Two-point ray tracing based on Eikonal solvers

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Abstract

This paper presents two-point ray tracing algorithm in anisotropic media and in isotropic media with salt intrusions of complicated interfaces. The main peculiarity of this approach is the use of finite difference solutions of Eikonal equation. The traditional ways of two point ray tracing are shooting and bending (Peryra V., Lee W.H.K., Keller H.B., 1980). In comparison to these well-known approaches the proposed algorithm is more stable and computationally cheaper.

This approach is extremely effective as two point ray tracing method for specific acquisition geometries widely used for 3D Offset and Walk-away VSP. Really, for this statements a moderate number of receivers (a few tens usually) are used, so one should apply nonlinear eikonal solver for a few ten times as well. Another important advantage is the fact that time of computations does not depend on the complexity of a model.

In order to illustrate the range of applicability we present numerical results for anisotropic Gullfaks model and in well known Sigsbee2a model that contains salt body with very rough interface. The experiments demonstrate the approach provides stable two point ray tracing algorithm for a medium with complex geology in a presence of anisotropy layers and salt intrusions. Numerical experiments prove that the method finds a ray for any source/receiver pair in a presence of anisotropy and salt intrusions. The last property is really important in many applications.

Introduction

Nowadays two-point ray tracing is widely used in many ray based seismic application (for example, ray based tomography). Efficient solution of this problem is a challenge in anisotropic media and in media containing salt bodies. A natural way to solve two point ray tracing problem is shooting method. The essence of shooting methods is repeated solution of the Cauchy problem for the source while ray does not hit receiver (Peryra V., Lee W.H.K., Keller H.B., 1980). Poor stability in environments with large speed contrasts, but also in the presence of anisotropy, and the large computational costs do not allow the application of those algorithms for two-point ray tracing in complex media (Thurber, Kissling, 2000). More widespread methods are ray bending methods. They allow to select geometry of ray for a fixed pair of source-receiver by minimizing the travel time between them (Moser et.al., 1992). These methods are more stable than shooting methods in a presence of high contrasts. But in a sense of computational time they may cost much in complex media with salt bodies and in media containing anisotropic layers.

On the other hand if we talk about solving the eikonal equation then the finite difference methods (Sethian, J.A., 1996, Qian, J., and Symes, W., 2002) of this equation do not have such problems as existing methods of two-point tracing. Mathematical justification of the numerical solution of the eikonal equation using finite-difference scheme is given in the paper of French mathematician Lions (1982). He shows that generalized solution is stable with respect to the parameters of the medium, source position, etc. It can be found by using the finite-difference schemes. The results of the Lions (1982) refer to the isotropic case. But it is also shown (see for example Qian, J., and Symes, W., 2002) that the eikonal equation can be solved using the finite-difference scheme describing the propagation of quasi-longitudinal waves in anisotropic media.

As mentioned above, finite difference methods for solving the eikonal equation devoid of the shortcomings of existing two-point ray tracing methods. Therefore it is natural to use these methods for solving the eikonal equation. After we solve eikonal equation by the finite difference scheme we have spatial time derivatives. *The key of our approach is to use those spatial time derivatives when we solve two-point ray tracing problem.* This allows us to solve Cauchy problem in order to solve two-point ray tracing problem. Solving Cauchy problem is more stable and easy than solving the boundary value problem. Therefore this approach provides computationally stable and cheap two-point ray tracing algorithm in environments with high contrast and in the presence of anisotropy.

Method

The first stage of the proposed two-point ray tracing algorithm is solving the eikonal equation by the finite difference (FD) algorithm:

$$(\nabla \tau)^2 = \frac{1}{V^2(\mathbf{p},\mathbf{x})}, \quad (1)$$

where τ - is travel time at the point $\mathbf{x} = (x, y, z)$, $\mathbf{p} = \nabla \tau$ and $V(\mathbf{p},\mathbf{x})$ is phase velocity. In the present work we consider P – waves in isotropic media and qP – waves in transversal isotropic media.

There are two classes of FD eikonal solvers: point expanding algorithms and box expanding methods. Point expanding methods propagate time field in all directions from the given source point. Usually point expanding schemes are based on upwind first order finite difference eikonal approximation and special recount algorithm (Van Trier J. and Symes W.W. 1991). The most popular is Fast Marching Method (FMM) proposed by (Sethian J.A., 1996) being based on Dijkstra's algorithm (Dijkstra E.W.,1959). Fast marching method is of the first order and requires $O(N \ln N)$ operations to compute time field in N grid points. This method is applicable practically in any complex isotropic velocity model, in particular it computes travel times in high contrast models with salt bodies. The essential drawback is that FMM doesn't work in anisotropic media.

Box expanding methods continue time field, preset on the surface, to the depth. The eikonal equation is rewritten in the form:

$$\tau_z = H(\tau_x, \tau_y, V(x,y,z)). \quad (2)$$

The gradient of the travel time field $\mathbf{p} = \nabla \tau$ is not continuous in the general case. The modern approach is to approximate right and left horizontal spatial derivatives $\tau^{\pm}_{x,y}$ by using so called essentially non - oscillating (ENO) or weighted essentially non - oscillating (WENO) approximations. And then by using Godunov's like flux function to solve Riemann problem and to get the values $\tau_{x,y}$ for the equation (2) (Qian and Symes, 2002). Then equation (2) is integrated by Runge-Kutta (RK) methods. ENO-WENO RK finite-difference schemes may be up to the forth order and they require $O(N)$ operations. They can be modified for transversal isotropic media (Qian and Symes, 2002) as well. Unfortunately, this approach is suitable only for computations of travel times corresponding to down-going rays. To avoid this limitation one may change from time to time the "computing" direction from vertical to horizontal. Such algorithm is called "Down and Out" (DNO) and is proposed in (Kim and Cook, 1999). Unfortunately DNO doesn't work when rays change their directions from down going to up going many times.

Formally two-point ray tracing is a boundary value problem for the system of ordinary differential equations:

$$\begin{aligned} \frac{d\bar{x}}{d\tau} &= \frac{\partial H(\bar{x},\bar{p})}{\partial \bar{p}}, \ \bar{x}_1 = \bar{x}_s; \\ \frac{d\bar{p}}{d\tau} &= -\frac{\partial H(\bar{x},\bar{p})}{\partial \bar{x}}, \ \bar{\mathrm{x}}_2 = \bar{x}_r. \end{aligned} \quad (3)$$

Function $H(\bar{x},\bar{p})$ is known as the Hamilton-Jacobi function. In complex media solution of this problem is often unstable and the existing methods do not converge frequently.

We propose an approach that reduces the boundary value problem (3) to the Cauchy problem by using finite-difference solutions of the eikonal equation. Let us present this approach in more details.

After calculations of travel times at each grid point by the mentioned above finite-difference schemes the vector fields $\nabla \tau(\vec{x})$ is computed as well. This means that at the same grid points the phase velocity vector $V_{qp}(\tau_x, \tau_z)$ is known also. After that we can recover the derivatives $\bar{p} \equiv \nabla \tau$ at any point in the medium by using bilinear interpolation for each elementary grid triangle. Thus in the problem (3) the second equation is already solved (we know vector $\vec{p}$ everywhere!). It remains to "extract" the information directly about the ray, that is to solve the following Cauchy problem:

$$\frac{d\bar{x}}{d\tau} = \frac{\partial H(\bar{x},\bar{p})}{\partial \bar{p}}, \ \bar{x}_2 = \bar{x}_r. \quad (4)$$

For a transversely isotropic medium the system of equations (4) as follows:

$$\begin{aligned} \frac{dx}{d\tau} &= \frac{1}{2\rho}\left[(\mathrm{c}_{11}+\mathrm{c}_{44})\mathrm{p}_1 + \frac{(\mathrm{c}_{44}-\mathrm{c}_{11})^2|p|^2 p_1 + 0.5k_1 p_3^2 p_1}{\sqrt{(\mathrm{c}_{44}-\mathrm{c}_{11})^2|p|^4 + k_1|p|^2 p_3^2 + k_2 p_3^4}}\right]; \\ \frac{dz}{d\tau} &= \frac{1}{2\rho}\left[(\mathrm{c}_{33}+\mathrm{c}_{44})\mathrm{p}_3 + \frac{(\mathrm{c}_{44}-\mathrm{c}_{11})^2|p|^2 p_3 + 0.5k_1(p_3^3 + |p|^2 p_3) + k_2 p_3^3}{\sqrt{(\mathrm{c}_{44}-\mathrm{c}_{11})^2|p|^4 + k_1|p|^2 p_3^2 + k_2 p_3^4}}\right], \end{aligned} \quad (5)$$

where

$$\begin{aligned} k_1 &= 4(\mathrm{c}_{13}+\mathrm{c}_{44})^2 - (\mathrm{c}_{44}-\mathrm{c}_{33})^2 - 3(\mathrm{c}_{44}-\mathrm{c}_{11})^2 + (\mathrm{c}_{33}-\mathrm{c}_{11})^2; \\ k_1 &= 2(\mathrm{c}_{44}-\mathrm{c}_{33})^2 - 4(\mathrm{c}_{13}+\mathrm{c}_{44})^2 + 2(\mathrm{c}_{44}-\mathrm{c}_{11})^2 - (\mathrm{c}_{33}-\mathrm{c}_{11})^2. \end{aligned} \quad (6)$$

Results

We present now the results of numerical experiments implemented for two fairly well known realistic models. The first model is Gullfaks model. Fig.1 shows the wave fronts being simulated in this model by the WENO-RK3 method. In Fig.2 there are shown the rays in isotropic and anisotropic versions of the Gullfaks model. They are calculated on the basis of constructed fronts. In Fig.2 gray colour denotes the anisotropic layers. Anisotropy parameters in the upper layer are such that $\varepsilon = 0.05, \delta = 0.05$ while in the lower layer: $\varepsilon = 0.20, \delta = 0.15$. These experiments were designed to demonstrate the applicability and feasibility of the proposed approach in anisotropic media and isotropic media with salt intrusions with intricate form. It should be emphasized that the cost of this approach is the same for both isotropic and anisotropic media. In the case of application of shooting or bending the computations in anisotropic media are much more expensive in comparison with isotropic ones.

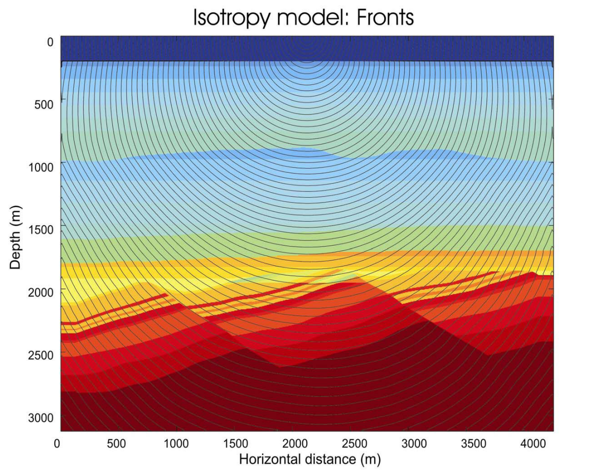

Fig.1 *Fronts calculated by WENORK3 in the isotropic Gullfaks model.*

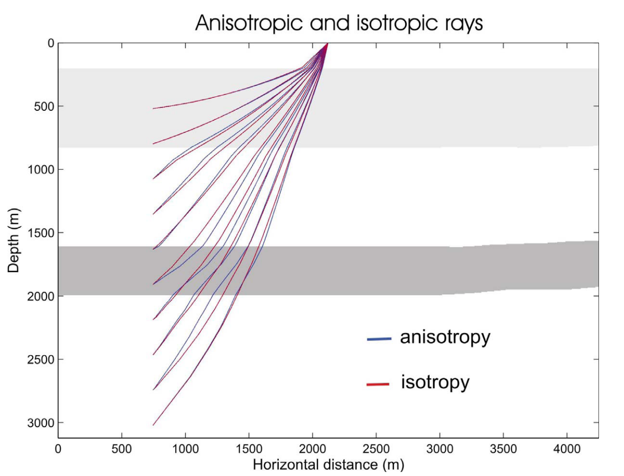

Fig.2 *Rays computed by the presented approach for isotropic (red) anisotropic (blue) Gullfaks model.*

In order to estimate advantages of presented approach for two-point ray tracing let us consider Fig.3. We present there two types of the rays:

1) Black rays – computed with the approach presented above based on the using of “precomputed” vector $\vec{p}$, that is by numerical resolution of equations (4); as one can these rays connect source and receivers in almost perfect way;

2) Red rays – computed with the standard backward propagation of the ray from receivers to a source by numerical resolution of equation (3); there is obvious deviation of rays from the target due to error accumulation.

Our conclusion is that the proposed approach yields a stable solution whereas shooting method shows a very high sensitivity to the initial direction.

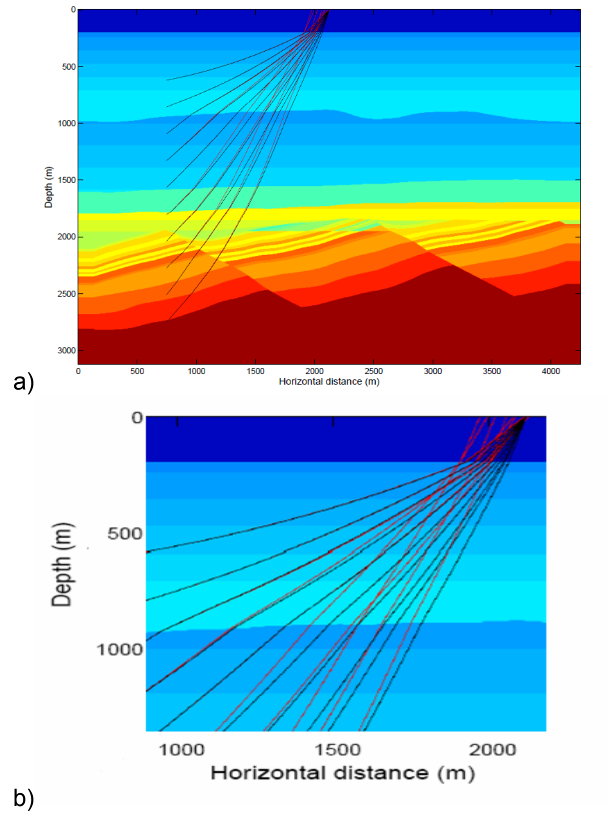

Fig.3 *Rays computed on the basis of the calculated fronts (black) and rays that are computed by shooting method (red) in the anisotropic model Gullfaks (a) and the same, but zoomed view in the source vicinity (b).*

Another model is Sigsbee2a that contains water layer sediments and salt inclusion which is a stumbling block for tracing algorithms. Figure 4 displays the rays that are computed by solving the Cauchy problem using the calculated fronts before. It has to be emphasized that the results obtained in this way (the rays) correspond to waves that give the first arrivals. However please pay attention to the behaviour of rays in a given situation. After passing through the salt all the rays are concentrated together in a very narrow neighbourhood of a single ray. The size of this vicinity is close to the machine precision. So many of these rays will not be available for the calculation by the shooting type of methods. The proposed method calculates all these rays and their computational time cost is practically the same for every ray.

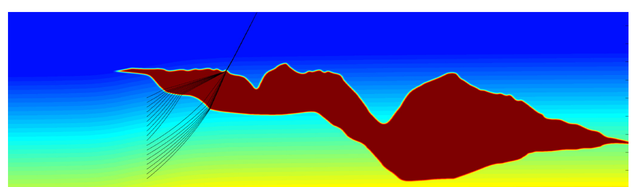

Fig.4 *Two-point ray tracing on the basis of the calculated fronts in the model Sigsbee2a.*

Conclusions

We have proposed and tested a novel approach to the problem of two-point ray-tracing. In this approach the boundary value problem for ordinary differential equations is reduced to the Cauchy problem. This is achieved through the use of pre-calculated wave fronts, i.e. rays are actually in some sense "extracted" from the vector field being a final finite-difference solution of the eikonal equation. The key point in the proposed approach is application of finite-difference schemes for resolution of the eikonal equation. For an isotropic medium there are quite efficient algorithms, in particular fast-marching method. As far as to anisotropic media is concerned we developed finite-difference schemes of Godunov's type and have used them successfully.

Until now poor stability and high computational costs do not allow the full application of traditional algorithms for two-point ray-tracing in complex environments. Particularly it is difficult to implement a two-point ray-tracing through the salt either in the presence of anisotropy. The proposed method allows us to find consistently the rays corresponding to the first arrivals into complex anisotropic media containing high-contrast boundaries. Very important aspect of this approach is the fact that computation time does not depend on the complexity of the model. Described above results allows us to hope that the proposed method will provide additional benefits to those applications in which you want to do a two point tracing. In its own turn this procedure is the engine for tomographic inversion providing valuable information about velocity model.

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