The scattering of electromagnetic waves by vertical faults problem revisited and the difficulties therein for its extension to the three dimensional case

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Abstract

We revisit the problem of the 2D scattering of electromagnetic plane waves by vertical faults to analyse the convergent or asymptotic character as well as the consistency in terms of even and odd functions of a proposed solution in the form of a Neumann's series as an extension to the Born's approximation. We also address the problems faced with the extension of the proposed solution to a simple 3D case.

Introduction

The two dimensional problem of the scattering of electromagnetic waves by lateral contrast of conductivities has received much attention from geophysical researchers since the work of Sommerfeld (1896). In one of the researches, Sampaio and Fokkema (1992) - from now on labeled SF - proposed a solution for the scattering due to a vertical fault, based on a recursion of a Neumann's series and as an extension to the Born's approximation, previously employed by Weaver (1963).

In that paper, some questions remained unanswered. So, Heimer and Rijo (2001) employed numerical computation to address the question of convergence of the series and concluded that the solution for the field diverges when the series exceeds the 15-th order. Subsequently, Guimarães and Rijo (2003) proposed an alternative solution, because they understood that the solution described by SF lacked consistency in terms of even and odd fuctions.

We will analyse the existent problems in the adaptation of the SF solution to a simple three dimensional model and revisit the 2D problem to address the two issues raised by Rijo and his associates.

Definition of the 3D problem

Let the half-space $z < 0$ be the air and the half-space $z > 0$ consist of four homogeneous and isotropic media separated by two vertical orthogonal faults $z > 0$, $x = 0$ and $z > 0$, $y = 0$. To develop the solution of the problem, it will be useful to divide the air into four equal and homogeneous parts, labeled 1 to 4 in the counterclockwise direction. Let also an electromagnetic plane wave to propagate in the air in the positive z direction, with an x-oriented primary electric vector $E_x^P = e^{-ik_0 z}$, where: $i = \sqrt{-1}$; $\kappa_0^2 = -i\mu_0\sigma_0\omega$, $\Re(\kappa_0) > 0$, $\Im(\kappa_0) < 0$; ω is the angular frequency; μ_0 represents the magnetic permeability, and σ_0 represents the total current conductivity of the air.

The solution of the three-dimensional Helmholtz wave equation for the x component of the electric vector in each one of the eight media yields:

$$
\begin{aligned}
E_{x1} &= E_x^P + E_{x1}^R + E_{x1}^S,\ x > 0,\ y > 0,\ z < 0,\\
E_{x2} &= E_x^P + E_{x2}^R + E_{x2}^S,\ x > 0,\ y < 0,\ z < 0,\\
E_{x3} &= E_x^P + E_{x3}^R + E_{x3}^S,\ x < 0,\ y < 0,\ z < 0,\\
E_{x4} &= E_x^P + E_{x4}^R + E_{x4}^S,\ x < 0,\ y > 0,\ z < 0,\\
E_{x5} &= E_{x5}^T + E_{x5}^S,\ x > 0,\ y > 0,\ z > 0,\\
E_{x6} &= E_{x6}^T + E_{x6}^S,\ x > 0,\ y < 0,\ z > 0,\\
E_{x7} &= E_{x7}^T + E_{x7}^S,\ x < 0,\ y < 0,\ z > 0,\\
E_{x8} &= E_{x8}^T + E_{x8}^S,\ x < 0,\ y > 0,\ z > 0,
\end{aligned} \tag{1}
$$

In equations 1, E_{xm}^R, for $m = 1, \ldots, 4$ and E_{xn}^T, for $n = 5, \ldots, 8$ represent, respectively, the reflected and the transmitted x componentes of the eletric field. So

$$
E_{xm}^R = \frac{\sqrt{\mu_{(m+4)}\sigma_0} - \sqrt{\mu_0\sigma_{(m+4)}}}{\sqrt{\mu_{(m+4)}\sigma_0} + \sqrt{\mu_0\sigma_{(m+4)}}} e^{+i\kappa_0 z} = R_{xm}\, e^{+i\kappa_0 z}, \tag{2}
$$

$m = 1, \ldots, 4$, and

$$
E_{xn}^T = \frac{2\sqrt{\mu_n\sigma_0}}{\sqrt{\mu_n\sigma_0} + \sqrt{\mu_0\sigma_n}} e^{-i\kappa_n z} = T_{xn}\, e^{-i\kappa_0 z}, \tag{3}
$$

$n = 5, \ldots, 8$.

Equations 2 and 3 describe only the pure reflections and transmissions that occur infinitely remote from both faults on the horizontal boundaries, respectively between medium 1 and medium 5, medium 2 and medium 6, medium 3 and medium 7, and medium 4 and medium 8. Employing Maxwell's equation we can also describe in a straightforward manner the correspondent y components of the magnetic vector,

$$
\begin{aligned}
H_{y1} &= H_y^P + H_{y1}^R + H_{y1}^S,\ x > 0,\ y > 0,\ z < 0,\\
H_{y2} &= H_y^P + H_{y2}^R + H_{y2}^S,\ x > 0,\ y < 0,\ z < 0,\\
H_{y3} &= H_y^P + H_{y3}^R + H_{y3}^S,\ x < 0,\ y < 0,\ z < 0,\\
H_{y4} &= H_y^P + H_{y4}^R + H_{y4}^S,\ x < 0,\ y < 0,\ z < 0,\\
H_{y5} &= H_{y5}^T + H_{y5}^S,\ x > 0,\ y > 0,\ z > 0,\\
H_{y6} &= H_{y6}^T + H_{y6}^S,\ x > 0,\ y < 0,\ z > 0,
\end{aligned}
$$

$$
\begin{aligned}
H_{y7} &= H_{y7}^T + H_{y7}^S, x<0, y<0, z>0,\\
H_{y8} &= H_{y8}^T + H_{y8}^S, x<0, y>0, z>0,
\end{aligned} \tag{4}
$$

In equations 4, the y-oriented primary magnetic vector is $H_y^P = \sqrt{\frac{\sigma_0}{\mu_0\omega}} e^{-i(\kappa_0 z + \frac{\pi}{4})}$; the reflected y components of the magnetic field are given by

$$
H_{ym}^R = -\sqrt{\frac{\sigma_0}{\mu_0\omega}} \frac{(\sqrt{\mu_{(m+4)}\sigma_0} - \sqrt{\mu_0\sigma_{(m+4)}})}{(\sqrt{\mu_{(m+4)}\sigma_0} + \sqrt{\mu_0\sigma_{(m+4)}})} e^{+i(\kappa_0 z - \frac{\pi}{4})}, \tag{5}
$$

$m = 1,\ldots,4$, and the transmitted y components of the magnetic field are given by

$$
H_{yn}^T = \sqrt{\frac{\sigma_n}{\mu_n\omega}} \frac{2\sqrt{\mu_n\sigma_0}}{(\sqrt{\mu_n\sigma_0} + \sqrt{\mu_0\sigma_n})} e^{-i(\kappa_n z + \frac{\pi}{4})}, \tag{6}
$$

$n = 5,\ldots,8$.

The solution of the three-dimensional wave equation also yields the scattered components, identified in equations 1 and 4 with the superscript S. For the present three dimensional model there are scattered components for the x-oriented, the y-oriented and the z-oriented electric and magnetic field vectors, even though only the x-oriented electric field vector and the y-oriented magnetic field vector have primary, reflected, and transmitted components. Employing double unilateral Fourier cossine transforms we may write them as follows:

$$
\begin{aligned}
E_{\eta n}^S(x,y,z,\omega) = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty \Big(& A_{\eta n}(\alpha,\beta)\cos(\alpha x)\cos(\beta y) e^{-u_n|z|}\\
&+ B_{\eta n}(\alpha,\beta)\cos(\alpha z)\cos(\beta x) e^{-u_n|y|}\\
&+ C_{\eta n}(\alpha,\beta)\cos(\alpha y)\cos(\beta z) e^{-u_n|x|}\Big) d\alpha d\beta,\\
H_{\eta n}^S(x,y,z,\omega) = \frac{4}{\pi^2}\int_0^\infty\int_0^\infty \Big(& D_{\eta n}(\alpha,\beta)\cos(\alpha x)\cos(\beta y) e^{-u_n|z|}\\
&+ F_{\eta n}(\alpha,\beta)\cos(\alpha z)\cos(\beta x) e^{-u_n|y|}\\
&+ G_{\eta n}(\alpha,\beta)\cos(\alpha y)\cos(\beta z) e^{-u_n|x|}\Big) d\alpha d\beta.
\end{aligned} \tag{7}
$$

In equations 7: $n = 1,2,\ldots,8$; $\eta = x,y,z$; and $u_n = \sqrt{\alpha^2 + \beta^2 - \kappa_n^2}$. Notice that $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \kappa_0$ and $\kappa_m^2 = -i\mu_m\sigma_m\omega$, $\Re(\kappa_m) > 0$, $\Im(\kappa_m) < 0$, for $m = 5,\ldots,8$. So, the solution of the wave equations generates 144 unknown kernel functions. The problem reduces to find their solution.

Analysis of the scattered components

To determine the functions A,B,C,D,F, and G, we must apply the continuity of four tangential components of the fields at each one of the twelve boundaries, which yields only 48 equations. We may assume that the fundamental uknowns are the x components of the electric and the magnetic field and employ Maxwell's equations to relate the y and z components to the x components of the fields via the following identities:

$$
\begin{aligned}
\left(1 - \frac{1}{\kappa_n^2}\frac{\partial^2}{\partial x^2}\right) E_{yn}^S &= -\frac{1}{\kappa_n^2}\frac{\partial^2 E_{xn}^S}{\partial x \partial y} + \frac{1}{\sigma_n}\frac{\partial H_{xn}^S}{\partial z},\\
\left(1 - \frac{1}{\kappa_n^2}\frac{\partial^2}{\partial x^2}\right) E_{zn}^S &= -\frac{1}{\kappa_n^2}\frac{\partial^2 E_{xn}^S}{\partial x \partial z} - \frac{1}{\sigma_n}\frac{\partial H_{xn}^S}{\partial y},\\
\left(1 - \frac{1}{\kappa_n^2}\frac{\partial^2}{\partial x^2}\right) H_{yn}^S &= -\frac{\sigma_n}{\kappa_n^2}\frac{\partial E_{xn}^S}{\partial z} - \frac{1}{\kappa_n^2}\frac{\partial^2 H_{xn}^S}{\partial x \partial y},\\
\left(1 - \frac{1}{\kappa_n^2}\frac{\partial^2}{\partial x^2}\right) H_{zn}^S &= +\frac{\sigma_n}{\kappa_n^2}\frac{\partial E_{xn}^S}{\partial y} - \frac{1}{\kappa_n^2}\frac{\partial^2 H_{xn}^S}{\partial x \partial z}.
\end{aligned} \tag{8}
$$

This reduces the number of necessary unknowns to 48, but the derivatives will change some of the scattered expressions to unilateral Fourier sine transforms for either α or β or both of them. However, unilateral Fourier cossine ($\mathscr{F}(\alpha,\beta)$) and sine ($\mathscr{F}^\alpha(\alpha,\beta)$) transforms relate to each other via single or double Hilbert transforms.

$$
\begin{aligned}
\mathscr{F}^\alpha(\alpha,\beta) &= -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathscr{F}(\eta,\beta)}{\alpha - \eta} d\eta,\\
\mathscr{F}^\beta(\alpha,\beta) &= -\frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathscr{F}(\alpha,\xi)}{\beta - \xi} d\xi,\\
\mathscr{F}^{\alpha\beta}(\alpha,\beta) &= +\frac{1}{\pi^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\mathscr{F}(\eta,\xi)}{(\alpha-\eta)(\beta-\xi)} d\eta d\xi.
\end{aligned} \tag{9}
$$

Application of Maxwell's equations and Hilbert transforms will set, after application of the boundary conditions, a system of 48 integral equations with 48 unknowns.

Identities for the scattered components

After applying equations 7 in equations 8 with the help of equations 9 and considering the properties of the Fourier transform we obtain the following twelve identities:

$$
\begin{aligned}
\left(1 + \frac{\alpha^2}{\kappa_n^2}\right) A_{yn} &= -\frac{\alpha\beta}{\pi^2\kappa_n^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{A_{xn}(\chi,\xi)}{(\alpha-\chi)(\beta-\xi)} d\chi d\xi\\
&\quad - \frac{u_n \mathrm{sgn}(z)}{\sigma_n} D_{xn},\\
\left(1 + \frac{\beta^2}{\kappa_n^2}\right) B_{yn} &= +\frac{u_n \mathrm{sgn}(y)\beta}{\pi\kappa_n^2}\int_{-\infty}^{+\infty} \frac{B_{xn}(\alpha,\xi)}{\beta-\xi} d\xi\\
&\quad + \frac{\alpha}{\pi\sigma_n}\int_{-\infty}^{+\infty} \frac{F_{xn}(\chi,\beta)}{\alpha-\chi} d\chi,\\
\left(1 - \frac{u_n^2}{\kappa_n^2}\right) C_{yn} &= +\frac{u_n \mathrm{sgn}(x)\alpha}{\pi\kappa_n^2}\int_{-\infty}^{+\infty} \frac{C_{xn}(\chi,\beta)}{\alpha-\chi} d\chi\\
&\quad + \frac{\beta}{\pi\sigma_n}\int_{-\infty}^{+\infty} \frac{G_{xn}(\alpha,\xi)}{\beta-\xi} d\xi,\\
\left(1 + \frac{\alpha^2}{\kappa_n^2}\right) A_{zn} &= +\frac{u_n \mathrm{sgn}(z)\alpha}{\pi\kappa_n^2}\int_{-\infty}^{+\infty} \frac{A_{xn}(\chi,\beta)}{\alpha-\chi} d\chi\\
&\quad - \frac{\beta}{\pi\sigma_n}\int_{-\infty}^{+\infty} \frac{D_{xn}(\alpha,\xi)}{\beta-\xi} d\xi,\\
\left(1 + \frac{\beta^2}{\kappa_n^2}\right) B_{zn} &= -\frac{\alpha\beta}{\pi^2\kappa_n^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{B_{xn}(\chi,\xi)}{(\alpha-\chi)(\beta-\xi)} d\chi d\xi\\
&\quad + \frac{u_n \mathrm{sgn}(y)}{\sigma_n} F_{xn},\\
\left(1 - \frac{u_n^2}{\kappa_n^2}\right) C_{zn} &= +\frac{u_n \mathrm{sgn}(x)\beta}{\pi\kappa_n^2}\int_{-\infty}^{+\infty} \frac{C_{xn}(\alpha,\xi)}{\beta-\xi} d\xi\\
&\quad - \frac{\alpha}{\pi\sigma_n}\int_{-\infty}^{+\infty} \frac{G_{xn}(\chi,\beta)}{\alpha-\chi} d\chi,\\
\left(1 + \frac{\alpha^2}{\kappa_n^2}\right) D_{yn} &= -\frac{\alpha\beta}{\pi^2\kappa_n^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{D_{xn}(\chi,\xi)}{(\alpha-\chi)(\beta-\xi)} d\chi d\xi\\
&\quad + \frac{u_n \mathrm{sgn}(z)\sigma_n}{\kappa_n^2} A_{xn},\\
\left(1 + \frac{\beta^2}{\kappa_n^2}\right) F_{yn} &= +\frac{u_n \mathrm{sgn}(y)\beta}{\pi\kappa_n^2}\int_{-\infty}^{+\infty} \frac{F_{xn}(\alpha,\xi)}{\beta-\xi} d\xi
\end{aligned}
$$

$$-\frac{\alpha\sigma_n}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{B_{xn}(\chi,\beta)}{\alpha-\chi}d\chi,$$

$$\left(1-\frac{u_n^2}{\kappa_n^2}\right)G_{yn}=+\frac{u_n\mathrm{sgn}(x)\alpha}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{G_{xn}(\chi,\beta)}{\alpha-\chi}d\chi -\frac{\beta\sigma_n}{\pi^2\kappa_n^2}\int_{-\infty}^{+\infty}\frac{C_{xn}(\alpha,\xi)}{\beta-\xi}d\xi,$$

$$\left(1+\frac{\alpha^2}{\kappa_n^2}\right)D_{zn}=+\frac{u_n\mathrm{sgn}(z)\alpha}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{D_{xn}(\chi,\beta)}{\alpha-\chi}d\chi +\frac{\beta\sigma_n}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{A_{xn}(\alpha,\xi)}{\beta-\xi}d\xi,$$

$$\left(1+\frac{\beta^2}{\kappa_n^2}\right)F_{zn}=-\frac{\alpha\beta}{\pi^2\kappa_n^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{F_{xn}(\chi,\xi)}{(\alpha-\chi)(\beta-\xi)}d\chi d\xi -\frac{u_n\mathrm{sgn}(y)\sigma_n}{\kappa_n^2}B_{xn},$$

$$\left(1-\frac{u_n^2}{\kappa_n^2}\right)G_{zn}=+\frac{u_n\mathrm{sgn}(x)\beta}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{G_{xn}(\alpha,\xi)}{\beta-\xi}d\xi +\frac{\alpha\sigma_n}{\pi\kappa_n^2}\int_{-\infty}^{+\infty}\frac{C_{xn}(\chi,\beta)}{\alpha-\chi}d\chi=0. \quad (10)$$

Equations 10 imply that, under the quasi-static assumption, the horizontal component of the magnetic field may not be constant at $z=0$, because if the x and y components of the D's, F's, and G's terms are zero, then all the other scattered terms vanish. This happens partially due to the fact that for a 3D scattering we cannot assume any separation between the TE and TM modes.

Boundary conditions

We shall now employ equations 1 and 7 to illustrate the algebraic difficulties of the problem with the examples of the continuity of: E_x, at $y=0$, $x<0$, and $z<0$; and of E_y, at $x=0$, $y<0$, and $z<0$. Straightforward applications of the boundary condition and of inverse Fourier transformation in z for E_x yields that:

$$2\pi\int_0^{+\infty}\left\{\{B_{x3}(\eta,\beta)-B_{x4}(\eta,\beta)\}\cos(\beta x)d\beta + \left\{C_{x3}(\alpha,\eta)e^{-v_3|x|}-C_{x4}(\alpha,\eta)e^{-v_4|x|}\right\}d\alpha\right\} = \frac{i\pi^2\kappa_0}{\eta^2-\kappa_0^2}(R_{x4}-R_{x3})+\int_0^{+\infty}\int_0^{+\infty}\left\{\frac{4u_4}{\eta^2+u_4^2}A_{x4}(\alpha,\beta) -\frac{4u_3}{\eta^2+u_3^2}A_{x3}(\alpha,\beta)\right\}\cos(\alpha x)d\alpha\, d\beta, \quad (11)$$

$v_n=\sqrt{\alpha^2+\eta^2-\kappa_n^2}$, $n=3,4$. This equation contains the known reflection terms and only the kernels assumed as fundamental. This situation will also hold for the other 15 boundary conditions for E_x and H_x.

The same doesn't occur with E_y, because the applications of the boundary condition and of the inverse Fourier transformation in z for it yields that:

$$2\pi\int_0^{+\infty}\left\{\{C_{y2}(\alpha,\eta)-C_{y3}(\alpha,\eta)\}\cos(\alpha y)d\alpha + \left\{B_{y2}(\eta,\beta)e^{-t_2|y|}-B_{y3}(\eta,\beta)e^{-t_3|y|}\right\}d\beta\right\} = +\int_0^{+\infty}\int_0^{+\infty}\left\{\frac{4u_3}{\eta^2+u_3^2}A_{y3}(\alpha,\beta) -\frac{4u_2}{\eta^2+u_2^2}A_{y2}(\alpha,\beta)\right\}\cos(\beta y)d\alpha\, d\beta, \quad (12)$$

$t_n=\sqrt{\eta^2+\beta^2-\kappa_n^2}$, $n=2,3$. In this last equation, it is necessary to employ the first three equations of equation array 10, in order to substitute the kernels of the E_y component by those of the E_x and H_x components. A similar situation will hold for the other 31 equations of the y and z components of the electromagnetic field.

Application of inverse x and y Fourier transformation, respectively, in equations 11 and 12 will improve their algebra. Even with this and similar improvements, the 48 boundary conditions will produce a cumbersome system of integral equations, which will require special methods for its solution.

Revisitation of the 2D problem

The solution prescribed by SF employs single unilateral Fourier cossine transforms for both the x and z coordinates, and every function representative of the electromagnetic field components is valid only within one of the four quarter spaces. As such, they are neither even (f_e) nor odd (f_o) functions: they are causal (f_c) functions.

According to Papoulis (1962), $f_c(u)=2f_e(u)=2f_o(u)$, for $u>0$, and $f_c(u)=0$, for $u<0$ in the one dimensional case. It is straightforward to apply it to the two dimensional case. Therefore, it is not correct to simply state that a reflected field, expressed as $E^{(r)}(z)=Re^{i\kappa z}$ for $z<0$, is neither even nor odd, because for $z>0$, $E^{(r)}(z)=0$. This is equivalent to state that: $E^{(r)}(z)=E_e^{(r)}(z)+E_o^{(r)}(z)$, $-\infty<z<+\infty$; $E_e^{(r)}(z)=E_o^{(r)}(z)=Re^{i\kappa z}/2$, $z<0$; and $E_e^{(r)}(z)=-E_o^{(r)}(z)=Re^{i\kappa z}/2$, $z>0$.

Obviously, the solution defined by Guimarães and Rijo (2003) doesn't represent a correction to the SF solution, because it complies to the Born's approximation and differs from the solution of Weaver (1963) because it interchanges the roles of the x and the z coordinates and employs two scattering terms instead of only one.

The question of convergence of the series is far more difficult. We will make a heuristic approach to analyse it. Equations 51 and 52 of SF define, for the quasi-static case, that: $f_{0,1}^{(0)}(\beta)=g_{0,1}^{(0)}(\beta)=g_{0,2}^{(0)}(\beta)=0$ and

$$f_1^{(0)}(\beta)=\frac{2iA(u_1-u_2)}{\pi u_1^2u_2}. \quad (13)$$

Consequently, it results from equations 29-32 of SF that:

$$g_{0,i}^{(1)}(\beta)=\frac{2A\left(\sqrt{\beta^2}-\sqrt{u_i^2+\kappa_j^2}\right)}{\pi(u_0+u_i)\sqrt{\beta^2}\sqrt{u_i^2+\kappa_j^2}} \quad (14)$$

and $f_{0,1}^{(2n-1)}(\beta)=f_1^{(2n-1)}(\beta)=g_{0,i}^{(2n)}(\beta)=0$, $n\geq 1$. In equations 13 and 14, $A=\omega\mu_0 H$, $u_i=\sqrt{\beta^2-\kappa_i^2}$, $i=1,2$, $j=2,1$, $j\neq i$. We also may verify at once that $f_1^{(0)}(\beta)$ and $g_{0,i}^{(1)}(\beta)$ assume the following asymptotic expressions:

$$f_1^{(0)}(\beta>>|\kappa_j|)=\frac{2A}{\pi}\frac{(\kappa_2^2-\kappa_1^2)}{2\beta^4}, \quad (15)$$

$$g_{0,i}^{(1)}(\beta >> |\kappa_j|) = \frac{2A}{\pi}\frac{\kappa_j^2}{4\beta^4}. \quad (16)$$

The asymptotic value of $f_1^{(0)}$ is twice that of $g_{(0,i)}^{(1)}$ because of the occurrence of the addition term (u_0+u_i) in the denominator of equation 14. Though explicit for large values of β, this two fold ratio applies for any value of the integration variable. Addition terms of the type (u_i+u_j) are present in equations 29-32 of SF, in such a way that every function of a consecutive larger order will incorporate another one of those terms. Therefore, we have that: (i) $O(f_1^{(2n)}) = \beta^{-4}$, $n \geq 0$; (ii) $O(g_{0,i}^{(2n+1)}) = \beta^{-4}$, $n \geq 0$; (iii) $O(g_{0,i}^{(2n-1)}) = 2\,O(f_1^{(2n)})$, $n \geq 1$; and (iv) $O(f_1^{(2n)}) = 2\,O(g_{0,i}^{(2n+1)})$, $n \geq 1$. As a result, the order of each series is a geometrical progresson of ratio 1/4 and

$$\lim_{N\to\infty}\sum_{n=0}^{N} O\left(f_1^{(2n)}\right) = \frac{4}{3}f_1^{(0)}, \quad (17)$$

$$\lim_{N\to\infty}\sum_{n=0}^{N} O\left(g_{0,i}^{(2n+1)}\right) = \frac{4}{3}g_{0,i}^{(1)}. \quad (18)$$

These results don't prove the convergence of the series for every value of the integration variable, but they show that the terms of each series decrease in magnitude as their order increases. So, if the series are not convergent, they are asymptotics. Under the asymptotic condition, we must determine the maximum order of the series beyond which it may no longer represent the function under investigation. Also, we may express $R_{0,i}$ and T_i in the following forms:

$$R_{0,i}\,e^{+i\kappa_0 z} = \int_0^\infty g_{0,i}^{(0)}(\alpha)\cos(\alpha x)e^{+u_0 z}, \quad (19)$$

$$T_i\,e^{-i\kappa_i z} = \int_0^\infty g_i^{(0)}(\alpha)\cos(\alpha x)e^{-u_i z}, \quad (20)$$

which results that $g_{0,i}^{(0)}(\beta) = R_{0,i}\delta(\beta)$ and $g_i^{(0)}(\beta) = T_i\delta(\beta)$. This means that the zeroth-order g's terms of the scattered functions may substitute the reflection and the transmission terms of the laterally homogeneous case, and imposes each iteration to consist of a $g^{(2n-1)}$ term followed by an $f^{(2n)}$ term, $n \geq 1$. Doing otherwise, the result will either oscillate or show a discontinuity of the electric field at the boundary.

One should be cautious with results from numerical integration. This is specially true in the present case because we deal with repeated improper integrals of complex functions and the kernels are analytic functions with poles and branch points. To obtain $f_1^{(m)}$ it is necessary to perform m recurrent integrations and each integration should avoid or circumvent the singularities. We can achieve this, not always easily, for analytical integration by modifying the countour of integration on the complex plane. Not for recurrent numerical integrations. They will accumulate errors due to approximation and interpolation, specially near singularities of the integrands.

Concluding remarks

The Neumann's series solution of SF is consistent with the physics of the problem and represents, at least, an approximate asymptotic solution for the 2D EM scattering problem of the vertical fault. The question as to whether the series is convergent or not represents an interesting problem that remains unanswered.

The extension of the SF procedure to the 3D case consists of a formidable problem, which will require the application of sophisticated matrix theory to approach its complete solution. Nevertheless, the present partial development enhances the kind of pitfalls that may be present in forward 3D EM scattering models. As such, it should be an alert to those dealing with this type of problem.

The partial development also suggests how to improve the 2D SF solution and, hopefully, clarify its variance with Rijo's results employing finite element. According to Papoulis (1962) we must employ only unilateral Fourier cossine transforms instead of sine transforms to express causal functions at the boundaries. So, if we apply this principle to both terms of the scattered H_x and H_z components of the SF solution and use Hilbert's transform, we obtain the following correction for equations 13 and 14:

$$f_1^{(0)}(\beta) = \frac{iA}{\pi\,u_1\,u_2}\left\{\frac{2(u_1-u_2)}{u_1} + \frac{\sqrt{\beta^2}(\kappa_1 u_1 - \kappa_2 u_2)}{\kappa_1\kappa_2(u_1+u_2)}\right\}, \quad (21)$$

$$g_{0,i}^{(1)}(\beta) = \frac{A}{\pi(u_0+u_1)}\left\{\frac{2\left(\sqrt{\beta^2}-\sqrt{u_i^2+\kappa_j^2}\right)}{\sqrt{\beta^2}\sqrt{u_i^2+\kappa_j^2}} + \frac{iu_i\left(\kappa_i\sqrt{\beta^2}-\kappa_j\sqrt{u_i^2+\kappa_j^2}\right)}{\kappa_1\kappa_2\sqrt{u_i^2+\kappa_j^2}\left(\sqrt{\beta^2}+\sqrt{u_i^2+\kappa_j^2}\right)}\right\}, \quad (22)$$

and all the other terms of the series will change accordingly. With this improvement, the SF solution may continue to calibrate or serve to compare solutions from other techniques.

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