Reverse time migration by interpolation and pseudo-analytical method

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Abstract

Within the seismic method, in order to obtain an accurate seismic image, it is necessary to use some processing techniques, such as the seismic migration. The reverse time migration (RTM) is considered the most accurate migration technique, although its application is restricted due to the enormous computational effort required. Trying to balance the processing cost with the image's quality and confiability, several numeric methods are used to perform the migration. This work presents two different ways of performing the reverse time migration using the complete wave equation: RTM by interpolation and RTM by the pseudo-analytical method. The first migrates the data with different constant velocities and interpolate the results. The second uses modifications in the spacial derivatives in order to compensate errors from the second order time derivatives approximation. The methods applicability was tested by the migration of a bidimensional pre- and pos-stack synthetic data. A real pre-stack data was migrated successfully and is also presented.

Introduction

Since the seismic method entered the digital era, several migration methods have been developed aiming to produce the most accurate subsurface image with the least computational cost. The reverse time migration (RTM) introduced by Whitmore (1983), McMechan (1983), Baysal et al (1983) and Faria (1986) is considered the most accurate migration method. However, its use has been restricted due to the high computational cost required.

The reverse time migration uses the complete wave equation to march from the moment the seismic data is registered on the surface to previous times until the initial time, when the image from the geology by where the seismic wave passed through is reconstructed.

Several strategies can be used to solve the complete wave equation. The proposition of method for that is the core of this work, in which are presented two ways to deal with the equation, the RTM by interpolation and by the pseudo-analytical method.

In the RTM by interpolation, the pressure field interpolation is done considering a certain quantity of constant velocities chose using the procedure proposed by Bagaini et al (1995). After that, the interpolation is done introducing a weight function calculated according to the velocity field.

The RTM by the pseudo-analytical method (Etgen et al., 2009) does a modification in the spatial derivatives in order to compensate the errors of the second order time derivative approximation by finite differences. This scheme is widely applied to solve the time derivatives but it is known that it can introduce numerical errors even when accurate spacial operators are used. The modified spacial derivatives form a pseudo-differential operator which is simplified assuming a constant velocity medium. In this way, it is easily calculated in the wavenumber domain.

The methods validation is done by the migration of synthetic pre- and pos-stack data. A real pre-stack dataset from Gulf of Mexico is also migrated and presented.

Exact solution of the wave equation

Given the constant density wave equation:

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}-\frac{1}{v(\mathbf{x})^2}\frac{\partial^2 P}{\partial t^2}=0 \tag{1}$$

where $P=P(x,y,z,t)$ is the pressure field and $v=v(x,y,z)$ is the medium velocity. To evaluate the Eq. (1) analytically in function of time, it should be rewritten as:

$$\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2}=v^2(\mathbf{x})\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)P(\mathbf{x},t) \tag{2}$$

where the position vector is defined by $(\mathbf{x})=(x,y,z)$ so the propagation velocity is given by $v(\mathbf{x})$ and the pressure field expressed by $P(\mathbf{x},t)$.

Following the deduction presented by Pestana and Stoffa (2009), an operator can be defined calling $v^2(\mathbf{x})\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)=-L^2$, and obtaining the differential equation below:

$$\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2}=-L^2P(\mathbf{x},t) \tag{3}$$

Given the initial conditions $P(\mathbf{x},t=0)=P_0$ and $\frac{\partial P(\mathbf{x},t)}{\partial t}|_{t=0}=\dot{P}_0$, it follows that:

$$P(t)=P_0\cos(Lt)+\frac{sin(Lt)}{L}\dot{P}_0 \tag{4}$$

Using the solution in Eq.(4), the fields $P(t+\Delta t)$ e $P(t-\Delta t)$ can be evaluated. Summing then and using trigonometric relations, it follows that:

$$P(\mathbf{x},t+\Delta t)+P(\mathbf{x},t-\Delta t)=2\cos(L\Delta t)P(\mathbf{x},t) \tag{5}$$

Isolating the left side terms of the Eq.(5), the time extrapolation of the wave field can be done; that results in direct modeling or reverse time migration, depending on the way the extrapolation is done in time.

RTM by interpolation

The central idea of this method is to use a solution for the complete wave equation with a constant velocity field in the time extrapolation, and then doing an interpolation based on the true velocity field.

After a Fourier transform, the time marching of the wavefield is done by different velocities on the wavenumber domain, and then interpolated back on the space domain. In this way, the method is a Fourier migration to the complete wave equation with variable velocities conceptually similar to the phase-shift plus interpolation (PSPI) method, commonly used with the unidirectional equation.

The spacial derivatives of the wave equation are computed implicitilly on the Fourier domain not suffering from dispersion problems in high frequencies, which occurs in a finite differences scheme. This procedure permits bigger steps in time without causing trouble to the stability condition, which implies in less computational cost.

Departing from Eq.(3) and following the procedures in Raymond (1991), we can define a pseudo-differential operator to the jth derivative of P in relation to t, which is given by:

$$\partial_t^j P \approx \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}[iL(\mathbf{x},\mathbf{k})]^j\,\varphi(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{k} \tag{6}$$

where $\mathbf{k} = (k_x,k_y,k_z)$ is the wavenumber vector correspondent to $\mathbf{x}$ and $\varphi(\mathbf{k})$ is defined by the equation:

$$\varphi(\mathbf{k}) = \int_{-\infty}^{\infty} P(\mathbf{x})e^{-i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{x} \tag{7}$$

The Taylor series expansion of the wavefield $P(\mathbf{x},t+\Delta t)$ around a known field $P(\mathbf{x},t)$ is given by:

$$P(\mathbf{x},t+\Delta t) = P(\mathbf{x},t) + \sum_{j=1}^{\infty}\frac{\partial_t^j P(\mathbf{x},t)}{j!}(\Delta t)^j \tag{8}$$

where the ^{j} denotes the jth time derivative.

Rewriting Eq. (8) and inserting Eq. (6) in it, results:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t) = P(\mathbf{x},t)+\\ &+\textstyle\sum_{j=1}^{\infty}\frac{\Delta t^j}{j!}\left[\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}[iL(\mathbf{x},\mathbf{k})]^j\,\varphi(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{k}\right]\end{aligned} \tag{9}$$

It's known that the Taylor series expansion of the exponential function e^x is given by $e^x = \sum_{j=0}^{\infty}\frac{x^j}{j!}$. Using this relation and expanding the sum in Eq.(9), it can be observed that the field $P(\mathbf{x},t+\Delta t)$ can be written as:

$$P(\mathbf{x},t+\Delta t) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}e^{[iL(\mathbf{x},\mathbf{k})\Delta t]}\varphi(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{k} \tag{10}$$

Evaluating and summing the fields $P(\mathbf{x},t+\Delta t)$ and $P(\mathbf{x},t-\Delta t)$ according to the solution viewed in Eq. (10):

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)+P(\mathbf{x},t-\Delta t) =\\ &\frac{1}{(2\pi)^3}\textstyle\int_{-\infty}^{\infty}\varphi(\mathbf{k},t)e^{i(\mathbf{k}\cdot\mathbf{x})}\left[e^{iL(\mathbf{x},\mathbf{k})\Delta t}+e^{-iL(\mathbf{x},\mathbf{k})\Delta t}\right]d\mathbf{k}\end{aligned} \tag{11}$$

or:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)+P(\mathbf{x},t-\Delta t) =\\ &\frac{1}{(2\pi)^3}\textstyle\int_{-\infty}^{\infty}\varphi(\mathbf{k},t)\cdot 2cos\,[L(\mathbf{x},\mathbf{k})\Delta t]\,e^{i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{k}\end{aligned} \tag{12}$$

In the constant velocity case, Eqs. (12) and (5) are exactly the same solution to the acoustic wave equation. And it is worth point that the pseudo-differential operator $L(\mathbf{x}) = v(\mathbf{x})\sqrt{-\nabla^2}$ in the space domain, when represented in the wavenumber domain, has the following form:

$$L(\mathbf{x},\mathbf{k}) = v(\mathbf{x})\sqrt{k_x^2+k_y^2+k_z^2} \tag{13}$$

which is exactly the same pseudo-differential operator derived in Zhang and Zhang (2009).

It can be notice that the time extrapolation is done by the multiplication of the spatial Fourier transformed of the wavefield by a cosine function whose argument depends on the wavenumber and velocity. In this way, the procedure can be interpreted as a spatial phase change applied in the Fourier domain.

The cosine function can be approximated by a series of two separable terms:

$$2\,\cos[L(\mathbf{x},\mathbf{k})\Delta t] \approx \sum_{j=0}^{n} a_j(\mathbf{x})b_j(\mathbf{k}) \tag{14}$$

where n is the number of terms, $a_j(\mathbf{x})$ and $b_j(\mathbf{k})$ are real functions depending on the separated terms **x** e **k** respectively.

Thus, the Eq. (12) can be rewritten:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)+P(\mathbf{x},t-\Delta t) \approx\\ &\textstyle\sum_{j=0}^{n} a_j(\mathbf{x})\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\varphi(\mathbf{k},t)b_j(\mathbf{k})e^{i(\mathbf{k}\cdot\mathbf{x})}d\mathbf{k}\end{aligned} \tag{15}$$

In the solution given by Eq. (15), the terms a_j are weight functions for each of the reference velocities $(a_j(v(\mathbf{x})))$ and are computed using the optimal reference velocities. To find that velocities, it has been used the procedure proposed by Bagaini et al (1995) in which the reference velocities $(v_1,v_2,...,v_n)$ are computed using the velocity distribution entropy criteria in the $[v_m,v_M]$ interval, where v_m is the minimum velocity of the whole field and v_M is the maximum velocity. The terms b_j are given by $b_j(\mathbf{k}) = \cos(v_n\sqrt{k_x^2+k_y^2+k_z^2}\Delta t)$ for each step of the time march.

Lastly, the migration procedure can be seen as:

$$P(\mathbf{x},t-\Delta t) = P(\mathbf{x},t+\Delta t) + \sum_{j=0}^{n} a_j(\mathbf{x})FFT^{-1}b_j(\mathbf{k})FFT P(\mathbf{x},t) \tag{16}$$

From Eq.(16) it can be noticed that, for each step of the time march, the method requires a fast Fourier transform

(FFT) and n inverse Fourier transform (FFT^{-1}). In order words, the computational cost to obtain $P(\mathbf{x},t-\Delta t)$ is proportional to the number of fast Fourier transforms, which is related with the quantity of reference velocities (v_j's).

RTM by the pseudo-analytical method

Numerical methods that use a second order time approximation to solve the derivatives usually introduce errors in its discretization, even if very accurate methods to solve the spacial derivatives are used. To solve this question, it is common the use of higher order time approximations or optimized methods.

The pseudo-analytical method (Etgen et at., 2009; Pestana et al., 2010) instead of searching for more precise approximations for the time derivatives, does a modification on the spatial derivatives to compensate the errors caused by the second order time approximation. The modified spatial derivatives form a pseudo-differential operator.

To compute the operator in an efficient way, the pseudo-analytical method simplifies it assuming a constant velocity medium, which results in a formula that can be calculated in the wavenumber domain. The combination of diverse operators for different constant velocities leads to a variable velocity migration.

Starting at Eq. (5), it is summed $-2P(\mathbf{x},t)$ to both sides of the equation:

$$P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=2\left[\cos(L\Delta t)-1\right]P(\mathbf{x},t) \tag{17}$$

Rewriting it in a more convenient way and in a form similar to the second order finite difference time derivative:

$$\begin{aligned}&\frac{P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)}{\Delta t^2}=\\&v^2(\mathbf{x})\frac{2\left[\cos(L\Delta t)-1\right]P(\mathbf{x},t)}{v^2(\mathbf{x})\Delta t^2}\end{aligned} \tag{18}$$

Using the second order Taylor expansion of the time derivative seen in Eq. (3), comparing to Eq. (18) and considering a constant velocity (v_0), a pseudo-Laplacian operator $F(K)$ is defined:

$$F(K)=\frac{2[cos(L_0\Delta t)-1]}{v_0^2\Delta t^2}\approx\frac{2}{v_0^2\Delta t^2}\left[-\frac{(L_0\Delta t)^2}{2}+\frac{(L_0\Delta t)^4}{24}-...\right] \tag{19}$$

Reminding that $L_0^2=-v_0^2\nabla^2$ and $K=\sqrt{-\nabla^2}$, the operator $F(K)$ can be rewritten according to the equation below:

$$F(K)\approx -K^2+\frac{v_0^2\Delta t^2}{12}K^4-... \tag{20}$$

It can be noticed that the operator $F(K)$ is exactly the Fourier transform of the Laplacian operator $-K^2$ when the time interval Δt tend to zero. Thus, the velocity v_0 that appears in the higher terms is seen as a compensation velocity which is constant to each pseudo-Laplacian operator. The pseudo-analytical method introduced by Etgen e Brandsberg-Dahl (2009) is here called the zeroth order approximation and is given by the following equation:

$$P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=v(\mathbf{x})^2\Delta t^2F(K)P(\mathbf{x},t) \tag{21}$$

As the operator $F(K)$ varies little with v_0, it can be used the combination of several pseudo-Laplacian operators to better accommodate the velocity variations.

Applying the Taylor series expansion in Eq. (18), the wavefield propagation equation can be rewritten:

$$P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=-(L\Delta t)^2P(\mathbf{x},t)+O(v\Delta tK) \tag{22}$$

where $O(v\Delta tK)$ represents the terms with higher order time derivatives of $P(\mathbf{x},t)$.

Replacing Eq. (5) on Eq. (22):

$$O(v\Delta tK)=2\left[\cos(L\Delta t)-1+\frac{(L\Delta t)^2}{2}\right]P(\mathbf{x},t) \tag{23}$$

With the value of $O(v\Delta tK)$ obtained in Eq. (23), Eq. (22) can be used to derive the second order pseudo-analytical method expression:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=\\&-(L\Delta t)^2P(\mathbf{x},t)+F_2(K)P(\mathbf{x},t)\end{aligned} \tag{24}$$

where the second order pseudo-Laplacian operator $F_2(K)$ is given by the expression:

$$F_2(K)=\frac{2}{(v_0\Delta t)^4}\left[\cos(L_0\Delta t)-1+\frac{(L_0\Delta t)^2}{2}\right] \tag{25}$$

It can be noticed that the first term on the right side of the Eq. (24) is exactly the pseudo-spectral method expression approximated by a second order time derivative. The second term acts as a correction term.

Obviously, if evaluated in the time domain, the Eq. (24) would be written as:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=\\&-(v\Delta t)^2\nabla^2P(\mathbf{x},t)+FFT^{-1}F_2(K)FFTP(\mathbf{x},t)\end{aligned} \tag{26}$$

With the same technique used in the second order approximation, the pseudo-analitycal fourth order approximation can be derived. The migration expression follows:

$$\begin{aligned}&P(\mathbf{x},t+\Delta t)-2P(\mathbf{x},t)+P(\mathbf{x},t-\Delta t)=\\&-(L\Delta t)^2P(\mathbf{x},t)+\frac{1}{12}(L\Delta t)^4P(\mathbf{x},t)+(L\Delta t)^6F_4(K)P(\mathbf{x},t)\end{aligned} \tag{27}$$

where F_4 is the pseudo-Laplacian operator given by:

$$F_4(K)=\frac{2}{(v_0\Delta t)^6}\left[\cos(L\Delta t)-1+\frac{(L\Delta t)^2}{2}-\frac{(L\Delta t)^4}{24}\right] \tag{28}$$

Results

In order to test the applicability of the proposed methods, synthetic bidimensional pre- and pos-stack data were migrated. Besides that, the migration of a real pre-stack data was also performed.

To perform the pos-stack migration, it was used the salt dome model from SEG/EAEG of which velocity field is showed in Figure 1 and the migration parameter in Table 1. For poststack seismic data, RTM is performed by pushing the recorded wavefield backward in time into the subsurface with half of the velocity of the medium. At time t=0, the image time, the back propagated recorded wavefield is captured to construct the image.

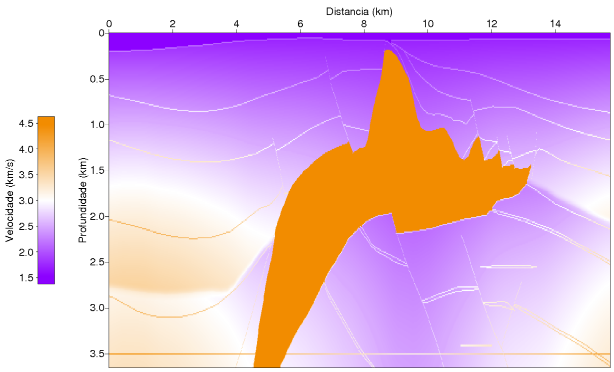

Figure 1: Salt dome SEG/EAGE model velocity field.

Δx	12,19 m
Δz	12,19 m
v_{min}	1,524 km/s
v_{max}	4,480 km/s

Table 1: Salt dome SEG/EAGE model parameters.

Figure 2 shows the result of the RTM by interpolation using 15 different velocities and Figure 3 the RTM result by the pseudo-analitycal method with a fourth order approximation of the pseudo-Laplacian operator with $v_0 = v_{min}$. Both results were obtained with the dataset with time sample interval of $\Delta t = 0,004s$.

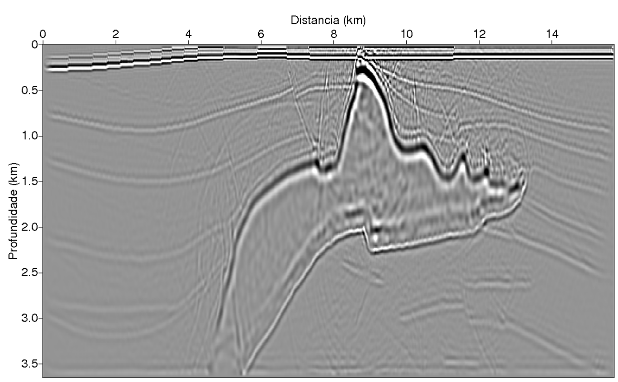

Figure 2: Migrated salt dome SEG/EAGE dataset using the RTM by interpolation with 15 velocities.

The pre-stack migration was tested applying it to the Marmousi model. The image condition considered is the crosscorrelation between the descending and ascending wavefields, created by the sources and receivers respectively. The model's velocity field is presented on Figure 4 while the parameters can be seen on Table 2.

The migrated result by the interpolation method is presented on Figure 5 while the one migrated by the

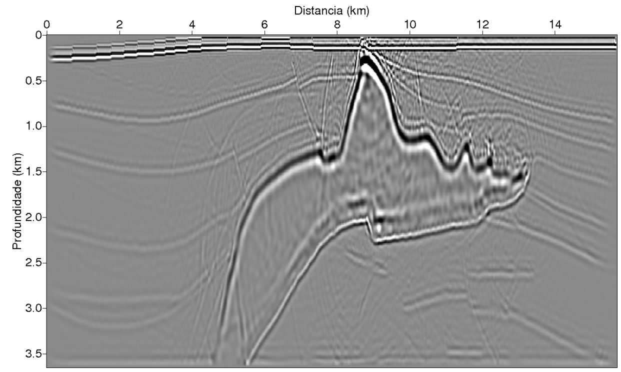

Figure 3: Migrated salt dome SEG/EAGE dataset using the RTM by the pseudo-analitycal method with second order approximation of the pseudo-Laplacian operator.

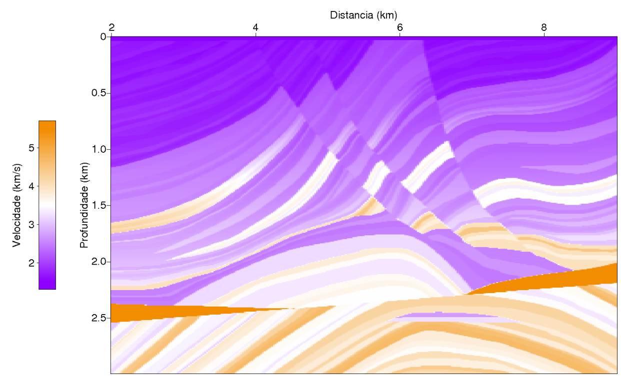

Figure 4: Marmousi velocity field.

Δx	25 m
Δz	8 m
f_{max}	35 Hz
v_{min}	1,5 km/s
v_{max}	5,5 km/s

Table 2: Marmousi model parameters

pseudo-analitycal with a second order approximation for the pseudo-Laplacian appears on Figure 6.

At last, a 2D real line acquired at the central region of Gulf of Mexico, in the Mississippi canyon area, has been migrated. This is one of the most oil and gas productive areas around the world and, according to Chowdhury and Borton (2007), where the hydrocarbon trapping is strongly related to the presence of salt.

The data acquisition was made by the end-on technique with a 180 receivers in the line. It were recorded 1001 shots. The data parameters are indicated on Table 3 while the velocity field used in the migration process appears on Figure 7.

The result of the RTM by interpolation with 5 velocities can be seen on Figure 8. Figure 9 shows the result from the pseudo-analytical method using a second order approximation of the pseudo-Laplacian operator.

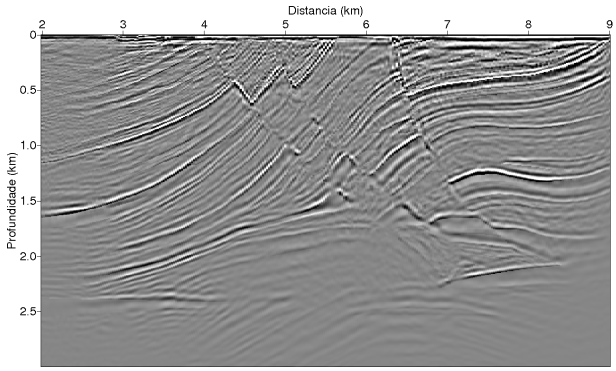

Figure 5: Marmousi migrated result by interpolation method with 10 velocities.

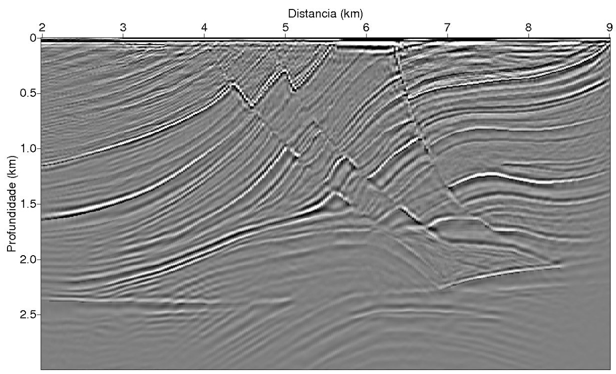

Figure 6: Marmousi migrated result by pseudo-analytical method with the pseudo-Laplacian operator second order approximation.

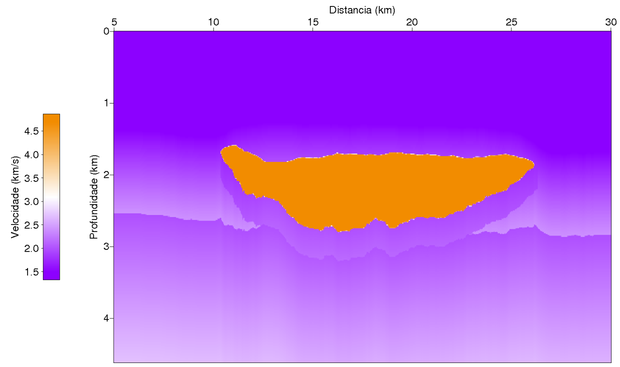

Figure 7: Gulf of Mexico velocity field.

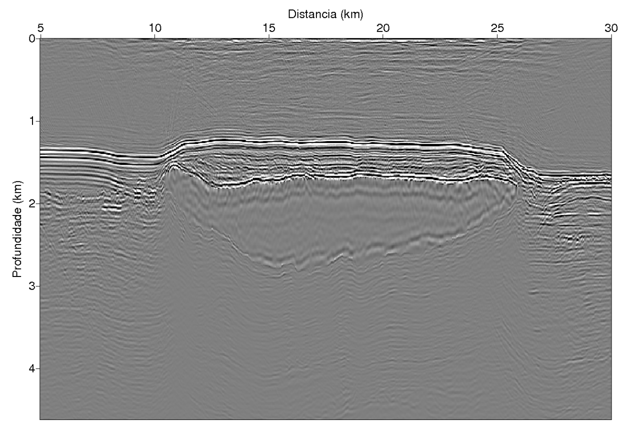

Figure 8: Gulf of Mexico migrated result by interpolation with 5 velocities.

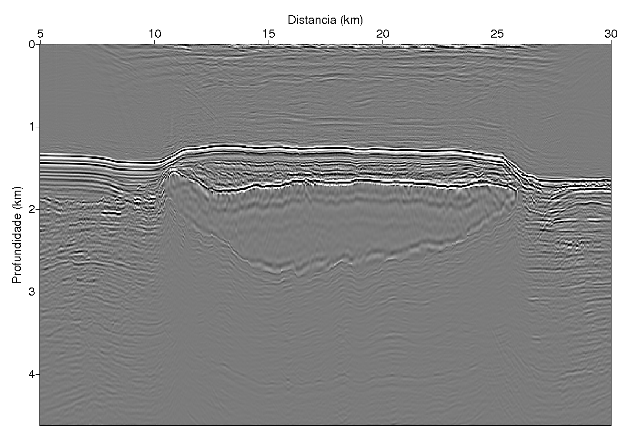

Figure 9: Gulf of Mexico migrated result by pseudo-analytical method using a second order approximation of the pseudo-Laplacian operator.

Conclusions

Both methods have succeed on the migration of synthetic and real data. The application on synthetic data is interesting because the geological model is known precisely and, consequently, the velocity field too. In salt dome SEG/EAGE and Marmousi models, both interpolation and the pseudo-analytical method could accurately reproduce the synthetic model.

The salt dome SEG/EAGE model is characterized by strong vertical and lateral velocity variations. For this reason, the migration by interpolation required a greater number of velocities to obtain a satisfactory result. This obviously requires a greater computational expense impairing the methods' efficiency. In the Marmousi model, characterized by the geological complexity, the interpolation could be applied accurately with less velocities. The pseudo-analytical method was also well succeeded in imaging the model and its use has been recommended once it has a lower computational cost.

Alternatives to the presented methods can include the use of polynomials instead of operators approximations by Taylor series in the pseudo-Laplacian in the case

Δx	26,67 m
Δz	13,21 m
f_{max}	30 Hz
v_{min}	1,485 km/s
v_{max}	4,000 km/s

Table 3: Gulf of Mexico data parameters.

of pseudo-analytical method. As for the interpolation, variations in the way we distribute the velocities in the model already exist in the literature and can be tested.

Finally, the migration of real data from the Gulf of Mexico proved the presented methods applicability in two-dimensional data. Its application in 3D data must still be implemented and tested.

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