Finite-difference migration by wave extrapolation in three dimensions

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Abstract

Three-dimensional wave-equation migration techniques are still quite expensive because of the huge matrices that need to be inverted. Several techniques have been proposed to reduce this cost by splitting the full 3D problem into a sequence of 2D problems. We compare the performance of splitting techniques for 3D Finite-Difference (FD) migration techniques in terms of image quality and computational cost. The FD method is complex Padé FD, and the compared splitting techniques are two and alternating four-way splitting, i.e., splitting into the coordinate directions at one depth and the diagonal directions at the next depth level. From numerical examples in inhomogeneous media, we conclude that alternate four-way splitting yields better results than the two-way splitting, with the same cost.

Introduction

In three dimensions, migration methods based on solving the one-way wave equation, besides facing problems to image dip reflectors and handle evanescent waves, are still computationally expensive. For the problems of imaging dip reflectors and evanescent waves, we use the complex Padé approximation. Because the resolution of three-dimensional problem is computationally expensive, over the years various techniques have been developed in order to reduce costs and still maintain the quality of the migration method that you are using. A commonly used technique is splitting.

For the case of splitting in two directions, we face the problem of numerical anisotropy, i.e., the migration operator acts differently in different directions, resulting in positioning errors of reflectors in the situation where the direction of the dip reflector is far from the directions of migration plans. To correct this problem we use the correction of Li (1991). Without changing the basic principle of applying subsequent 2D FD migrations in the x and y directions, the Li correction is an extrapolation of the residual field by a phase shift. When splitting is applied alternately in four directions (the horizontal coordinates and the diagonals), we may still face problems of numerical anisotropy and, consequently, of positioning errors of steeply dipping reflectors. Therefore, we also tested the application of a Li correction in this case.

Our goal in this work is to evaluate the behavior of 3D FD migration operators using the complex Padé approximation, the technique of splitting into two or four alternating directions, as well as the Li correction. For that purpose, we compare the results obtained by FD migration of synthetic data from the SEG/EAGE salt model.

Theory

According to the hypothetical model of the exploding reflector, a migration consists only in repositioning the seismic wave recorded at the source position to depth at time $t=0$. Thus, we see that we need only the waves that propagate upward to perform a migration. The acoustic wave equation for a homogeneous medium is given by (Leontovitch et al., 1964)

$$\nabla^2 p(\mathbf{x},t) = \frac{\partial^2 p(\mathbf{x},t)}{\partial x^2} + \frac{\partial^2 p(\mathbf{x},t)}{\partial y^2} + \frac{\partial^2 p(\mathbf{x},t)}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 p(\mathbf{x},t)}{\partial t^2}, \tag{1}$$

where ∇ is the Laplacian operator, $p(\mathbf{x},t)$ is the scalar wave field, $\mathbf{x}=(x,y,z)$ and v is the speed of the medium, considered constant. We define the Fourier transform in horizontal coordinates x and y and time t as

$$P(k_x,k_y,z,\omega) = \int_{-\infty}^{\infty} p(\mathbf{x},t) e^{i(k_x x + k_y y - \omega t)} dt dx dy, \tag{2}$$

and its inverse in the same variables as

$$p(\mathbf{x},t) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} P(k_x,k_y,z,\omega) e^{-i(k_x x + k_y y - \omega t)} d\omega dk_x dk_y. \tag{3}$$

Applying the Fourier transform as in (2) in equation (1) and factoring the resulting equation, we can write the one-way wave equation for upgoing waves as

$$\left[\frac{\partial}{\partial z} - \frac{(-i\omega)}{v}\sqrt{1-\frac{v^2}{\omega^2}(k_x^2+k_y^2)}\right] P(k_x,k_y,z,\omega) = 0. \tag{4}$$

Considering that the velocity is locally constant, i.e., the propagation of waves at each location $\mathbf{x}$ is well described by a constant speed, but the value of this constant velocity depends on $\mathbf{x}$, we rewrite the above equation as

$$\frac{\partial P(\mathbf{x},\omega)}{\partial z} = \frac{(-i\omega)}{v(\mathbf{x})}\sqrt{1+\frac{v(\mathbf{x})^2}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)} P(\mathbf{x},\omega), \tag{5}$$

which is the upgoing one-way wave equation for media with moderate velocity varations.

Complex Padé approximation

To actually use equation (5) in migration, we need to approximate the square-root operator by some numerically executable expression. Using a Taylor approximation may present some difficulties inherent in such series. Often the convergence of the series is extremely slow, or else, its radius of convergence does not include areas of particular interest to the problem being studied. On the other hand, Padé series enable us to start from a power series and obtain much more information than the series itself can provide us directly (Navarro et al., 1999).

Padé approximants are rational functions, i.e., quotients of two polynomials, which represent an expansion. These approximants are characterized by two positive integers L and M degrees of the numerator and denominator, respectively, rational function, and are represented by the notation $[L/M]$. Explicitly, an Padé approximant is defined by

$$[L/M] = \frac{P_L(x)}{Q_M(x)}\ , \quad L, M \geq 0. \tag{6}$$

Using the method of Padé approximants in equation (5), we can rewrite the square root operator as

$$\sqrt{1+Z} \approx 1 + \sum_{n=1}^{N} \frac{a_n Z}{1+b_n Z}, \tag{7}$$

where the coeficientes a_n and b_n are real Padé coeficients (Bamberger et al., 1988) given by

$$a_n = \frac{2}{2N+1}\left(\text{sen}^2 \frac{n\pi}{2N+1}\right) \quad \text{and} \quad b_n = \cos^2\left(\frac{n\pi}{2N+1}\right). \tag{8}$$

The Padé approximation is widely used because it allows an efficient numerical implementation and a larger error order than other approximation methods. However, when looking for images with good accuracy for wide angles of propagation, equation (7) becomes inappropriate. The reason is that for angles near $90°$, the argument Z becomes very close to -1. And when $Z < -1$, the left side of the equation (7) is complex while the right side is still real, causing an inconsistency in the approximation (Amazonas et al., 2007). Physically, this means that the representation of equation (7) cannot properly handle evanescent waves. This feature is responsible for the unstable behavior of the algorithm in the presence of strong lateral velocity variations.

In the complex plane, a Padé expansion with real coefficients corresponds to approximations of the square root with branch cut along the negative real axis. To overcome the problems with evanescent waves, Millinazzo et al. (1997) proposed the introduction of a rotation angle α, which rotates the branch cut, improving the representation of the evanescent part of spectrum, and therefore the stability of the approach.

Considering the rotation of the branch cut on the complex plane, the representation of the square root has the form

$$\sqrt{1+Z} \approx C_0 + \sum_{n=1}^{N} \frac{A_n Z}{1+B_n Z}, \tag{9}$$

where the complex Padé coeficientes are given by

$$A_n = \frac{a_n e^{-i\alpha/2}}{[1+b_n(e^{-i\alpha}-1)]^2}\ , \qquad B_n = \frac{b_n e^{-i\alpha}}{1+b_n(e^{-i\alpha}-1)}\ , \tag{10}$$

$$C_0 = e^{i\alpha/2}\left[1 + \sum_{n=1}^{N} \frac{a_n(e^{-i\alpha}-1)}{[1+b_n(e^{-i\alpha}-1)]}\right] \approx 1. \tag{11}$$

The constant C_0 is the closer to one the more terms N are used in the approximation. Since it does not depend on the argument Z, it can be immediately replaced by one in the algorithm.

FD migration with the complex Padé approximation

This method approximates the operator of the one-way wave equation (5) by a Padé series (Bamberger et al., 1988; Claerbout, 1985). Its complex version is obtained replacing the real coefficients a_n and b_n by the corresponding complex coefficients A_n and B_n, resulting in

$$\frac{\partial P(\mathbf{x},\omega)}{\partial z} = \frac{(-i\omega)}{v(\mathbf{x})}\left[1 + \sum_{n=1}^{N} \frac{A_n \frac{v^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}{1+B_n \frac{v^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}\right] P(\mathbf{x},\omega). \tag{12}$$

Like its real version, this operator can be implemented in cascade, considering each term of the summation independently. Thus, we first need to solve the differential equation

$$\frac{\partial P(\mathbf{x},\omega)}{\partial z} = \frac{(-i\omega)}{v(\mathbf{x})} P(\mathbf{x},\omega), \tag{13}$$

followed by equations resulting from the terms inside the summation, i.e.,

$$\frac{\partial P(\mathbf{x},\omega)}{\partial z} = \frac{(-i\omega)}{v(\mathbf{x})}\left[\frac{A_n \frac{v^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}{1+B_n \frac{v^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)}\right] P(\mathbf{x},\omega), \tag{14}$$

for $n = 1, \ldots, N$.

To solve this system of equations, we consider each solution of the previous equation the initial condition for the following equation. The first equation (13) has an analytic solution for models for which the velocity satisfies $v(\mathbf{x}) = v(x,y)$ within a layer of size Δz. Then, we can express the wavefield at level $z+\Delta z$ depending on the wavefield at the previous level, z, as

$$P(x,y,z+\Delta z,\omega) = P(x,y,z,\omega)\exp\left[\frac{(-i\omega)}{v(\mathbf{x})}\Delta z\right]. \tag{15}$$

The differential equations associated with the summation are given by

$$\frac{\partial P(\mathbf{x},\omega)}{\partial z} + B_n \frac{v^2(\mathbf{x})}{\omega^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\partial P(\mathbf{x},\omega)}{\partial z} = -A_n \frac{v(\mathbf{x})}{(-i\omega)}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right) P(\mathbf{x},\omega)\ . \tag{16}$$

The advantages of using the complex version is that this technique does not have limitations towards the velocity model variations. It can handle evanescent waves more appropriately. The downside remains the same as in the real version of the FD method, which is the difficulty of imaging steep-dip reflectors.

Splitting

The numerical solution of the differential equation (17) involves the inversion of a huge matrix, which is a very costly process. Individual solution in the coordinate directions [suppressing the x or y derivatives in equation (17)] are orders of magnitude less expensive. So, the idea arose to approximately execute the full 3D operator of equation (17) as a sequence of 2D processes (Brown, 1983). This approach is called splitting.

Generalizing this idea, Ristow et al. (1997) proposed splitting in various directions, which consists in approximating a 3D operator in a sequence of 2D operators, usually along the lines of horizontal coordinates, the inline and crossline directions and some additional directions away from the coordinate axis. The price to pay for the increased efficiency of the method is reduced accuracy in the directions not used for the 2D operators.

Correction for two way splitting (Li correction)

To compensate the errors caused by the use of splitting into the coordinate directions (below referred to as two-way splitting) and still preserve the efficiency of the FD method, Li (1991) proposed the application of a phase correction operator, implemented either as a phase-shift term or using the PSPI method. This difference operator is obtained by evaluating the error between the true and approximate operators.

The idea of this correction is to carry out the conventional 2D FD migration in both directions where the splitting was done plus a residual wavefield extrapolation by the phase-shift method (when the lateral velocity variation is small), or the PSPI method (when the lateral variations of the wave speed are more significant).

Because the error is supposedly small in each step of the extrapolation, the effect of the compensation process is similar to the residual migration, in which case the waves propagate very little at each step. Therefore, it is reasonable to replace the true velocity $v(\mathbf{x})$ by a reference velocity v_r and apply the correction only at a reduced number of depth steps.

For the choice of these reference velocities, several methods exist. In this work, we employ a modified version of the method of Lloyd (1982). The main idea behind this method is to iteratively improve the values of a chosen set of reference velocities by looking at statistical values in each region (mean, median and variance), and then change the region boundaries in each iteration as an attempt to find the best solution based on some criterion. In our modified version, reference velocities too close to each other are eliminated and values too far from each other are split at the end of the process.

Results

We tested these implementations of FD migration for the case of zero offset by their impulse responses in the EAGE/SEG model salt. We used the following modeling parameters: The source wavelet is a Ricker pulse with central frequency of 15 Hz; the migration grid spacing is $\Delta x = \Delta y = \Delta z = 20$ m (to avoid numerical dispersion, we treat the model as if the spacing was 10 m); the delta pulse to be migrated is symmetrically positioned with its center at $t = 1.1$ s.

To represent the results, we use vertical cuts parallel to the yz and xz planes at $x = y = 8.32$ km, and horizontal cuts at $z = 1.1$ km and $z = 2.7$ km depths. In order to use Lloyd's method for choosing the reference velocities for each level, we calculate the average slowness using all points (x, y) of the velocity model. Because it is an iterative method, the number of reference velocities is variable, limited to at most 10 velocities at each level.

Unless otherwise specified, we use in our numerical tests the following parameters: three terms ($N = 3$) in the complex Padé approximation, rotation angle $\alpha = 45°$, and application of the Li correction at every 6 steps. We compare the results varying these parameters individually. As a reference, we use the result of the FFDPI method (Biondi, 2002) to evaluate our results (see Figure 1).

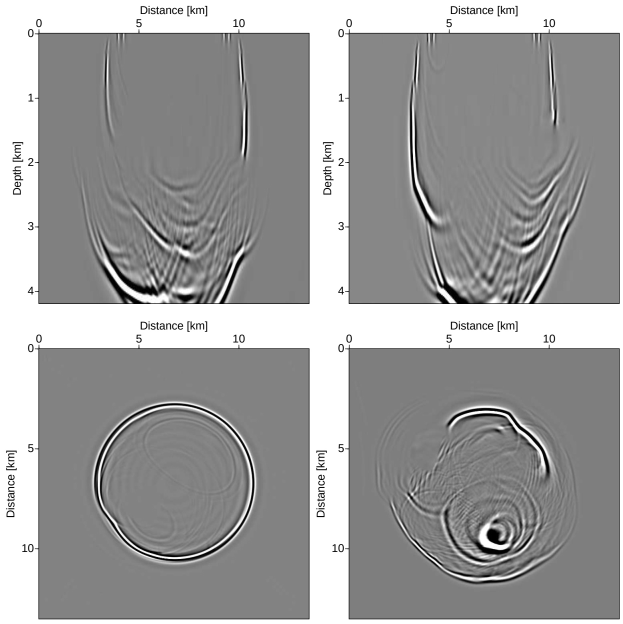

Figure 1: Impulse response obtained by FFDPI migration. 4 cuts represented as $x = 8.32$ km (top left), $y = 8.32$ km (top right), $z = 1.1$ km (bottom left) and $z = 2.7$ km (bottom right).

Despite the numerical dispersion visible in these figures, they show no signs of numerical anisotropy, so that the positioning of the events can be considered reliable.

In Figure 2 we see the result of complex Padé FD migration with three terms and without Li correction. In the horizontal sections, we see a deviation from circular to diamond shape and the appearance of artifacts, indicating the strong numerical anisotropy caused by splitting. We also note the the wavefront lag behind their true position (compare to Figure 1).

Next, we investigate the dependence on the number of terms in real and complex FD migration. As a general observation, we find that the optimum rotation angle depends on the number of terms used. Below, we show only the results with the best rotation angle.

Figures 3 and 4 show the results of real and complex Padé FD migration, both with 1 term, and Li correction. Comparing Figures 3 and 4 we note the appearance of artifacts in the real version in vertical cuts, and blurred events in the complex version. The artifacts present in the shallowest horizontal cut are a consequence of applying

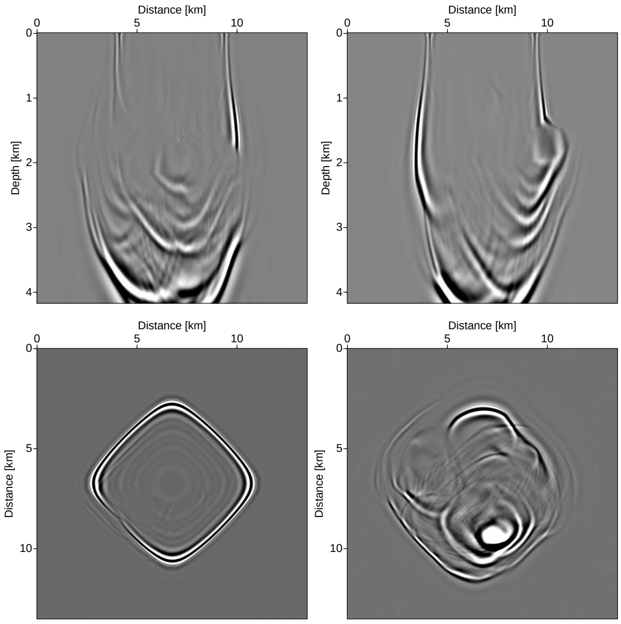

Figure 2: Impulse response obtained by FD migration without Li correction and complex Padé approximation.

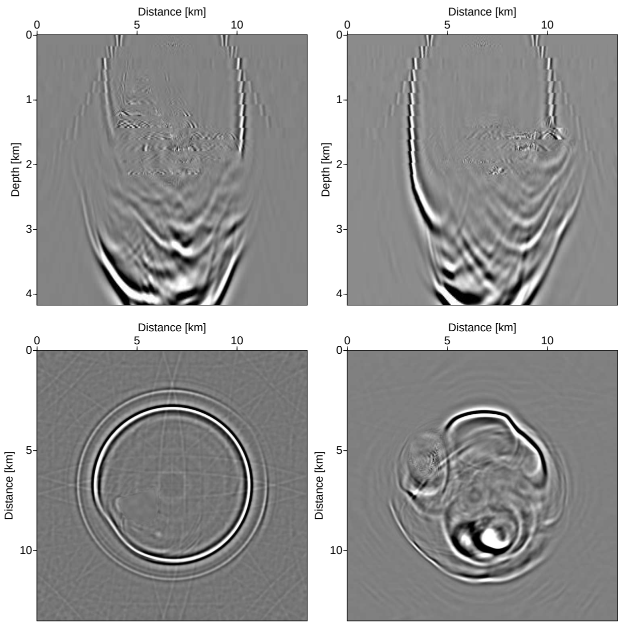

Figure 3: Impulse response obtained by FD migration using real Padé approximation with 1 term and Li correction.

the Li correction. However, there are less artifacts in the complex version than in the real one. The comparison with the results in Figure 2 shows that Li correction actually improves the positioning of the events. Delays in the diagonal directions were fixed, recovering an almost perfect circular shape.

Figures 5 and 6 show the corresponding results with 2 terms and Li correction. Comparing Figures 5 and 6 with Figures 3 and 4 we note that with 2 terms we have less artifacts in the real version, and more marked events in the complex version in the vertical cuts.

In Figures 7 and 8 we see the corresponding results obtained with 3 term. Comparing Figures 3 and 4 we note

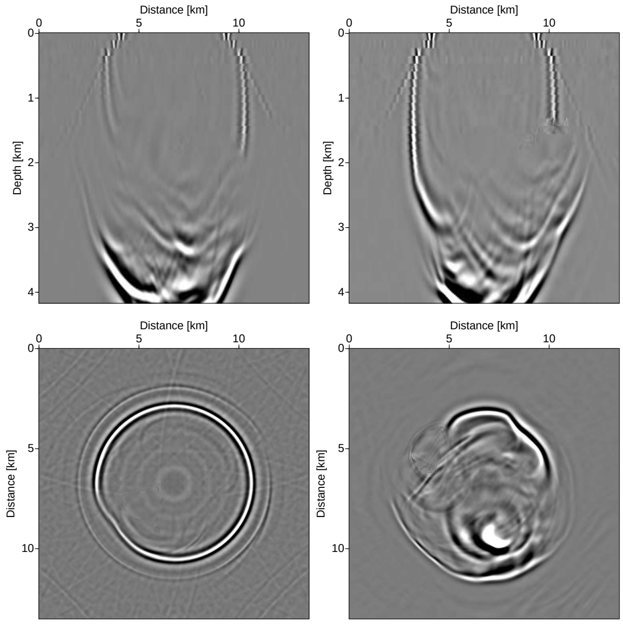

Figure 4: Impulse response obtained by FD migration using complex Padé approximation with 1 term with rotation angle of 15° and Li correction.

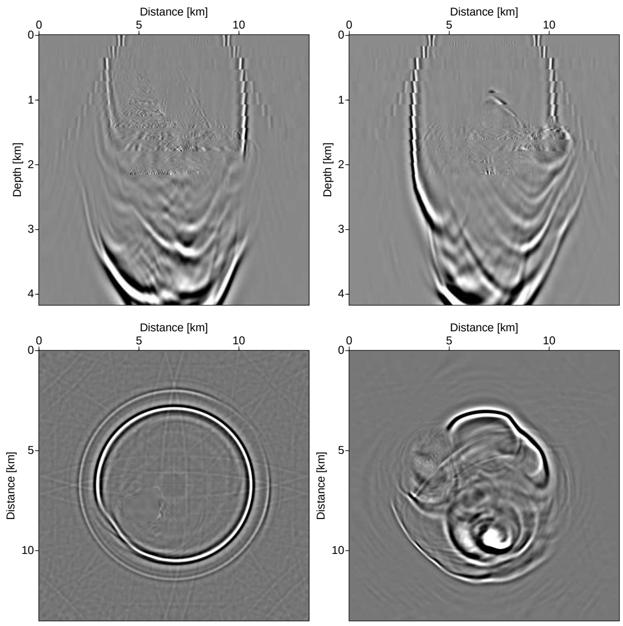

Figure 5: Impulse response obtained by FD migration using real Padé approximation with 2 terms and Li correction.

that we have less artifacts than when we use 1 term in the approximation. The 3-term real result attains approximately the same quality as the 2-term complex result.

Figure 9 depicts the results when using the mean velocity instead of the mean slowness for the reference velocity computation. As expected, the result is of inferior quality to that obtained with the mean slowness (Figure 8), because the latter corresponds to the real representation we can see in the wave equation.

In addition two the above experiments with two-way splitting, we also tested alternating four-way splitting, where the splitting is carried out in the coordinate directions

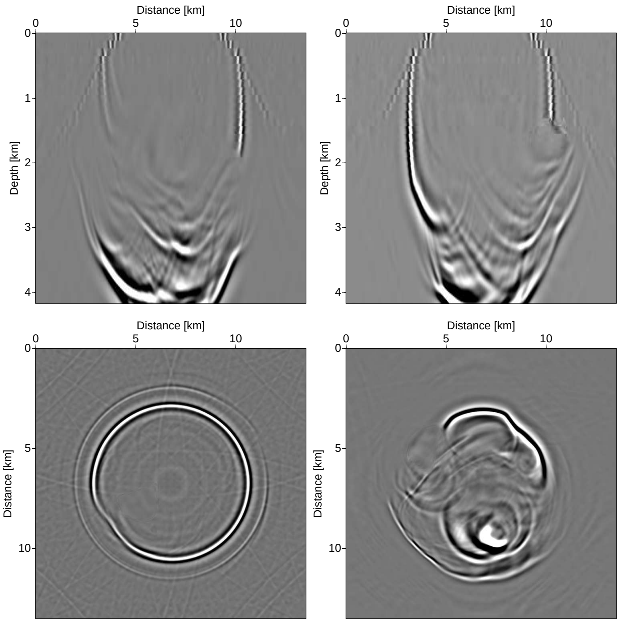

Figure 6: Impulse response obtained by FD migration using complex Padé approximation with 2 terms with rotation angle of 25° and Li correction.

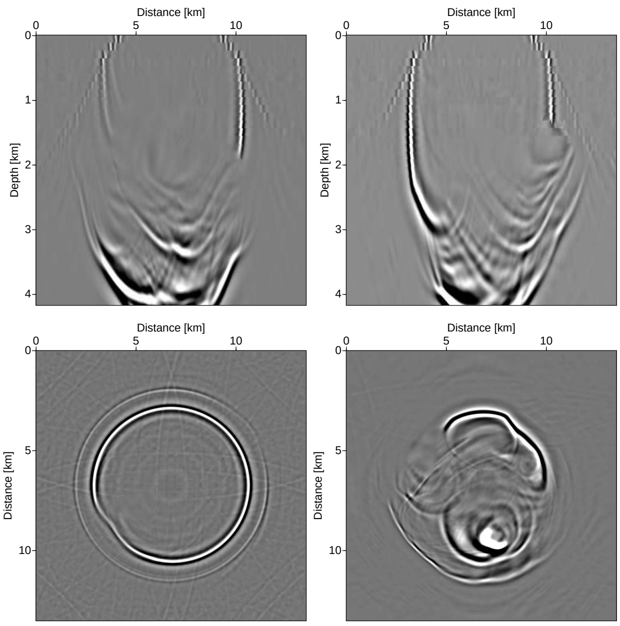

Figure 7: Impulse response obtained by FD migration using real Padé approximation with 3 terms and Li correction.

at one level and in the diagonal directions at the next level. Apart from minor effects, alternating four-way splitting has the same computational cost as two-way splitting. Figure 10 shows the results. We see that the problem of numerical anisotropy has improved, but not completely solved. Although we do not see artifacts in the horizontal cuts, we see a deviation from circular to octogonal shape. Other remaining positioning errors are evident when comparing Figure 10 with the corresponding Figure 1 obtained by FFDPI method. To fix the remaining anisotropy we also applied the Li correction (see Figure 11).

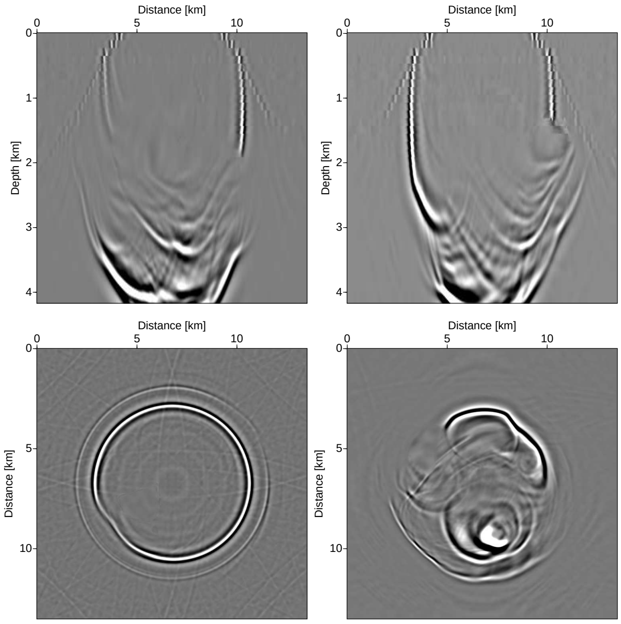

Figure 8: Impulse response obtained by FD migration using complex Padé approximation with 3 terms with rotation angle of 45° and Li correction.

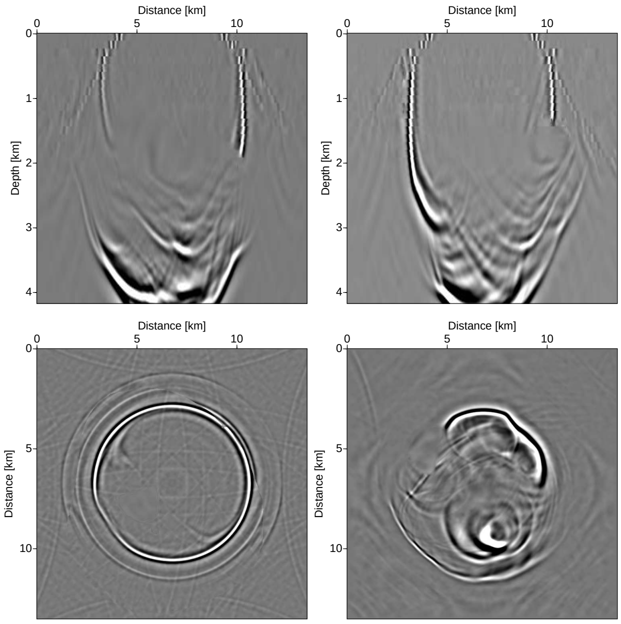

Figure 9: Impulse response obtained by FD migration using complex Padé approximation with 3 terms with rotation angle of 45°, mean velocity and Li correction.

Conclusions

From the numerical tests, we conclude that FD migration with two-way splitting plus Li correction showed an excellent recovery of circular shape, but still showed some remaining phase errors. As a drawback, it must be noted that as a consequence of the Li correction, some migration artifacts emerged or were reinforced.

When comparing the real and complex versions of the Padé approximation, we found few differences. Using fewer terms in the Padé expansion stronger affected the real version than its complex counterpart. With respect to

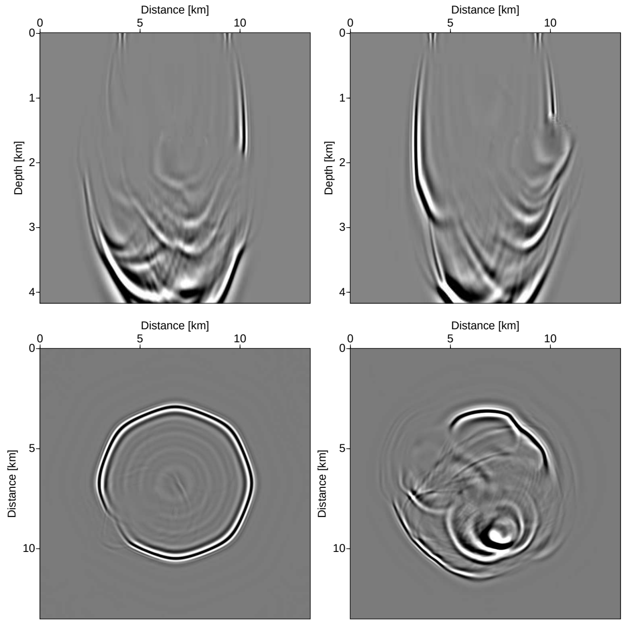

Figure 10: Impulse response obtained by FD migration using alternating four-way splitting and without Li correction.

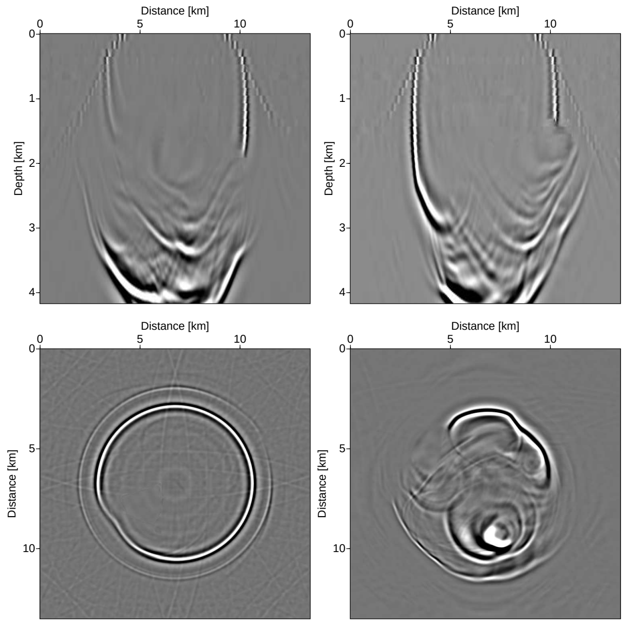

Figure 11: Impulse response obtained by FD migration using alternating four-way splitting and with Li correction.

the rotation angle of the branch cut in the complex Padé approximation, we found that the fewer number of terms are used the smaller the rotation angle must be.

The application of splitting in more than two directions is useful to reduce the numerical anisotropy, but is costly. Moreover, in the diagonal directions aliasing effects may arise because the distances of the grid points are a factor $\sqrt{2}$ higher. Other directions outside of the axes and diagonals complicate the problem because they demand resampling. One way to reduce the cost of splitting into four directions is the application in alternating directions, the coordinate directions at one depth step and the diagonals at the next. This alternating application has the same cost of splitting in two directions but has a very high quality. In our experiments, the quality was equivalent to full 4-way splitting. Thus, the numerical results in four directions presented in this paper exclusively use this alternate implementation. For not too deep reflectors, the remaining numerical anisotropy is acceptable. However, for larger migration distances, regardless of the splitting into two-way or alternating four-way directions Li correction should not be forgotten if accurated positioning is desired.

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