A simple SOC model for reversals

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Abstract

In this work we introduce a simple model for Earth's magnetic field reversals. The model consists in random nodes simulating vortices in the liquid part of the core which through a simple updating algorithm converge to a self organized critical state. The model presents reversals with time frequency distributions in the form of power laws (as supposed to be in reversals). Even with the more complex simulations (through magnetohydrodynamics) the description of reversals includes parameter values out of any range with physical sense. Thus it should not be expected that with a simple model as the present one we could obtain a detailed description of reversals. However, we hope to reach a profounder knowledge of reversals through some of the basic characteristic that are well reproduced.

Introduction

Reversals are, together with earthquakes, some of the more astonishing events in Earth. They are period during which the direction of the Earth's magnetic field reverses, i.e., the magnetic field that previous to a reversal points in one direction after the reversal will point in the opposite direction.

The main component of the Earth's magnetic field comes from the external liquid nucleus. The field produced at the interior of the planet has its main source in a complex system of internal currents that eventually produce geomagnetic reversals. Many of them have been documented. Periods between geomagnetic reversals present power law distribution functions, which can be the signature of some critical system as the mechanism of their source [1]. There are other mechanisms capable of producing power laws (for example, superposition of some distributions [2], critical systems [3] and non-extensive versions of statistical mechanics [4]) but we will not extend on them upon here. In Fig. 1 we present the sequence of geomagnetic reversals from 80 million years (My) to our days. The data was obtained from the most recent and complete record that we have found [5,6]. Fig. 2 presents the distribution of periods between reversals. It follows a power law,

$$f(t) = c.t^{d} \qquad (1)$$

where f (t) is the frequency distribution of periods between consecutive reversals, c is a proportionality constant and d is the exponent of the power-law (and also the slope of the graph in log-log plots). For the present case we have approximately -1.5 as slope value.

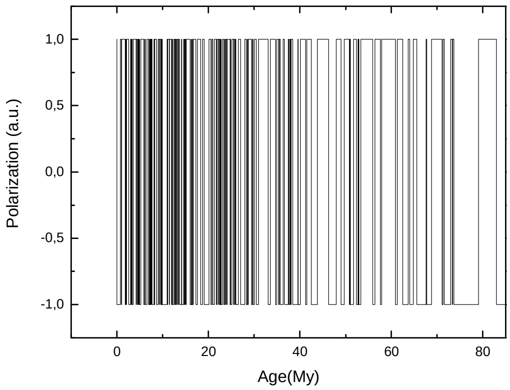

Figure 1.- A binary representation of geomagnetic reversals from 80 My ago to our days. We arbitrarily have assumed -1 as the current polarization. We have not plotted the period from 80 My to 165 My (for which also a record exists) for the sake of clarity. There was a period with no reversals from 80 to 120 My (the beginning appears in the plot). The period from 120 My to 165 My was very similar to the period from 0 to 40 My (in number of reversals and in the average duration of reversals). The data was obtained from Cande and Kent [5,6].

The magnetic field needs a finite time to change its direction (the time between the moment at which the dipolar component is no more the main one and the moment at which it is again the main one but building up in the opposite direction). This time is in the average around 5000 years. It is small if compared to the smallest interval between reversals already detected (10,000 years) and to the average interval between reversals. There are clustered periods of high activity from 40 My ago to our days and during the period 165-120 My (not shown), a period of low activity between 80 and 40 My ago and a period of almost null activity from 120 to 80 My. The sequence of Earth's magnetic field reversals seems to be a non-equilibrium process as can be inferred from Fig. 1 where the average time interval between successive reversals seems to increase with geological time. Unfortunately, the reversal series is unique and relatively small, there is no other similar.

Contrary to the record of reversals (known from 160 My up to nowadays) the record of magnetic field intensities exists just for around 10 My [7]. Fig. 3 shows the distribution of Earth's dipole values for the last 10 My. The distribution of dipole values for actual reversals (Fig. 3) follows a function between a bi-normal and a bi-log-normal distribution. However, the number of experimental

points is also small and there will certainly be changes in these facts when new measurements become available.
Geomagnetic reversals have called a great attention since their discovery and this interest remains nowadays. Works devoted to the study of the time distribution of geomagnetic reversals include, among many others, statistical studies on reversals distributions and correlations [1,8-10], modeling of the problem [11-13] and experimental studies [5,6,14-16]. Gaffin [8] pointed out that long-term trends and non-stationary characteristics of the record could hamper a formal detection of chaos in geomagnetic reversal record.
It is our opinion that actually it is very difficult to detect in a consistent manner that geomagnetic reversals present any systematic characteristic at all, without mattering which this characteristic could be (including chaos). Its nature remains an open question.
The study of geomagnetic reversals can contribute to the comprehension of important processes in Earth's evolution.This includes from the internal dynamics of the planet to processes involving the evolution of living organisms. The Earth is continuously bombarded by charged particles mainly born at the Sun. Large fluxes of those particles are incompatible with life. Fortunately, only a tiny fraction of them hits the Earth's surface. The most part is deviated by the geomagnetic field. However, during reversals the field intensity decreases drastically which means a more intense bombarding. The "magnetic shield" is reduced. Those increases in the flux of particles could have interfered in life cycles of the planet.

Here we profit from all the accumulated experience mentioned before. However, our model follows lines quite different from theirs. At the best of our knowledge, it is the first SOC model representing reversals and it has also the peculiarity of reaching a single stationary state with two equally probably opposite magnetization states.

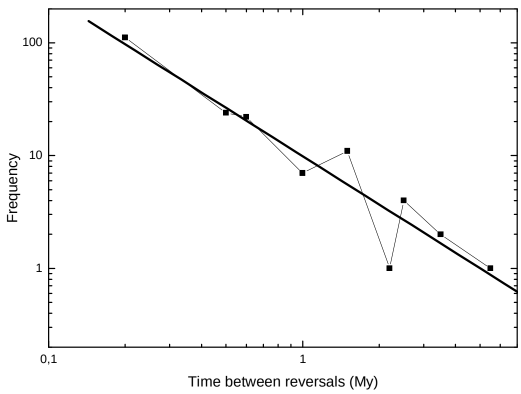

Figure 2.- Frequency distribution of periods between reversals from 80 My to our days. The slope of the solid line is around -1.5. The data was obtained from Cande and Kent [5,6].

The rest of the paper is organized as follows: first, we present the model, later on we present the results of our simulations as well as a comparative study with experimental results and a discussion on their possible connection with previous works on the statistics of geomagnetic reversals. Finally, we present our conclusions and some possible trends for future works.

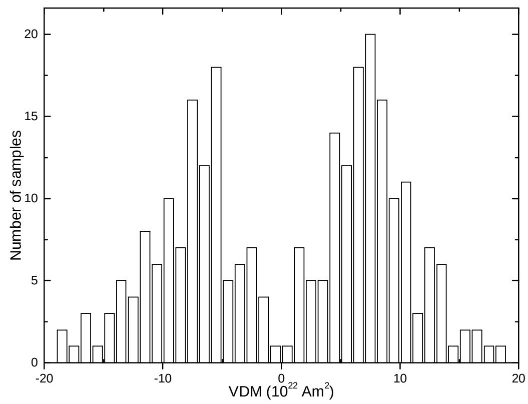

Figure 3.- Distribution of virtual dipole moment (VDM) values for samples not older than 10 My. Negative values of VDM correspond to reverse polarization and are represented by 119 samples. Positive values of VDM represent normal polarization and are represented by 142 samples. The data was obtained from Kono and Tanaka [10].

The Model

We simulate the Earth's liquid core and the electric current structures on its volume by nodes distributed on an LxL square lattice, where L=100, 200 and 300. This gives different numbers of nodes on each simulation from where we have checked for finite size effects. We are exploring in sets of equally spaced points at the Earth's equator. To each of these nodes we have initially assigned a random value between -1 and 1 to simulate both, the accumulated magnetic energy at each of the simulated positions and the magnetic moment orientation. We have looked then for the lowest absolute value through the whole system and changed it and its four nearest neighbors by new random values between -1 and 1. With this we simulate a more or less continuous energy flux to the core bulk (this is the reason to pick the lowest value) and the possible absorption of smaller vortices. In this way we also simulate the creation of new vortices. At the same time, the assignment of new random values, to the lowest in absolute and its neighbors, works as a continuous release of energy out of the system.

This process is repeated several times (usually between 10^6 and 10^8 times) to obtain stationary distributions for the quantities we are interested in. We have used periodic boundary conditions in our square system.

We define the magnetization M for the system as:

$$M = \frac{1}{N}\sum_{i=1}^{N} s_i \qquad (2)$$

where the sum runs over all the nodes and N = LxL is the total number of nodes. It can take values between -1 and 1 (corresponding to all nodes in the -1 value and to all nodes in the 1 value, respectively).

Constructed in the way we have done, our model qualifies as a Bak-Sneppen one [17]. The Back-Sneppen model probably is the simplest model presenting self-organized criticality, i.e., the tendency to a stationary critical state without necessity of a fine tuning.

The Bak-Sneppen model is a general model that has found applications in a large number of fields among which we can mention evolution [17], the brain [18], the cosmic rays spectrum [19] and X-rays bursts at the Sun's surface [20].

Many scientific efforts have been devoted to characterize the Bak-Sneppen model from several points of view. Examples of them are: its correlations from detrended fluctuation analysis [21], damage spreading on it [22], and its behavior under reduction to near zero dimension [23].

Results

Note that within the simplified model here introduced it should not be expected a detailed description of the system, but just some specific details and, in particular, the class of universal behavior displayed by the real system that the model represents.

Beginning with an arbitrary distribution of accumulated magnetic energy at each node, the subsequent activity will be completely uncorrelated in space and time. But as times goes by (and then, the mean accumulated energy increases in absolute value as a consequence of selecting and changing the lowest absolute values) it will be more and more likely that near neighbors are consecutively changed. After a transient the system reaches a steady state characterized by a well-like distribution for the accumulated energies and a couple of thresholds, ± E_c for the distribution of the lower barriers (see Figures 4 and 5). The distribution of lower energies vanishes above + E_c and below - E_c.

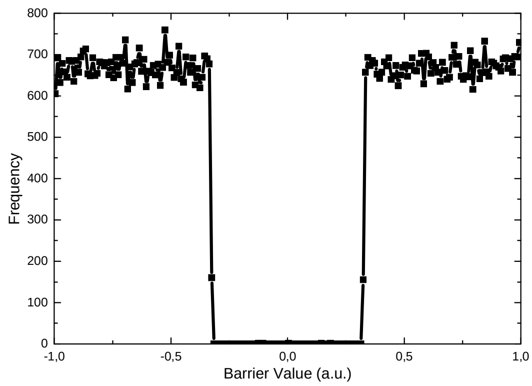

Figure 4.- Distribution of the nodes values at the stationary state. It has a well-like form with vertical walls at ± 3.5 approximately.

As mentioned before, in the stationary state the events become correlated also in space. In actual reversal this correlation could be associated to jerks but we have not done a systematic study on this correlation in the present work. However it will be the subject of forthcoming works because the straight line it follows (not shown) is one of the fingerprints of the critical state on which we believe the system we are simulating is. As in the rest of the simulations the result does not depend on the initial conditions indicating that the critical state is a global attractor for the dynamics, hence it is self-organized.

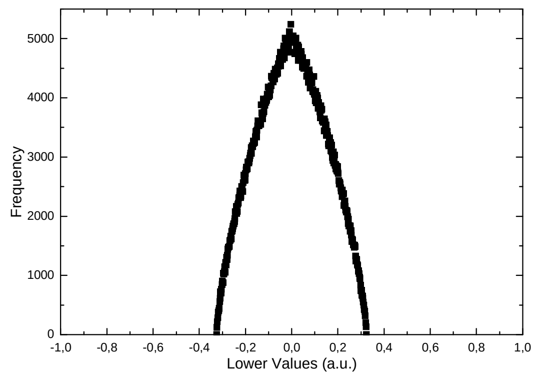

Figure 5.- Distribution of the lower absolute values for nodes at the stationary state. It has an inverse V-like shape with vertical walls at ± 3.5 approximately.

In Figure 6 we present the dependence of the magnetization on time for a short run. Note that in this particular case there is a preponderancy of positive values for the magnetization. However, positive and negative values are equivalent. The important fact is that the model presents reversals.

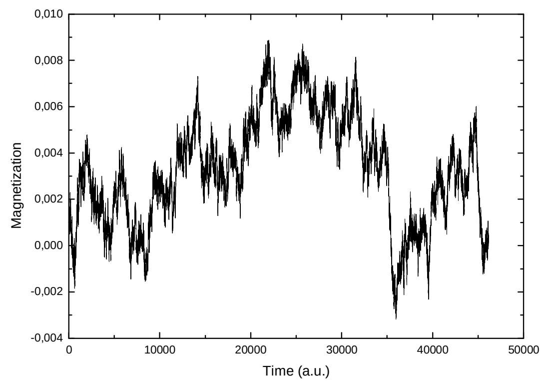

Figure 6.- Magnetization (see text) versus time for a 100x100 system during a short simulation (~45000 time steps).

Figure 7 presents the distribution of inter-reversals times, i.e., the distribution of times during which the magnetization keeps its sign unaltered. It follows a power-law with a slope ~ -1.65, very close to the value ~ -1.5 for actual reversals.

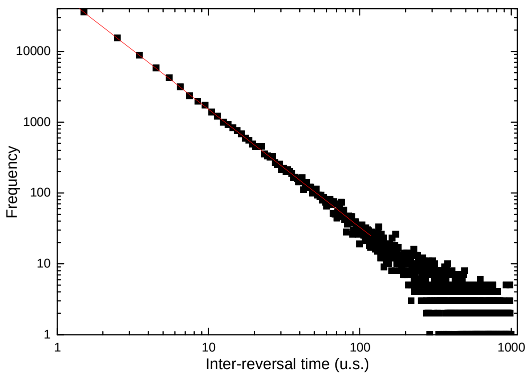

Figure 7.- Distribution of time between consecutive reversals (both from positive to negative and from negative to positive). The slope is ~ -1.6.

Finally, Figure 8 presents the distribution function of nodes values for the data presented in Figure 6. The asymmetry is clear between positive and negative values of magnetizations. Actually, it seems to be not a stationary distribution function because for finite systems there is always a probability for the system to enter a long "avalanche" (a long period of time with the same magnetization sign) able to change any supposedly stationary distribution.

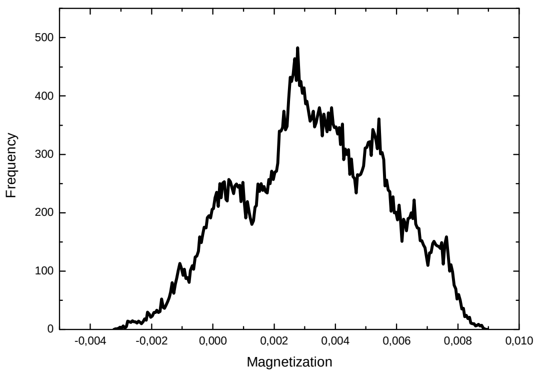

Figure 8.- Distribution function of magnetization values corresponding to Figure 1. Note a preponderancy of positive values in this example (see text). Note also that the magnetization M remains in values much lower in absolute value than the limit value 1.

Conclusions

We have introduced a two dimensional self-organized critical model to simulated Earth`s magnetic field reversals, one of the more fascinating geophysical phenomena. The model presents reversals, i.e. changes in the sign of magnetization. We have obtained power law distributions for several relevant quantities, similar to some results of experimental works on reversals. This could imply that the Earth`s liquid core could be in a critical state where the greater possible duration of periods between consecutive reversals is limited just by the size of the system (i.e. the volume of the Earth's liquid core). Elements to be introduced in future works include among others a careful analysis on the spatial distribution of consecutive activity and its relations with jerks and a quantitative estimative on the actual ratio between maximum observed magnetization and the maximum magnetization that the systems might support.

Acknowledgments

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