An acoustic wave equation for pure P wave in 2D TTI media

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Abstract

In this paper, a pure P wave equation for an acoustic 2D TTI media is derived. Compared with conventional TTI coupled equations, the resulting equation is unconditionally stable due to the complete isolation of the SV wave mode. To avoid numerical dispersion and produce high quality images, the rapid expansion method (REM) is employed for numerical implementation. Synthetic results validate the proposed equation and show that it is a stable algorithm for modeling and reverse time migration (RTM) in a TTI media for any anisotropic parameter values.

Introduction

The isotropic acoustic assumption for seismic processing may not always be appropriate. This fact has been recognized in areas such as the North Sea and the Canadian Foothills. Conventional isotropic methods may cause errors in anisotropic media, which result in low resolution and misplaced images of subsurface structures. Therefore, imaging of surveys in the presence of anisotropy requires a migration method that can handle general anisotropic media to obtain a significant improvement in image quality, clarity, and positioning.

Some ordering in the structure of rocks, such as fine-layering and parallel cracks can induce anisotropic effects in the wave propagation. VTI RTM is a good simplification to image such structures since they have similar properties with a VTI media (Crampin, 1984) However, the VTI assumption may not be satisfied for imaging under steeply dipping anisotropic overburdens. For example, if the sedimentary layering is not horizontal, such as shale masses overlying dipping salt flanks, the symmetry axis is most likely to be tilted. Ignoring the titled symmetry not only causes image blurring and mispositioning of the salt flank, but also degrades and distorts the base of salt and subsalt images (Zhang and Zhang, 2009). In other words, a local symmetry assumption instead of a global one is more realistic.

Alkhalifah (2000) started from the dispersion relation and proposed a pseudo-acoustic wave equation for TI media by setting the shear wave velocity along the symmetry axis to be zero. Based on Alkhalifah's pseudo-acoustic approximation, a number of variations of pseudo-acoustic wave equation have been developed to account for VTI media (Zhou et al., 2006a; Du et al., 2008; Duveneck et al., 2008). Assuming the symmetry axis is non-vertical and locally variable, extensions from VTI to TTI have been developed (Zhou et al., 2006b; Fletcher et al., 2008; Zhang and Zhang, 2008). This allows the anisotropy to conform to spatially variable structure.

The main problem of previously published methods is that they are not really free of shear waves. Because simply setting the shear wave velocity along the symmetry axis to be zero does not result in the vanishing of the shear wave phase velocity everywhere in an acoustic TI media (Grechka et al., 2004). The generated SV component is usually considered as numerical artifacts and may cause numerical instabilities in a TTI media. It is well known that a small smoothly tapered circular region with $\varepsilon = \delta$ setting around the source (Duveneck et al., 2008) can avoid shear wave artifacts generated from the source. However, contrasts existed in anisotropic parameter models elsewhere still produce shear wave artifacts. To avoid the undesired SV wave mode ultimately, different approaches have recently been proposed to model the pure P wave mode (Etgen and Brandsberg-Dahl, 2009; Liu et al., 2009; Pestana et al., 2011) for the VTI case.

In this paper, we construct a pseudo-differential wave equation for the P wave mode in 2D TTI media. Rather than following Alkhalifah's (2000) work, we start with a new derivation from the exact dispersion relation that was originally derived by Tsvankin (1996). Using a square root approximation, we obtain an approximation for the P wave dispersion relation. The rapid expansion method (REM) proposed by Pestana and Stoffa (2010) is chosen to propagate the wavefield in time since it has no numerical dispersion. Impulse responses for modeling using the new equation have been shown. RTM examples associated with the BP 2D TTI benchmark dataset are presented as well to validate the proposed algorithm. For comparison, results from the TTI coupled equations (Zhou et al., 2006b) solved by REM are also presented.

Formulations

Coupled Equations for 2D TTI Media

Start with the exact phase velocity expression (Tsvankin, 1996)

$$\frac{V^2(\theta)}{V_{p_0}^2} = 1+\varepsilon \sin^2\theta - \frac{f}{2} + \frac{f}{2}\left(1+\frac{2\varepsilon \sin^2\theta}{f}\right)\sqrt{1-\frac{2(\varepsilon-\delta)\sin^2 2\theta}{f\,(1+\frac{2\varepsilon\sin^2\theta}{f})^2}}, \quad (1)$$

where θ is the phase angle measured from the symmetry axis, V_{p_0} is the P wave velocity in the direction of symmetry axis, ε and δ are Thomsen's anisotropy parameters (Thomsen, 1986), and $f = 1 - V_{s_0}^2/V_{p_0}^2$ with the shear wave velocity along the symmetry axis denoted by V_{s_0}. Setting $V_{s_0} = 0$ (i.e., $f = 1$) and after some algebraic manipulations, TTI acoustic wave propagation can be formulated as a coupled 2nd-order PDE system. The 2D version reads (Zhou et al., 2006b)

$$\begin{cases} \dfrac{1}{V_{p_0}^2}\dfrac{\partial^2 p}{\partial t^2} = (1+2\delta)H_2(p+q)+H_1 p \\[2ex] \dfrac{1}{V_{p_0}^2}\dfrac{\partial^2 q}{\partial t^2} = 2(\varepsilon-\delta)H_2(p+q) \end{cases}, \quad (2)$$

where differential operators H_1 and H_2 are defined as

$$\begin{cases} H_1 = (\sin\phi\ \partial_x + \cos\phi\ \partial_z)^2 \\ H_2 = (\partial_x^2+\partial_z^2) - H_1 \end{cases}. \quad (3)$$

Here, p is the usual pressure wavefield, q is an introduced auxiliary wavefield to ease numerical computations, and ϕ is the dip measured to the vertical.

Pure P Wave Equation for 2D TTI Media

We revisit equation 1 and expand the square root to first order (i.e., $\sqrt{1+X} = 1+X/2$), then get an approximation of the P wave phase velocity

$$\frac{V^2(\theta)}{V_{p_0}^2} \approx 1+2\varepsilon\sin^2\theta - \frac{(\varepsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\varepsilon\sin^2\theta}{f})}. \quad (4)$$

Equation 4 (Pestana et al., 2011) is a good approximation for the P wave dispersion relation when

$$\left|\frac{2(\varepsilon-\delta)\sin^2 2\theta}{f\,(1+\frac{2\varepsilon\sin^2\theta}{f})^2}\right| \ll 1. \quad (5)$$

Since we have the following relations $\sin\theta = V(\theta)k_x/\omega$, $\cos\theta = V(\theta)k_z/\omega$, and $V^2(\theta) = \omega^2/(k_x^2+k_z^2)$, plugging them into equation 4 gives

$$\omega^2 = V_{p_0}^2\left[(1+2\varepsilon)k_x^2+k_z^2 - \frac{2(\varepsilon-\delta)k_x^2k_z^2}{F\ k_x^2+k_z^2}\right], \quad (6)$$

where $F = 1+2\varepsilon/f$. Setting $\varepsilon = 0$, equation 6 reduces to the same dispersion relation used by Etgen and Brandsberg-Dahl (2009) and Crawley et al. (2010)

$$\omega^2 = V_{p_0}^2\left[(1+2\varepsilon)k_x^2+k_z^2 - \frac{2(\varepsilon-\delta)k_x^2k_z^2}{k_x^2+k_z^2}\right]. \quad (7)$$

The above dispersion relation is true for VTI media with a vertical symmetry axis. The same relation for TTI media with arbitrary orientation of symmetry axis can be deduced from equation 7 through a variable change corresponding to a rotation of the z-axis in the counterclockwise sense. The wavenumber operators in the rotated coordinate system write

$$\begin{pmatrix}\hat{k}_x\\ \hat{k}_z\end{pmatrix} = \begin{pmatrix}\cos\phi & \sin\phi\\ -\sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}k_x\\ k_z\end{pmatrix}. \quad (8)$$

From equation 8 we have

$$\begin{cases} \hat{k}_x^2 = \cos^2\phi\ k_x^2 + \sin 2\phi\ k_xk_z + \sin^2\phi\ k_z^2 \\ \hat{k}_z^2 = \sin^2\phi\ k_x^2 - \sin 2\phi\ k_xk_z + \cos^2\phi\ k_z^2 \end{cases}. \quad (9)$$

Substituting k_x^2 and k_z^2 in equation 7 with $\hat{k}_x^2$ and $\hat{k}_z^2$, we formulate the P wave dispersion relation for TTI media

$$\begin{aligned}\omega^2 = V_{p_0}^2\Bigg[&k_x^2+k_z^2+(2\delta\sin^2\phi\cos^2\phi+2\varepsilon\cos^4\phi)\frac{k_x^4}{k_x^2+k_z^2}\\ &+(2\delta\sin^2\phi\cos^2\phi+2\varepsilon\sin^4\phi)\frac{k_z^4}{k_x^2+k_z^2}\\ &+(\ \delta\sin 4\phi-4\varepsilon\sin 2\phi\cos^2\phi)\frac{k_x^3k_z}{k_x^2+k_z^2}\\ &+(-\delta\sin 4\phi-4\varepsilon\sin 2\phi\sin^2\phi)\frac{k_xk_z^3}{k_x^2+k_z^2}\\ &+(-\delta\sin^2 2\phi+3\varepsilon\sin^2 2\phi+2\delta\cos^2 2\phi)\frac{k_x^2k_z^2}{k_x^2+k_z^2}\Bigg]. \quad (10)\end{aligned}$$

Multiplying both sides of equation 10 with the wavefield function $p(\omega,k_x,k_z)$ in the Fourier domain, followed by an inverse Fourier transform to both sides and then using the relation $i\omega \leftrightarrow \partial/\partial t$, we finally derive the pure P wave equation in the time-wavenumber domain for 2D TTI media

$$\begin{aligned}\frac{\partial^2 p}{\partial t^2} = V_{p_0}^2\Bigg[&k_x^2+k_z^2+(2\delta\sin^2\phi\cos^2\phi+2\varepsilon\cos^4\phi)\frac{k_x^4}{k_x^2+k_z^2}\\ &+(2\delta\sin^2\phi\cos^2\phi+2\varepsilon\sin^4\phi)\frac{k_z^4}{k_x^2+k_z^2}\\ &+(\ \delta\sin 4\phi-4\varepsilon\sin 2\phi\cos^2\phi)\frac{k_x^3k_z}{k_x^2+k_z^2}\\ &+(-\delta\sin 4\phi-4\varepsilon\sin 2\phi\sin^2\phi)\frac{k_xk_z^3}{k_x^2+k_z^2}\\ &+(-\delta\sin^2 2\phi+3\varepsilon\sin^2 2\phi+2\delta\cos^2 2\phi)\frac{k_x^2k_z^2}{k_x^2+k_z^2}\Bigg]p. \quad (11)\end{aligned}$$

Numerical Implementation

Equation 11 can be solved numerically by the rapid expansion method (REM). The REM for reverse time migration problem was proposed by Pestana and Stoffa (2010) for the isotropic case and has been successfully applied to the pure P wave equation (equation 6 or 7) for the VTI case as well (Pestana et al., 2011). It is based on the Chebyshev polynomial expansion (Tal-Ezer et al., 1987) and applied to seismic modeling by Kosloff et al. (1989). It has no dispersion and allows a better approximation for the 2nd-order time derivative when combined with the Fourier pseudo-spectral method to compute the spatial derivatives. The numerical implementation of equation 11 using REM is described in Appendix.

Numerical Examples

In all of the following 2D numerical examples, wave propagation is simulated in time using the REM. First, we use impulse responses to demonstrate the proposed algorithm. Figure 1 shows time snapshots of wave propagation in a 2D homogenous TTI media. A diamond-shaped SV wavefront arises from the middle of Figure 1a using the TTI coupled equations. Figure 1b is obtained by solving equation 11, and it is clear that it contains only the P wave component. Therefore, the proposed pure P wave equation offers a better modeling and migration alternative since it does not have the SV wave mode.

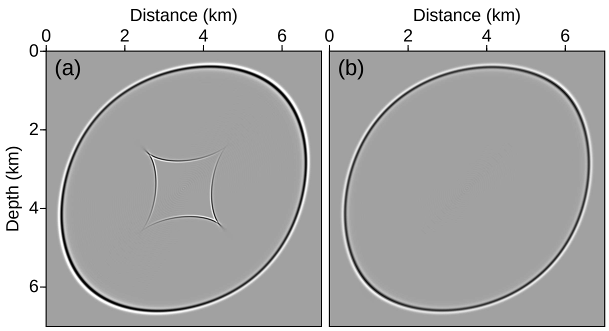

Figure 1: Wavefield snapshots in a homogeneous TTI media ($V_{p_0} = 3000\ m/s,\ \varepsilon = 0.24,\ \delta = 0.1,\ \theta = 45°$). (a) is obtained by solving the TTI coupled equations. (b) is from the solution of the pure P wave equation.

Second, we show some examples associated with the BP 2D TTI model. Figure 2 shows the parameter values for V_{p_0}, ε, δ and θ of a small region of the BP model. Figure 3a displays a forward modeling snapshot from the TTI coupled equations. Notice that numerical instabilities arise from the interaction of the SV wavefront with rapid variations in the tilt axis. Setting $\varepsilon = \delta$ around locations with a sharp dip contrast can stabilize wave propagation and the resulting snapshot is displayed in Figure 3b. This selective anisotropic parameter matching scheme (Yoon et al., 2010) is simple and easy to implement, but it greatly depends on the choice of selection and is tricky to implement for a complicated model. Besides, it alters wave kinematics. Instead, we use equation 11 to propagate the wavefield and the result is shown in Figure 3c. Here the remaining instabilities are due to the ringing effects in the wavefield. To remove the high frequency spatial oscillations and thus stabilize the computation, a band pass filter is applied to the wavenumber operators. The stabilized wavefield snapshot without ringing effects is displayed in Figure 3d. Another way to eliminate the ringing effects is to compute the spatial derivatives with an odd order on a staggered grid (Corrêa et al., 2002). Without ringing effects, equation 11 provides a stable result and is completely independent of variations in the anisotropic parameter models. Wave kinematic properties are well preserved also.

Finally, RTM results are presented. Figure 4a is obtained by solving the TTI coupled equations using the selective anisotropic parameter matching scheme. With the pure P wave equation given by equation 11, we get a RTM image shown in Figure 4b, which is comparable with Figure 4a but without any change to the anisotropic parameter values.

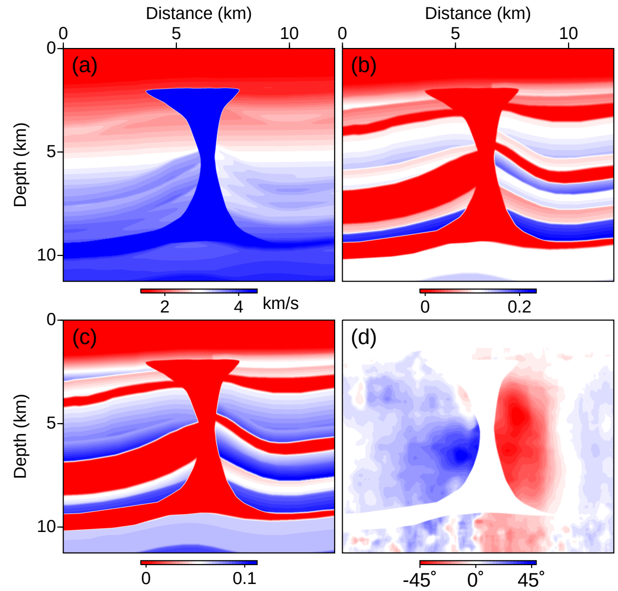

Figure 2: Partial region of the BP 2D TTI model: (a) V_{p_0}; (b) ε; (c) δ; and (d) θ.

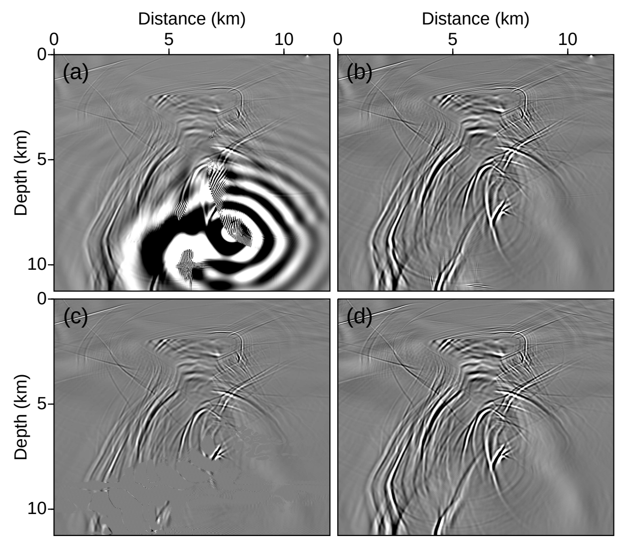

Figure 3: Wavefield snapshots in the BP 2D TTI model. (a) Wavefield snapshot from the TTI coupled equations. The wave propagation is unstable when a sharp contrast exists in the dip model. (b) The same as (a) but simply setting $\varepsilon = \delta$ in regions with rapid dip angle variations. The wavefield blow-up disappeared. (c) Wavefield snapshot from equation 11. The remaining instabilities are caused by ringing effects. (d) A stable wavefield snapshot is achieved using equation 11 with a band pass filter applied to the wavenumber operators k_x and k_z.

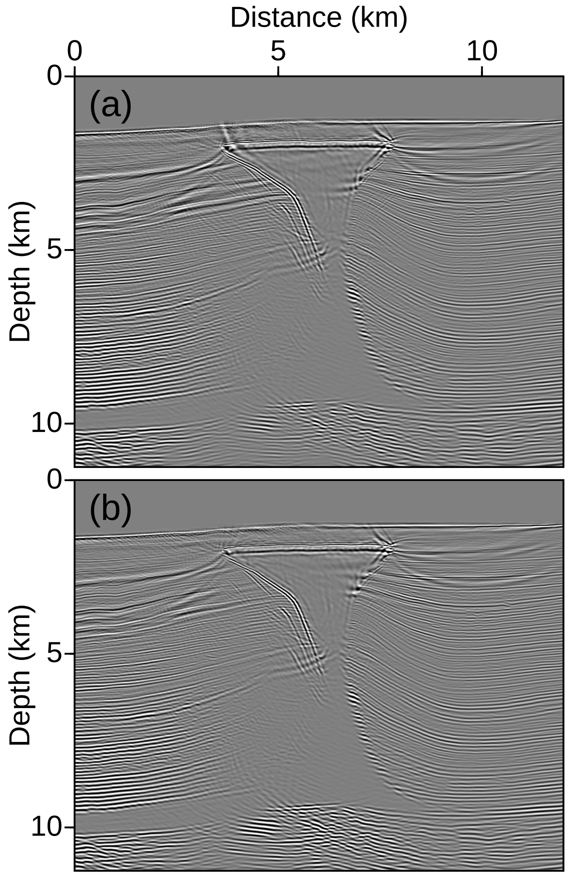

Figure 4: TTI RTM images of the partial BP model. (a) is corresponding to the solution of the TTI coupled equations, and (b) is achieved by solving the pure P wave equation.

Summary and Conclusions

The conventional TTI coupled equations lead to instability in general TTI media. Starting with the exact dispersion relation, a pseudo-differential equation for the P wave component in an acoustic 2D TTI media is derived. SV wavefront triplication is successfully removed. Numerical examples show that modeling and migration using the new equation provides a stable P wave propagation with any anisotropic parameter values. The REM is adopted in the numerical implementation and it provides accurate and nondispersive wave propagation. We conclude that a stable TTI RTM is achievable with the pure P wave equation proposed here, and an implementation with REM provides high quality TTI RTM images.

Appendix: Rapid expansion method (REM)

The solution of equation 11 is given by

$$p(t+\Delta t) = -p(t-\Delta t) + 2\cos(L\Delta t)\,p(t), \tag{12}$$

where pseudo-differential operator L is defined as

$$\begin{aligned} -L^2 = V_{p_0}^2 \Big[& k_x^2 + k_z^2 + (2\delta \sin^2\phi \cos^2\phi + 2\varepsilon \cos^4\phi)\frac{k_x^4}{k_x^2+k_z^2} \\ & +(2\delta \sin^2\phi \cos^2\phi + 2\varepsilon \sin^4\phi)\frac{k_z^4}{k_x^2+k_z^2} \\ & +(\ \ \delta \sin 4\phi - 4\varepsilon \sin 2\phi \cos^2\phi)\frac{k_x^3 k_z}{k_x^2+k_z^2} \\ & +(-\delta \sin 4\phi - 4\varepsilon \sin 2\phi \sin^2\phi)\frac{k_x k_z^3}{k_x^2+k_z^2} \\ & +(-\delta \sin^2 2\phi + 3\varepsilon \sin^2 2\phi + 2\delta \cos^2 2\phi)\frac{k_x^2 k_z^2}{k_x^2+k_z^2} \Big]. \end{aligned} \tag{13}$$

An efficient orthogonal polynomial series expansion for the cosine function in equation 12 was presented by Tal-Ezer et al. (1987)

$$\cos(L\Delta t) = \sum_{k=0}^{M} C_{2k}\, J_{2k}(R\Delta t)\, Q_{2k}(\frac{iL}{R}), \quad (M \to \infty) \tag{14}$$

where $C_{2k} = 1$ for $k = 0$ and $C_{2k} = 2$ for $k > 0$. R is chosen as the largest eigenvalue of L^2. J_{2k} is the Bessel function of the first kind order and Q_{2k} are the modified Chebyshev polynomials that satisfy the following recurrence relations

$$\begin{aligned} Q_0(\frac{iL}{R}) &= I, \\ Q_2(\frac{iL}{R}) &= I - \frac{L^2}{R^2}, \\ Q_{2k+2}(\frac{iL}{R}) &= 2(I - \frac{2L^2}{R^2})Q_{2k} - Q_{2k-2}. \end{aligned} \tag{15}$$

Here, I is the identity matrix.

For 2D isotropic wave propagation, the value of R is given by $R = \pi V_{max}\sqrt{1/\Delta x^2 + 1/\Delta z^2}$, where V_{max} is the highest P wave velocity in the grid. For anisotropic case, V_{max} should be replaced by $V_{max}(1+|\varepsilon|_{max})$ and $|\varepsilon|_{max}$ is the maximum absolute value from the ε model.

The summation in equation 14 is known to converge exponentially for $M > R\Delta t$, therefore the summation can be safely truncated using a value of M slightly greater than $R\Delta t$. Pestana and Stoffa (2010) have demonstrated that when $M = 1$, which means only two terms are kept in the summation, this approximation of the cosine function using the Chebyshev polynomials results in the 2nd-order in time finite-difference scheme. When $M = 2$, the L^4 operator term is included, this approximation is equivalent to the 4th-order finite-different scheme proposed by Dablain (1986) and Etgen (1986).

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