in the space domain.

To produce anisotropic wave propagation, without adding spurious waves, we can use expression 6 and in this case we have:

$$\varphi = \sqrt{v_{po}^2 k_z^2 + v_h^2 k_r^2 - \frac{(v_h^2-v_n^2)k_r^2 k_z^2}{k_z^2+F k_r^2}} \quad (11)$$

The solution of equation 10 is given by:

$$P(t+\Delta t) = e^{-i\Phi\Delta t}P(t) \quad (12)$$

Adding $P(t-\Delta t) = e^{i\Phi\Delta t}P(t)$ to equation 12 we obaitn

$$p(t+\Delta t)+p(t-\Delta t) = 2\cos(\Phi\Delta t)p(t) \quad (13)$$

Now we can revert to $p$ as the imaginary part is decoupled (cosine is real). Since cosine is an even function, its expansion contains only powers of $\Phi^2$, which is a differential operator. The cosine function can now be evaluated by the rapid expansion method (REM) (Pestana and Stoffa, 2010).

### Numerical results

In a modeling experiment, time snapshots of wave propagation in a homogeneous VTI medium ($v_{po} = 3000m/s$, $\varepsilon = 0.24$, and $\delta = 0.1$) are simulated with the source pulse in the center of the model. Figure and Figure correspond to the same time snapshots from the simulations using the rapid expansion method (REM) to the system of equations of Du et al. (2008) for the p and q wavefields. On these figures a diamond-shaped spurious SV-wave front can be seen. Figure shows the P-wave and Figure shows the SV component computed from the decoupled P- and SV-wave equations proposed by Liu et al. (2009) and also solved by REM. The system of equations introduced by Du et al. (2008) produce a strong unwanted spurious SV-wave and it is possible that SV-wave artifacts will contaminate RTM images. Using the decoupled P- and SV-waves, it is clear that Figure has only a P-wave component, while Figure has only a SV wave component. Therefore, a pure P-wave wave equation offers a better imaging alternative since it does not have the SV-wave artifacts.

To demonstrate the theory proposed here, we use the same anisotropic model parameters. In Figure we have the the same time snapshot from simulation by REM to the pure P-wave ( eq. 6 ) and in Figure we have the SV-wave component (eq. 7). Both were computed using $F = \frac{1+2\varepsilon-\gamma^2}{1-\gamma^2}$ and with $\gamma^2 = \frac{v_{so}^2}{v_{po}^2} = 1/4$. In Figure we also have the simulation of the pure P-wave, but with $F = 1$, which is the same dispersion relation used by Etgen and Brandsberg-Dahl (2009). The pure-P wave results presented in Figure  and Figure  are quite close. However, the SV-wave using the new SV-wave equation proposed here (eq. 7) is stable. In the new SV-wave equation, the shear wave velocity is not zero which removes the SV wavefront triplication and results in stable wave propagation (Tsvankin, 2001).

Next an anisotropic salt model is used to test the performance of imaging quality with the theory presented here and with the wave equations solved by REM. The input 2D synthetic dataset was generated from this model using elastic finite-difference modeling. The vertical P-wave velocity is shown in Figure . The $\varepsilon$ and $\delta$ parameters are shown in Figure  and Figure , respectively.

Figure  shows the anisotropic reverse time migration result of the p wavefield, using REM for Du et al. (2008)'s system of equations. This result was obtained using the correct model parameters with $v_{so} = 0$. For comparison, we show the isotropic reverse time migration results (Figure ) which was imaged using the P-wave velocity. In Figure , the anisotropic migration improves the image of the steeply dipping reflectors including the faulted bed and salt body. It also correctly images the reflector in the center of the section which is caused by variations only in the anisotropic parameters. This event nearly disappears in the isotropic image and generates some noise artifacts.

In Figure  we show the result of prestack RTM using our equation 6 with F computed with $\frac{v_{so}^2}{v_{po}^2} = 1/4$ and $\varepsilon$ equal to the maximum value in the model. Figure  is the prestack RTM image obtained using $F = 1$ as proposed by Etgen and Brandsberg-Dahl (2009). We see that these two images are very similar to the results obtained using the equations from Du et al. (2008). But our new equation and the one proposed by Etgen and Brandsberg-Dahl (2009) are computationally more efficient.

## Conclusions

A new procedure to derive individual P- and SV-waves for VTI media is presented and analyzed. The dispersion relation proposed by Alkhalifah (2000), which yields a kinematically good approximation of P-waves in a VTI medium, has with a major drawback in that it generates a spurious SV-wave. Here we separate the P- and SV modes into two wave equations based on the Tsvankin (1996) dispersion relations. The equations propose here can be effectively used in VTI media where $|\varepsilon - \delta|$ is small. The SV-wave equation obtained is now well-posed and the triplication in the SV wavefront, as documented by Tsvankin (2001), is removed and allows a stable wave propagation. We also showed also that the pure P-wave equation is equivalent to the P-wave derived by Liu et al. (2009) when we have $v_{so} = 0$, and that it reduces to the equation used by Etgen and Brandsberg-Dahl (2009) when the F factor, introduced in our equations, is equal to 1. The pure P-wave derived here was used in our implementation of 2D VTI reverse time migration using the rapid expansion method (REM). The REM solution provides accurate and nondispersive wave propagation and it was used to time-step the Du et al. (2008) system of equations and also our pure P scalar wave equation. We concluded that the RTM VTI media using our decoupled equation and REM for time extrapolation provides accurate images.

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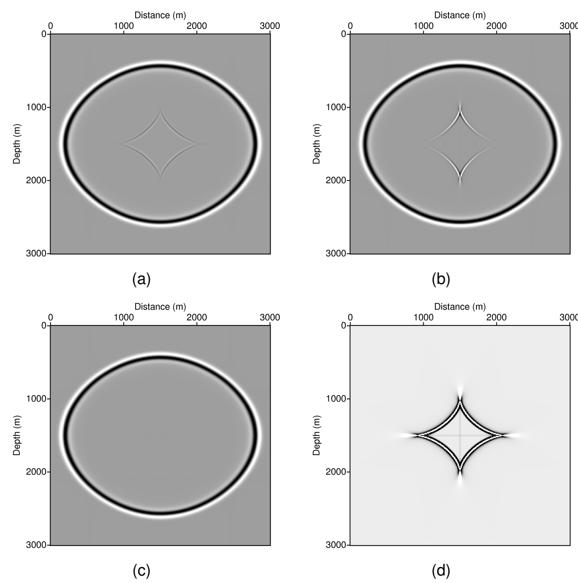



Figure 1: Impulse response: (a) P-wave wavefield and (b) q-wave wavefield, of the Du et al. (2008) equations solved by REM. They clearly show the spurious SV-wave artifacts. (c) P and S wavefields (d) from the decoupled P- and SV-wave equations proposed by Liu et al. (2009) also solved by REM.

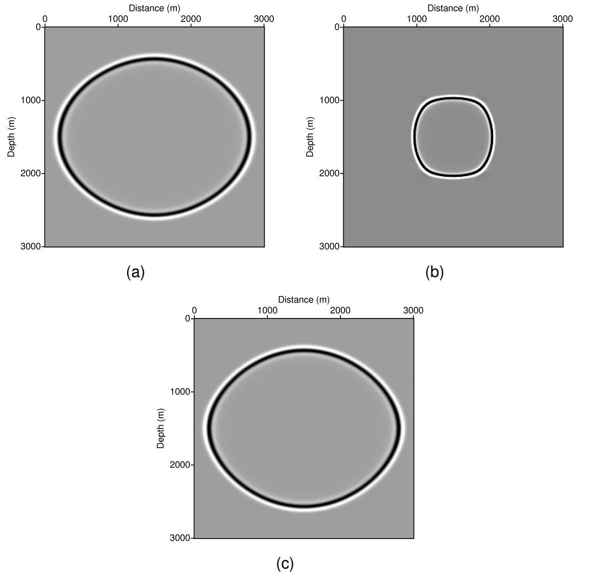



Figure 2: Impulse response: (a) Pure P-wave wavefield and (b) SV-wave wavefield using the method presented here. REM was used to solve equations 6 and 7, respectively. (c) P-wave wavefield solved by REM of the dispersion relation given by equation 6 with $F = 1$).

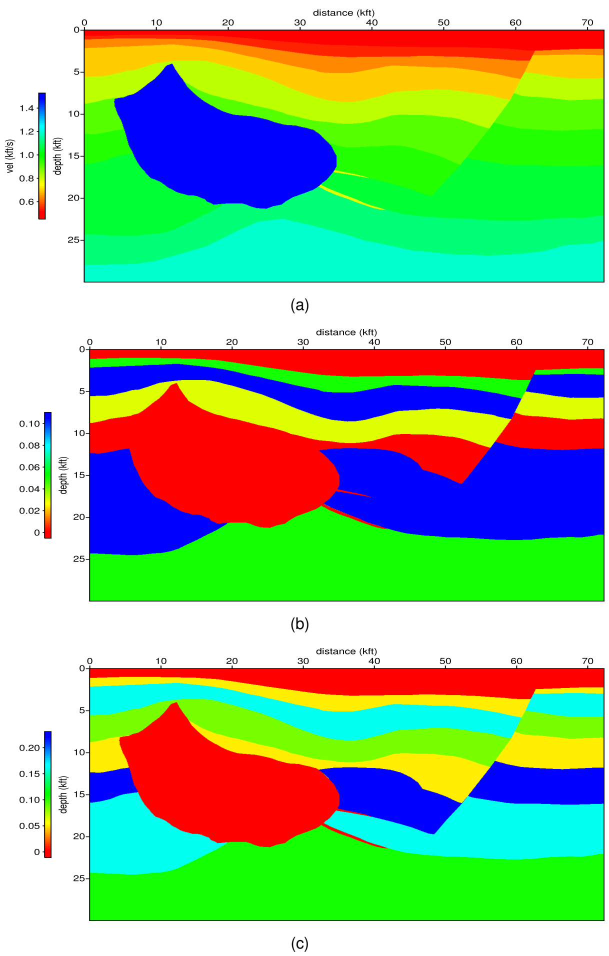



Figure 3: Salt model; (a) Velocity field; (b) delta and (c) epsilon.

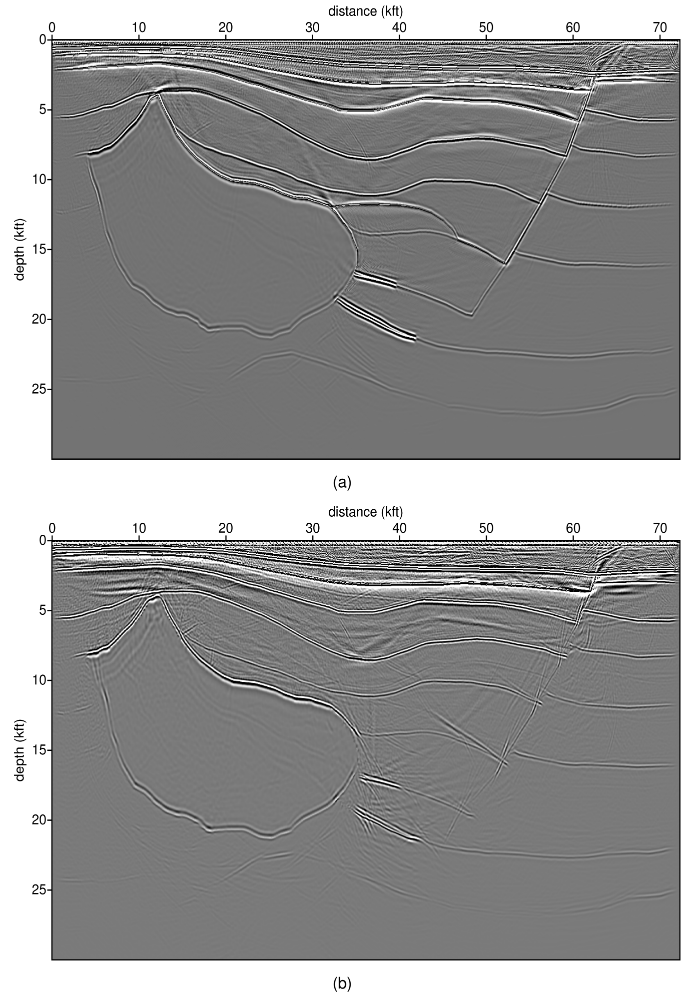
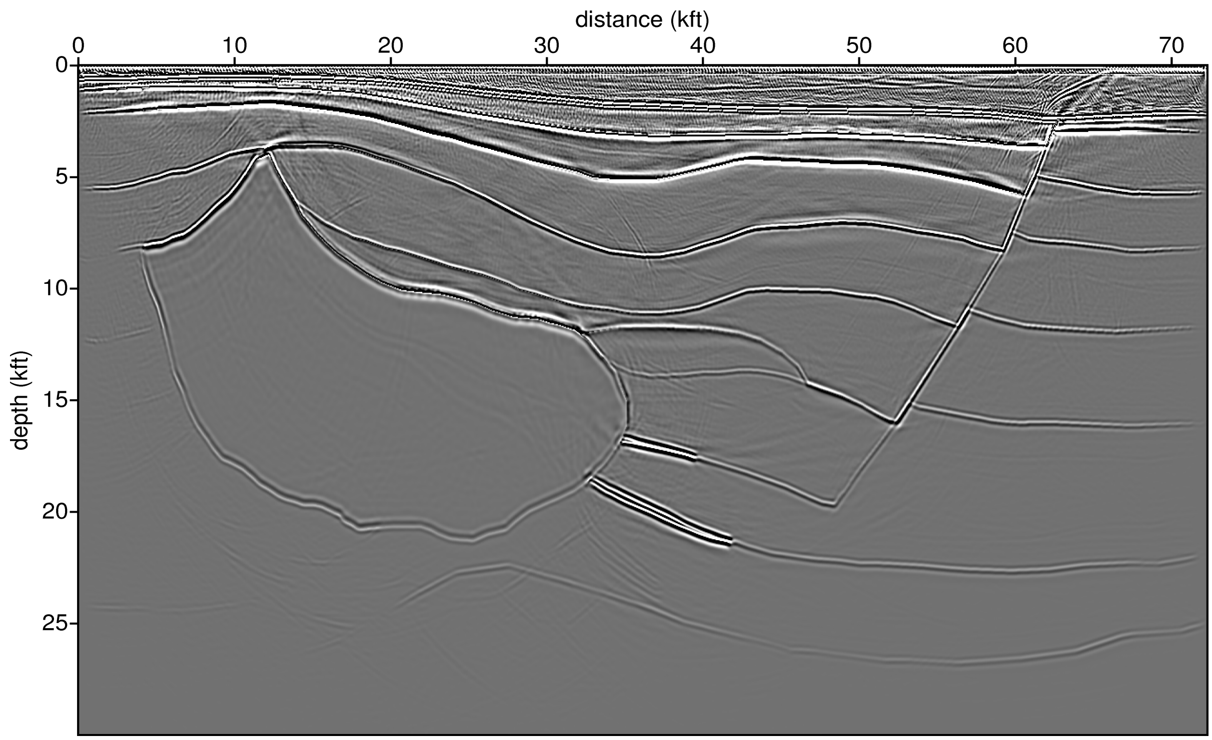
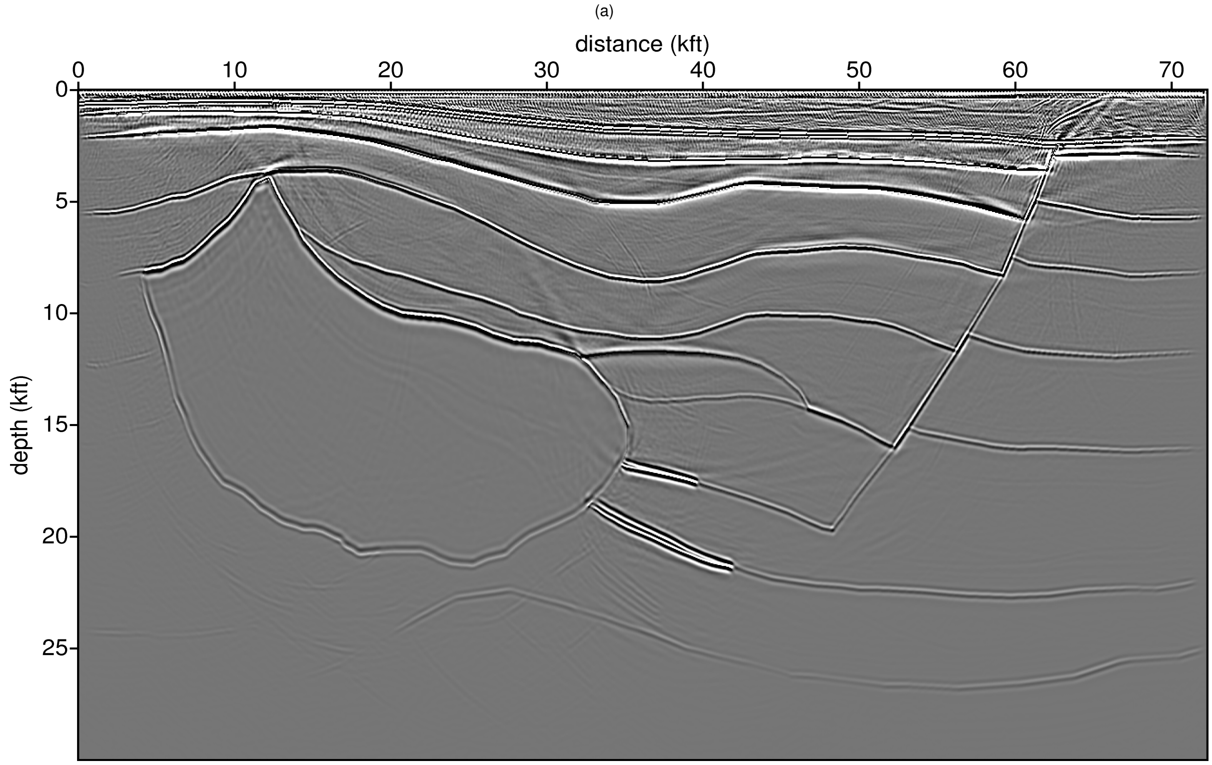



Figure 4: (a) Anisotropic reverse time migration by REM using the Du et al. (2008) equations and the correct VTI model parameters. (b) Isotropic reverse time migration also solved by REM.




(a)




(b)

Figure 5: (a) Anisotropic reverse time migration by REM our new pure-P wave equation ( eq. 6) and (b) by solution proposed by Etgen and Brandsberg-Dahl (2009) ( equation 6 with F=1.)

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