Evaluation of Acoustic Operators for VTI Seismic Modeling

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Abstract

In this paper, acoustic and pseudo-acoustic wave equations in vertical transverse isotropic media (VTI), developed by Tariq Alkhalifah (Alkhalifah, 2000), Klíe and Toro (Klíe and Toro, 2001) and Linbin Zhang (Zhang et al., 2005) are investigated. It is emphasized that the study seeks the understanding of the nature of wave propagation, related to those approaches as well as the phenomena that control it and its limitations. The seismic modeling is applied in order to illustrate the differences among the presented equations.

1. Introduction

The anisotropic seismic modeling is an essencial tool to study wave propagation characteristics in anisotropic media. Wave propagation in anisotropic media are inherently discribed by elastic wave equations, with P-wave and SV-wave modes coupled. To generate images of subsurface applying only primary energy of P-wave mode it is necessary to decouple P-wave and S-wave modes. As in isotropic media, in anisotropic media it is possible to decouple P and SV seismic waves (Yan and Sava, 2009), but the theory involved is more difficult than that for isotropic media, because it involves the directions of wave polarizations, which implies in the increasing of computational cost.

For this reason, Tariq Alkhalifah derived a simple two-way acoustic wave equation for anisotropic media from a dispersion relation approximation for transversely isotropic media (TI) with vertical symmetry axis (VTI) (Alkhalifah, 1998), which can be used for modeling only P-wave modes, based on setting to zero the shear wave velocity (V_{sz}) along the axis of symmetry (Alkhalifah, 2000).

Due to some limitations of the Alkhalifah approximation, Klíe and Toro (Klíe and Toro, 2001) developed a variant equation from Muir's dispersion relation (Conceição, 2011), but this equation involves space-time mixed derivatives. Later, Linbin Zhang (Zhang et al., 2005) derived an acoustic wave equation from Thomsen's dispersion relation, being that the features of this equation are the same as those of Klíe and Toro's equation.

Other authors have implemented alternative equations in order to remove mixed detivatives, for instance (Du et al, 2008; Duveneck, et al., 2008). In different way (Duveneck, et al., 2008) derived coupled first-order and second-order VTI wave equations starting from Hooke's law and the equations of motion with the vertical shear velocity again set to zero.

In the next topic it is presented a short discussion concerning anisotropic acoustic wave equations by Tariq Alkhalifah, Linbin Zhang and Klíe and Toro.

2. P-wave dispersion relations

P-wave phase velocity approximation is important in the derivation of acoustic approximations to the anisotropic wave equations. A commom way to obtain wave equations for P-wave modeling is by introducing the dispersion relation based on different P-wave velocity approximations.

From (Tsvanskin, 2001) the exact P-SV wave phase velocity for VTI media is described by:

$$\frac{V^2(\theta)}{V_{pz}^2} = 1+\epsilon\sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\sqrt{\left(1+\frac{2\epsilon\sin^2\theta}{f}\right)^2 - 2(\epsilon-\delta)\frac{\sin^2 2\theta}{f}} \qquad (1)$$

where

$$f = 1 - V_{sz}^2/V_{pz}^2 \qquad (2)$$

The positive sign in front of the radical corresponds to the P-wave, while the negative sign corresponds to the SV-wave.

Equation (1) describes P-wave phase velocity as function of Thomsen's anisotropic parameters (Thomsen, 1986) and P-wave (V_{pz}) and S-wave (V_{sz}) vertical velocities.

2.1 Alkhalifah's dispersion relation

Based on equation (1) Alkhalifah derived the simplified TI acoustic phase velocity approximation shown below,

$$\begin{aligned}\lim_{V_{sz}\to 0} V_{p,ph}^2(\theta) &= V_{pz}^2\Bigg(1+2\epsilon\sin^2\theta - \frac{1}{2} \\ &+\frac{1}{2}\sqrt{\left(1+2\epsilon\sin^2\theta\right)^2 - 2(\epsilon-\delta)\sin^2 2\theta}\Bigg)\end{aligned} \qquad (3)$$

applying $\vec{k} = \omega\vec{p}$ and $p_x = \frac{sen(\theta)}{V(\theta)}$, $p_z = \frac{cos(\theta)}{V(\theta)}$, one obtains:

$$k_z^2 = \frac{V_{pn}^2}{V_{pz}^2}\left(\frac{\omega^2}{V_{pn}^2} - \frac{\omega^2 k_x^2}{\omega^2 - 2\eta V_{pn}^2 k_x^2}\right). \qquad (4)$$

Equation (4) is the 2D Alhkalifah's dispersion relation, where V_{pn} is P-wave normal moveout velocity, η the anellipticity paramater (Alkhalifah, 1998) and ω is the angular frequency.

2.3 Muir's dispersion relation

Like performed previously, starting from Muir's P-wave phase velocity approximation (Muir and Dellinger, 1985),

$$V_{p,ph}^2(\theta) = V_{pe}^2(\theta) + \frac{(q-1)V_{px}^2 V_{pz}^2 \sin^2(\theta) cos^2(\theta)}{V_{pe}^2(\theta)} \qquad (5)$$

and using q parametrization (Fomel, 2004), $q = \frac{1}{1+2\eta}$,

$$V_{ph,p}^2(\theta) = V_{pe}^2(\theta) + \frac{V_{pz}^2(V_{pn}^2 - V_{px}^2)\sin^2(\theta) cos^2(\theta)}{V_{pe}^2(\theta)} \qquad (6)$$

where $V_{px}^2 = V_{pz}^2(1+2\epsilon)$ is the P-wave horizontal velocity and $V_{pe}^2(\theta) = V_{px}^2 \sin^2(\theta) + V_{pz}^2 \cos^2(\theta)$ is the elliptical velocity. The corresponding dispersion relation to equation (6) is,

$$k_z^2 = \frac{1}{\omega^2 V_{pz}^2}\left[(V_{px}^2 k_x^2 + V_{pz}^2 k_z^2)^2 + 2(1+\eta)V_{pz}^2 V_{pn}^2 k_x^2 k_z^2\right] - \frac{V_{px}^2}{V_{pz}^2} k_x^2. \qquad (7)$$

2.2 Thomsen's dispersion relation

Starting from weak anisotropic approximation (Thomsen, 1986),

$$V_p^2(\theta) = V_{pz}^2\left(1 + 2\delta \sin^2\theta \cos^2\theta + 2\epsilon \sin^4\theta\right) \qquad (8)$$

and applying again $\vec{k} = \omega\vec{p}$ and $p_x = \frac{sen(\theta)}{V(\theta)}$, $p_z = \frac{cos(\theta)}{V(\theta)}$,

$$k_z^2 = \frac{V_{pz}^2}{\omega^2}\left[(k_x^2 + k_z^2)^2 + 2\delta k_x^2 k_z^2 + 2\epsilon k_x^4\right] - k_x^2. \qquad (9)$$

Equation (9) is the last dispersion relation shown here. Figure 1 shows different P-wave phase velocities of the Greenhorn-shale anisotropy. According to figure 1(b), Alkhalifah's phase velocity approximation provides an appropriate estimate for angles below 30º, being that for angles larger than 30º does not exceed the relative error of 0.3%. On the other hand, Thomsen's approximation indicates increasing error between 0.1% and 1.25%, which decreases around 60º and 90º. Muir's approximation shows similar behaviour as Thomsen's approximation.

3. Acoustic VTI wave equations

3.1 Alkhalifah's Formulation

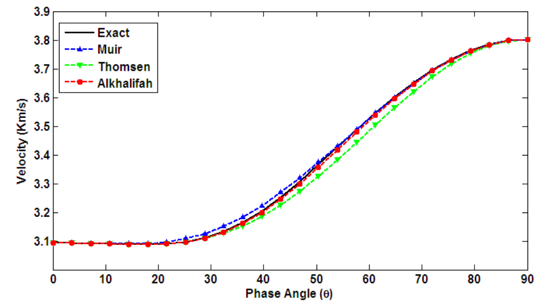
To achieve acoustic VTI wave equations, it is introduced the following operators,

$$\vec{\nabla}\Phi(\vec{r},t) = i\vec{k}\Phi(\vec{r},t) \;,\; \frac{\partial\Phi(\vec{r},t)}{\partial t} = -i\omega\Phi(\vec{r},t). \qquad (10)$$

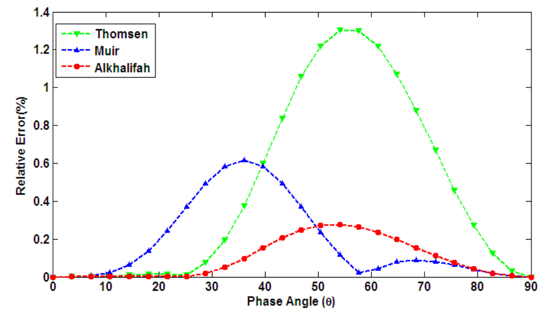
where $\Phi(\vec{r},t) = e^{i(\vec{k}.\vec{r}-\omega t)}$.

Now, multiplying both sides of equation (4) by $\Phi(\vec{r},t)$ and applying relation (10):

$$\frac{\partial^2 P(x,z,t)}{\partial t^2} = (1+2\eta)V_{pn}^2 \frac{\partial^2 P(x,z,t)}{\partial x^2} + V_{pz}^2 \frac{\partial^2 P(x,z,t)}{\partial z^2}$$
$$-2\eta V_{pn}^2 V_{pz}^2 \frac{\partial^4 \Phi(x,z,t)}{\partial x^2 \partial z^2}. \qquad (11)$$



(a) Phase velocity approximations



(b) Phase velocity relative error.

Figure 1: Phase velocity approximation and relative error of different phase-velocity approximations for the Greenhorn-shale. The parameters are $\epsilon = 0.255$, $\delta = -0.051$, $V_{pz} = 3094\, m/s$, $V_{sz} = 1509\, m/s$, $\rho = 2370\, Kg/m^3$.

$$P(x,z,t) = \frac{\partial^2\Phi(x,z,t)}{\partial t^2} \qquad (12)$$

Equations (11) and (12) together are called pseudo-acoustic wave equation, where $P(x,z,t)$ is the pressure field. The normal moveout velocity (V_{pn}) and anellipticity coefficient can be written as a function of the Thomsen's parameters such as (Alkhalifah, 1998): $V_{pn}^2 = V_{pz}^2(1+2\delta)$; $\eta = \frac{\epsilon-\delta}{1+2\delta}$.

The pseudo-acoustic stability restriction in Alkhalifah's formulation is $\epsilon \geq \delta$ or $\eta \geq 0$.

3.2 Klíe's Formulation

In a similar way as done previously, but using now equation (7), and again applying relation (10), follows that:

$$(1+2\eta)V_{pn}^2 \frac{\partial^2 P(x,z,t)}{\partial x^2} + V_{pz}^2 \frac{\partial^2 P(x,z,t)}{\partial z^2} = (1+2\eta)^2 V_{pn}^4 \frac{\partial^4 \Phi(x,z,t)}{\partial x^4}$$
$$+ V_{pz}^4 \frac{\partial^4 \Phi(x,z,t)}{\partial z^4} + 2V_{pz}^2 V_{pn}^2 (1+\eta) \frac{\partial^4 \Phi(x,z,t)}{\partial x^2 \partial z^2} \qquad (13)$$

$$P(x,z,t) = \frac{\partial^2\Phi(x,z,t)}{\partial t^2}$$

where the stability restriction is $\eta > -\frac{1}{4}$.

3.3 Zhang's Formulation

Finally, the last acoustic wave equation to VTI media studied in this paper was proposed by Zhang et al. (Zhang et al., 2005) in frequency domain. However in this work the same formulation will be shown in the time-space domain. So, multiplying again both sides of equation (9) by function $\Phi(x,z,t)$ and using the gradient and temporal derivative operators (10):

$$\frac{\partial^2 P(x,z,t)}{\partial x^2}+\frac{\partial^2 P(x,z,t)}{\partial z^2}=V_{pz}^2(1+2\epsilon)\frac{\partial^4\Phi(x,z,t)}{\partial x^4}$$
$$+V_{pz}^2\frac{\partial^4\Phi(x,z,t)}{\partial z^4}+2V_{pz}^2(1+\delta)\frac{\partial^4\Phi(x,z,t)}{\partial x^2\partial z^2} \quad (14)$$
$$P(x,z,t)=\frac{\partial^2\Phi(x,z,t)}{\partial t^2}$$

it should be observed that stability restriction is $|\epsilon|<<1$, $|\delta|<<1$.

4. P-Wave Modeling

4.1 Numerical Solution for Alkhalifah's Formulation

The equations (11) and (12) for pseudo-acoustic formulation are discretized applying second order central finite-difference in space and time, as follows:

$$\Phi(x,z,t)\Big|_{i,k}^{n+1} = 2\Phi_{i,k}^n-\Phi_{i,k}^{n-1}+\Delta t^2 P_{i,k}^n \quad (15)$$

$$P(x,z,t)\Big|_{i,k}^{n+1} = 2P_{i,k}^n-P_{i,k}^{n-1}+\Delta t^2\left(\frac{\partial^2 P}{\partial t^2}\right)_{i,k}^n \quad (16)$$

$$\left(\frac{\partial^2 P}{\partial t^2}\right)_{i,k}^n=a_1\left(\frac{P_{i+1,k}^n-2P_{i,k}^n+P_{i-1,k}^n}{\Delta x^2}\right)+a_2\left(\frac{P_{i,k+1}^n-2P_{i,k}^n+P_{i,k-1}^n}{\Delta z^2}\right)$$
$$-\frac{a_3}{\Delta x^2\Delta z^2}\left[4\Phi_{i,k}^n-2(\Phi_{i-1,k}^n+\Phi_{i,k-1}^n+\Phi_{i+1,k}^n+\Phi_{i,k+1}^n)\right.$$
$$\left.+\Phi_{i-1,k-1}^n+\Phi_{i+1,k-1}^n+\Phi_{i+1,k+1}^n+\Phi_{i-1,k+1}^n\right]$$

where, $a_1=(1+2\eta_{i,k})V_{pn(i,k)}^2$, $a_2=V_{pz(i,k)}^2$ and $a_3=2\eta_{i,k}V_{pn(i,k)}^2V_{pz(i,k)}^2$.

The numerical stability used by Alkhalifah are given by the Courant-Friedrichs-Lewy (CFL) condition (Alhalifah, 2000) as:

$$\Delta t<\frac{1}{\sqrt{2}}\,min\left(\frac{\Delta x}{max(V_{px}(x,z))},\frac{\Delta z}{max(V_{pz}(x,z))}\right) \quad (17)$$

4.2 Numerical Solution for Zhang and Klie's Formulation

In an analogous way we discretized system equations (13) by second order finite-difference scheme in space and time, such that:

$$b_1\left(P_{i+1,k}^n-2P_{i,k}^n+P_{i-1,k}^n\right)/\Delta x^2+$$
$$b_2\left(P_{i,k+1}^n-2P_{i,k}^n+P_{i,k-1}^n\right)/\Delta z^2=$$
$$b_3\left(\Phi_{i+2,k}^n-4\Phi_{i+1,k}^n+6\Phi_{i,k}^n-4\Phi_{i-1,k}^n+\Phi_{i-2,k}^n\right)/(\Delta x)^4+$$
$$b_4\left(\Phi_{i,k+2}^n-4\Phi_{i,k+1}^n+6\Phi_{i,k}^n-4\Phi_{i-1,k}^n+\Phi_{i,k-2}^n\right)/(\Delta z)^4+$$
$$b_5\left[4\Phi_{i,k}^n-2(\Phi_{i-1,k}^n+\Phi_{i,k-1}^n+\Phi_{i+1,k}^n+\Phi_{i,k+1}^n)\right. \quad (18)$$
$$\left.+\Phi_{i-1,k-1}^n+\Phi_{i+1,k-1}^n+\Phi_{i+1,k+1}^n+\Phi_{i-1,k+1}^n\right]/\Delta x^2\Delta z^2$$

where:

$$\begin{aligned} b_1 &= (1+2\eta_{i,k})V_{pn(i,k)}^2 \\ b_2 &= V_{pz(i,k)}^2 \\ b_3 &= (1+2\eta_{i,k})^2V_{pn(i,k)}^4 \\ b_4 &= V_{pz(i,k)}^4 \\ b_5 &= 2V_{pz(i,k)}^2V_{pn(i,k)}^2(1+\eta_{i,k}) \end{aligned}$$

Zhang's formulation, system equations (14), is discretized likewise equation (18), However, coefficients b's are given by:

$$\begin{aligned} b_1 &= 1 \\ b_2 &= 1 \\ b_3 &= V_{pz(i,k)}^2(1+2\epsilon_{i,k}) \\ b_4 &= V_{pz(i,k)}^2 \\ b_5 &= 2V_{pz(i,k)}^2(1+\delta_{i,k}) \end{aligned}$$

Rewriting equation (18) in matrix notation:

$$\mathbf{MP^n}=\mathbf{K\Phi^n} \quad (19)$$

$\mathbf{M}$ and $\mathbf{K}$ are coefficient matrices, while $\mathbf{P^n}$ e $\mathbf{\Phi^n}$ are vectors. Equation (19) provides solution for the pressure field $\mathbf{P^n}$ at the current time step n. Known the pressure field $P(x,z,t)$, $\Phi(x,z,t)$ at the next time step $n+1$ is given by equation (15). In summary, the implementation requires the numerical solution of a pentadiagonal linear system (Klíe and Toro, 2001). Once $P(x,z,t)$ is known in the domain, $\Phi(x,z,t)$ can be update at the new step, being it a recursive update process. The numerical stability for Klie's formulation are given (Klíe and Toro, 2001) by:

$$\left[(1+2\eta)V_{pn}^2+V_{pz}^2+\frac{2\eta V_{pn}^2V_{pz}^2}{(1+2\eta)V_{pn}^2+V_{pz}^2}\right]\frac{\Delta t^2}{h^2}\leq 1 \quad (20)$$

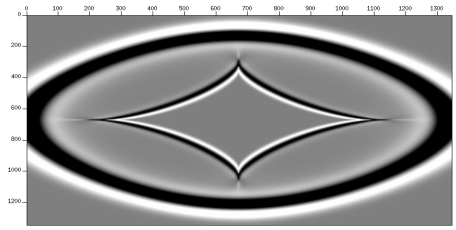
For Zhang's formulation, the numerical stability is the same as to Alkhalifah's equation.

5. Synthetic Data Examples

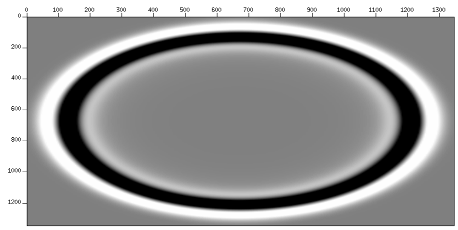
5.1 Alkhalifah's Formulation

The first example to be analyzed considers a homogeneous medium (Greenhorn shale), the same used for the evaluation of velocity approximations in section 2.

Figure (2) shows wavefronts for anisotropic and isotropic media at time 0.24s generated by a pressure source (Ricker's pulse, 60Hz) in VTI media specified by the vertical velocities $V_{pz}=3094\,m/s$, $V_{sz}=1509\,m/s$, Thomsen anisotropic coefficients, $\epsilon=0.255$, $\delta=-0.051$ and density $\rho=2370\,Kg/m^3$. The geological model has a dimension of 1.35 Km x 1.35 Km, grid spacing and time of $h=4.5$ m and $\Delta t=0.6$ ms respectively. Figure 2(a) displays a diamond-shape wavefront generated by Alkhalifah's formulation, this artifact represent a SV-wave present in Alkhalifah's formulation due to velocity approximation (3)(Grechka et al., 2004; Conceição, 2011). Because this event Alkhalifah's formulation is called Pseudo-Acoustic.



(a) Pressure field in anisotropic media



(b) Pressure field in isotropic media

Figure 2: Snapshots for the pressure field in homogeneous media - Pseudo-Acoustic Formulation.

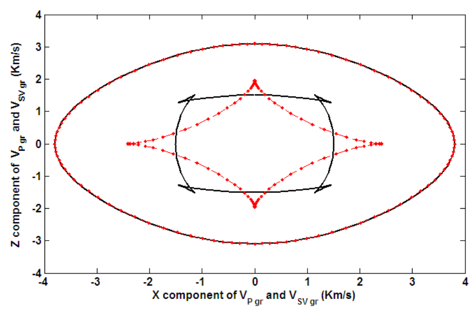


Figure 3: Group velocity wavefronts; black curves represents exact velocity and red curves Alkhalifah's approximation. Inner curves corresponds to SV-wave and outer curves to P-wave.

This occurs because the P-SV waves are coupled, so when Alkhalifah held simplification (3), which also occurs for the SV phase velocity, equation (21) becomes:

$$\lim_{V_{sz}\to 0} V^2_{sv,ph}(\theta) = V^2_{pz}\Bigg(1+2\epsilon\sin^2\theta-\frac{1}{2} \\ -\frac{1}{2}\sqrt{\left(1+2\epsilon\sin^2\theta\right)^2-2(\epsilon-\delta)\sin^2 2\theta}\Bigg) \tag{21}$$

Then, the calculation of wavefronts according to (Tsvankin, 2001), leads to:

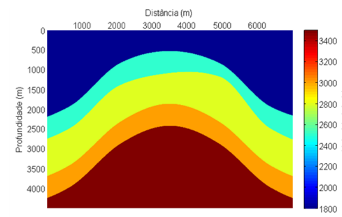
$$V^{gr}_{p,sv}(\phi) = V^{ph}_{p,sv}(\theta)\vec{n} + \frac{\partial V^{ph}_{p,sv}(\theta)}{\partial\theta}\frac{\partial\vec{n}}{\partial\theta} \tag{22}$$

where, $\vec{n} = (n_x, n_z) = (\sin(\theta), \cos(\theta))$ is the normal wavefront vector, $V^{gr}_{p,sv}$ and $V^{ph}_{p,sv}$ are group velocity and phase velocity of P-wave and SV-wave respectively. The wavefronts for equation (22) can be seen in figure (3), being that it is verified the appearance of the SV wave in figure 2(a).

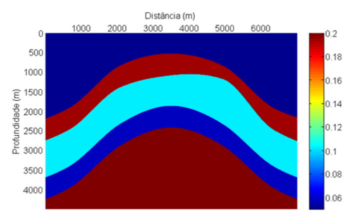
Another point mentioned by Grechka et al. (Grechka et al., 2004) is the extreme SV-wave anisotropy generated by Alkhalifah's formulation. From Tsvankin and Thomsen (Tsvankin and Thomsen, 1994) SV-wave anisotropy parameter is:

$$\sigma = \left(\frac{V_{pz}}{V_{sz}}\right)^2(\epsilon-\delta). \tag{23}$$

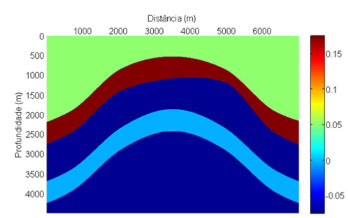
Thus, the Alkhalifah's formulation makes the SV-wave anisotropy parameter tend to infinite ($\sigma = \infty$). The example of figure 4 shows what happens in a complex media. In this example the source is placed in the first



(a) P-wave vertical velocity (V_{pz}) model.



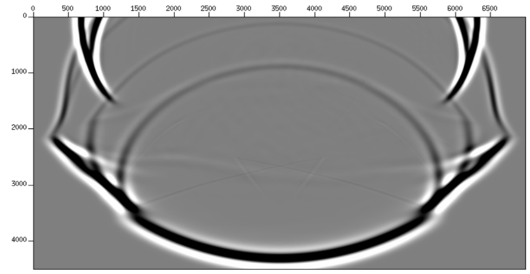
(b) Epsilon (ϵ) anisotropy model.



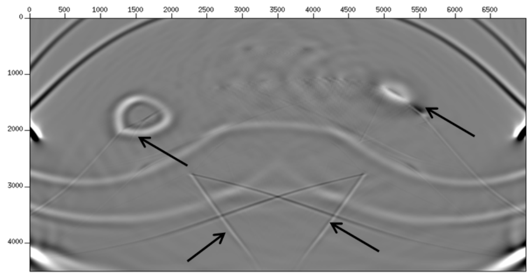
(c) Delta (δ) anisotropy model.

Figure 4: Anticlinal Model - P-wave vertical velocity and anisotropy coefficients.

isotropic layer; figure (5) shows snapshots for different times for pseudo-acoustic formulation. In Figure 5(b) it is observed spurious events generated by the SV-wave in the medium, while for the same snapshot generated by the VTI elastic formulation (Faria and Stoffa, 1994) it is not observed the presence of such an event. The creation of spurious artifacts found in the pseudo-acoustic equation can generate undesirable events, those events that should only be related to primary energy and P-wave modes.

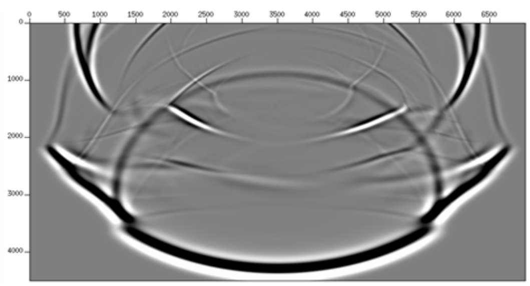


(a) Snapshot for the pressure field at $t = 1.62s$.

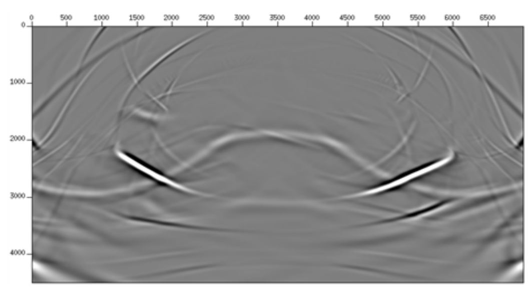


(b) Snapshot for the pressure field at $t = 2.27s$.

Figure 5: Snapshots for the pressure field at different times. The black arrows indicates spurious events generated by the presence of SV-wave.



(a) Snapshot for the vertical stress (σ_{zz}) field at $t = 1.62s$.

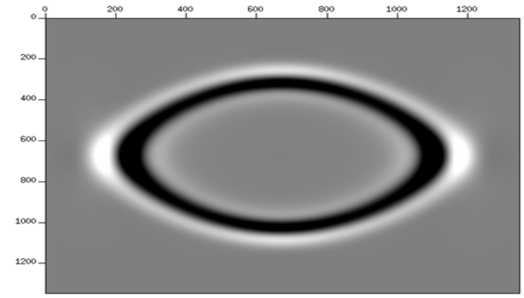


(b) Snapshot for the vertical stress (σ_{zz}) field at $t = 2.27s$.

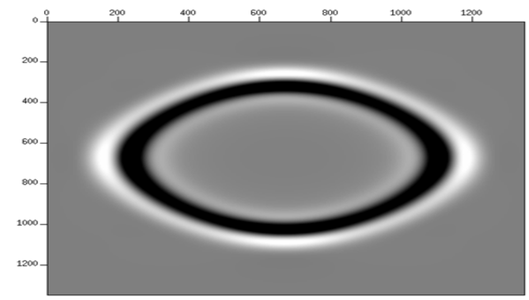
Figure 6: Snapshots for the vertical stress (σ_{zz}) at different times.

5.2 Zhang and Klie's formulations

For Zhang's formulation and Klie's formulation the same homogeneous medium (Greenhorn shale) will be employed in the analysis. Figure (7) shows the snapshots for both formulations. The formulation of Zhang and Klie definitely excludes the SV-wave in the pseudo-acoustic formulation, a fact borne out by plotting group wavefronts, figure (8), from the relations (5) and (8).



(a) Snapshot for the pressure field at 0.18s - Zhang's formulation



(b) Snapshot for the pressure field at 0.18s - Klíe's formulation

Figure 7: Snapshots for the pressure field in homogeneous media - Zhang and Klie's formulation

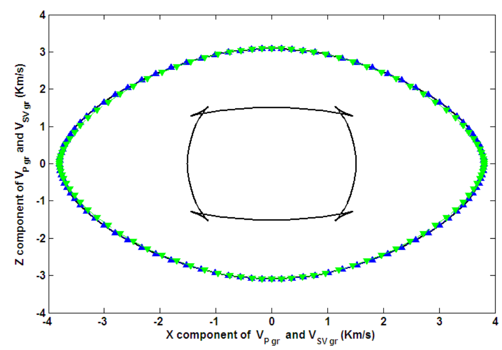


Figure 8: P-wave group velocity wavefronts; blue curve represents Muir's approximation and green curve Thomsen's approximation.

6. Summary and Conclusions

In this paper different approximations to model acoustic wave propagation in transverse isotropy media with vertical symmetry axis (VTI) were presented, namely the formulations proposed by Tariq Alkhalifah, Klie and Toro and Zhang. All wave equations were derived from approximations of phase velocity for the P-wave.

It was seen that the formulation proposed by Tariq Alkhalifah is not entirely acoustic (i.e., includes not only P-waves), creating regions with the presence of SV waves, and creating extreme anisotropy ($\sigma = \infty$). Both the extreme anisotropy, and the generation of SV waves, are due to the simplification made ($V_{sz} = 0$) for the approximation of analytic phase velocity, which had the purpose of obtaining an equation that simulates only the field of the P wave in anisotropic medium.

The generation of SV waves caused by pseudo-acoustic formulation evidently can produce relevant difficulties in imaging processes.

The equations proposed by Klie and later on by Zhang eliminate artifacts of the pseudo-acoustic approach, but with the disadvantage of requiring the solution of a linear system at each time step. However, the stability conditions $\eta > -\frac{1}{4}$ for the Klie's equation, $|\epsilon| \ll 1$ and $|\delta| \ll 1$ for the Zhang's equation make major changes on the anisotropy parameters.

7. References

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