Sensitivity to initial conditions in a simple model for Earth's magnetic field reversals

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Abstract

We have studied the sensitivity to initial conditions, at the critical state, of the bi-stable self-organized critical model introduced in a previous work by our group (Espirito Santo et al., this Conference) to model the reversals that the Earth's magnetic field has suffered along its whole history. The basic model which essentially consist in a set of random nodes simulating magnetic vortices in the interior of the Earth's liquid core, systematically removes those nodes with lower absolute values, and theirs neighbors. At the stationary state, two possible "magnetizations" are possible for the whole system: up and down (mimicking normal and reverse magnetizations, respectively). We show that the dependences correspond to power laws during a time interval depending on the simulated system size, after which present plateaus. However, in the limit N → ∞ the nontrivial power-law regime should last forever. As the number of elements increases so does the time needed to reach the plateau. From the dependence between those quantities we have obtained the dynamical exponent z.

Introduction

Motivated by the interest that reversals have triggered during the last decades and by their intrinsic importance for the human kind, Espirito Santo et al. [1] introduced a model that present self organized criticality.

The model simulates the Earth's liquid core and the electric current structures on its volume by nodes distributed on an LxL square lattice. It explores sets of equally spaced points at the Earth's equator. To each of these nodes it is initially assigned a random value between -1 and 1 to simulate both, the accumulated magnetic energy at each of the simulated positions and the magnetic moment orientation. The time step for the model is defined by looking for the lowest absolute value through the whole system and changing it and its four nearest neighbors by new random values between -1 and 1. With this it is simulated a more or less continuous energy flux to the core bulk (this is the reason to pick the lowest value) and the possible absorption of smaller vortices. In this way it is also simulated the creation of new vortices. At the same time, the assignment of new random values, to the lowest in absolute and its neighbors, works as a continuous release of energy out of the system.

This process is repeated several times (usually between 10^6 and 10^8 times) to obtain stationary distributions for the quantities we are interested in. In the original paper (and also here) periodic boundary conditions were used.

Constructed in the way it has been done, the model qualifies as a Bak-Sneppen one [2]. The Back-Sneppen model probably is the simplest model presenting self-organized criticality, i.e., the tendency to a stationary critical state without necessity of a fine tuning.

The Bak-Sneppen model is a general model that has found applications in a large number of fields among which we can mention evolution [2], the brain [3], the cosmic rays spectrum [4] and X-rays bursts at the Sun's surface [5].

Many scientific efforts have been devoted to characterize the Bak-Sneppen model from several points of view. Examples of them are: its correlations from detrended fluctuation analysis [6], damage spreading on it [7], and its behavior under reduction to near zero dimension [8].

The magnetization M for the system is defined as:

$$M = \sum_{i=1} S_i/N \qquad (1)$$

where the sum runs over all the nodes and N = LxL is the total number of nodes. It can take values between -1 and 1 (corresponding to all nodes in the -1 value and to all nodes in the 1 value, respectively).

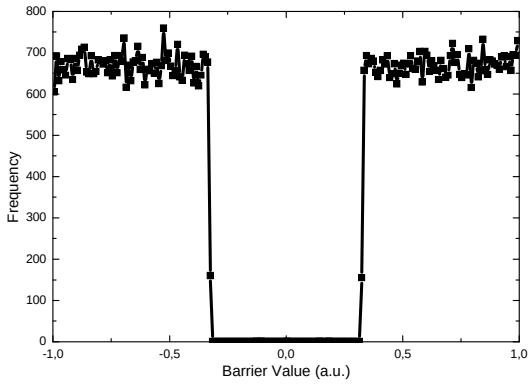

Figure 1.- Distribution of the nodes values at the stationary state. It has a well-like form with vertical walls at ± 3.5 approximately.

Beginning with an arbitrary distribution of accumulated magnetic energy at each node, the subsequent activity

will be completely uncorrelated in space and time. But as times goes by (and then, the mean accumulated energy increases in absolute value as a consequence of selecting and changing the lowest absolute values) it will be more and more likely that near neighbors are consecutively changed. After a transient the system reaches a steady state characterized by a well-like distribution for the accumulated energies and a couple of thresholds, $\pm E_c$ for the distribution of the lower barriers (see Figures 1 and 2). The distribution of lower energies vanishes above $+ E_c$ and below $- E_c$.

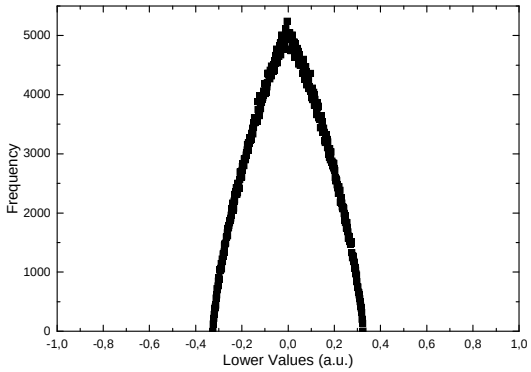

Figure 2.- Distribution of the lower absolute values for nodes at the stationary state. It has an inverse V-like shape with vertical walls at ± 3.5 approximately.

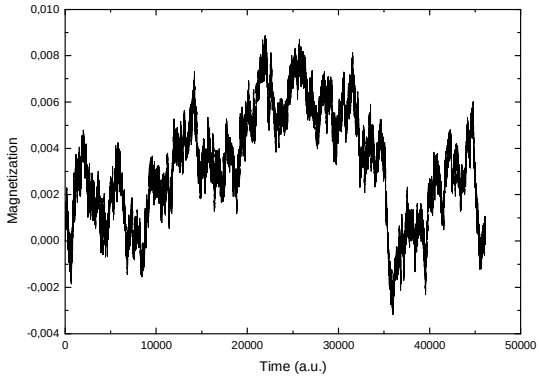

Figure 3.- Magnetization (see text) versus time for a 100x100 system during a short simulation (~45000 time steps).

Two main characteristic are relevant for the present study: first, that the simulations present reversals, i.e., changes in the sign of the magnetization defined in Equation (1); and second, that the distribution function for the inter-reversal time is a power-law,

$$f(t) = c.t^{d} \qquad (2)$$

where f (t) is the frequency distribution of periods between consecutive reversals, c is a proportionality constant and d is the exponent of the power-law (and also the slope of the graph in log-log plots), pointing to the possibility of the liquid core of the Earth be in a critical self-organized state (see figures 3 and 4). For the present case we have approximately -1.68 as slope value.

The rest of the paper is organized as follows: first, we present the Method, later on we present the Results of our simulations as well as a comparative study with experimental results and a discussion on their possible connection with previous works on the statistics of geomagnetic reversals. Finally, we present our conclusions and some possible trends for future works.

Method

The Hamming distance between two systems A and B composed by N elements each is usually defined by:

$$H = \sum_{i=1} \left| A_i - B_i \right| / N \qquad (3)$$

where H is the Hamming distance, A_i and B_i are the corresponding elements of each systems and the sum runs over the whole systems.

To calculate the sensitivity to initial conditions it would be ideal to initially perform the simulations for a single system which, once at the stationary state be duplicated. In the copy system a few nodes would be randomly changed and, after this, the Hamming distances between both systems (the original and the slightly modified one) calculated for each time step of the simulation. The sequence of generated random numbers, however, would have to be unique for both.

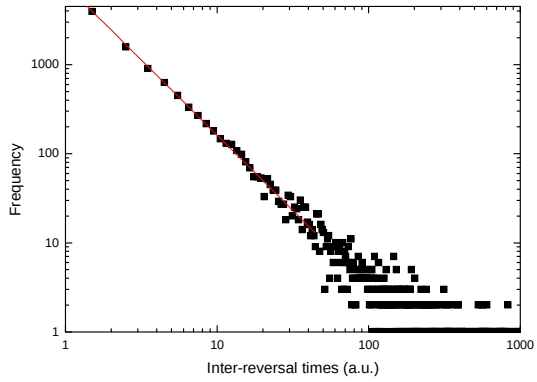

Figure 4.- Distribution of time between consecutive reversals (both from positive to negative and from negative to positive).

The peculiarities of our model, however, prevent us of doing so. Note that even if the system attains a single stationary state, this state is characterized by to different (and equiprobable) polarizations, which makes that the magnetization as a function of time follow a behavior changing of sign continuously. In this way, the Hamming distance oscillates without bringing to us any valuable information.

Furthermore, any tentative of extracting relevant information, for example, of the distribution of traditional Hamming distances will be frustrated because nodes are

random numbers and their difference distribution is a usual Gaussian.

Consequently, we have adopted an alternative strategy.

By looking at Figure 2 we note that the limiting values for the lower absolute values are reasonably well defined, even for the system sizes that we have used.

Given that at the stationary state the distribution of lower absolute values presents two thresholds above which it vanishes, we have studied the time dependence of lower absolute values as a function of time beginning at a completely disordered state.

We have done this type of study for several system sizes and from the dependence on it we have extracted a dynamical exponent z.

Results

The results for the time dependence of the lower absolute value in one simulation with system size 300x300 are presented in Figure 5.

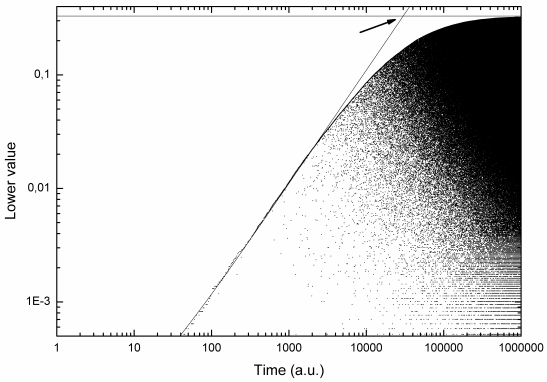

Figure 5.- Lower values (absolute values) as a function of time for a 300x300 system size. The initial state is a disordered one and initially tends to the stationary state following a power law. The arrow points to the interception between the stationary value level and the straight line of the power law. The interception corresponds to a value of approximately 30,000 times steps.

We note in Figure 5 that initially the lower absolute values growths with time following a power-law. After a given time the values can be found between zero and the thresholds ± 3.5 after which a plateau is established (note that we have plotted just positive values, i.e., the absolute value of lower nodes). To find in a unique fashion the time that for a given system size the simulation takes to attain the plateau we have determined the interception between the power-law extrapolation and the plateau extrapolation (as indicated by the arrow in Figure 5).

For the case shown in Figure 5, the time needed to reach the plateau was approximately 30,000 time steps. We have explored several system sizes (LxL where L = 25, 50, 100, 200 and 300).The result for the time needed to reach the plateau as a function of the system size is presented in Figure 6. From the slope we extracted the value of our dynamical exponent z = 0.96 ± 0.08.

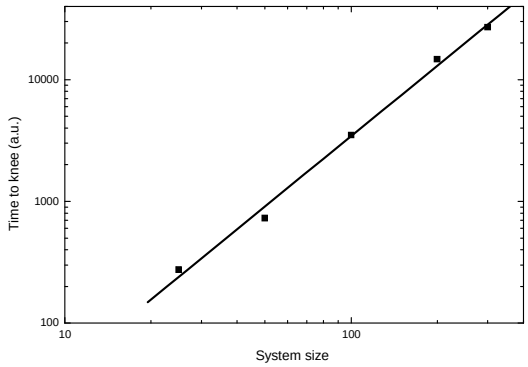

Figure 6.- Relaxation time as a function of system size. The systems were LxL with L = 25, 50, 100, 200 and 300. From the slope we extract the dynamical exponent z.

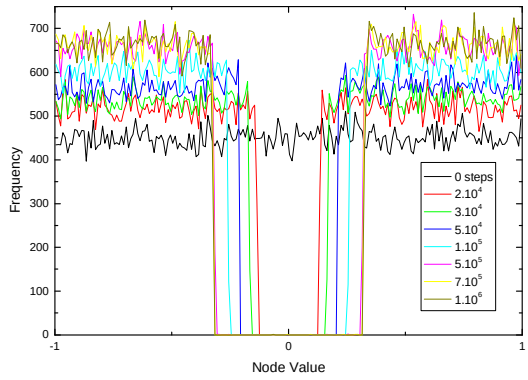

Figure 7.- Evolution of the distribution function of node values from a completely disordered state to the self-organized critical state as a function of time (see inset).

It is worth to mention here that, as can be noted in Figure 1, although the distribution function of nodes at the stationary state is more or less well defined, it is very noisy. So, trying to accompany the tendency of the systems to the stationary state by, for example, comparing (calculating the difference between) nodes distribution functions at different times, will give considerable errors. However, this is a potentially useful tool is we use large enough system sizes. To illustrate this, we present in Figure 7 the evolution of the distribution function of node values from a completely disordered state to the self-organized critical state as a function of time.

In Figure 8 we represent the absolute accumulated difference between the stationary distribution (gray in Figure 7) and the rest of the distributions. Although the curve seems to be soft, it is affected by considerable errors that can be avoided only through the realization of simulations with system sizes well above the ones we have used. We let this study for the future.

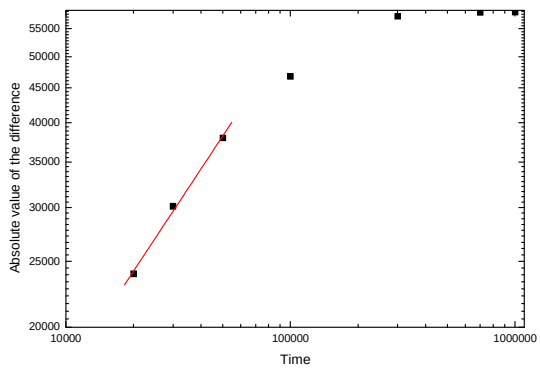

Figure 8.- Absolute value of the difference between the distributions in Figure 7 and the final state. From left to right $2x10^4$, $3x10^4$, $5x10^4$, $1x10^5$, $5x10^5$, $7x10^5$, and $1x10^6$ time steps. From the interpolation it can be extracted the time needed to reach the plateau.

Conclusions

We have obtained for a self organized critical model representing the Earth's magnetic field reversals the variation of the time needed to reach the stationary state as a function of system size. For finite systems sizes, N, a plateau is observed if enough time elapses, but, in the limit N $\rightarrow\infty$, the nontrivial power-law regime should last forever. As the systems size increases so does the time required to reach the plateau; from this dependence we have obtained the dynamical exponent z, by following a special methodology imposed by particularities of the model. We have done this in 1D (not shown) and 2D and from that we can asseverate that the result is robust in the sense that the tendency to the plateau is always attained through a power-law. The search for physical quantities that effectively bring us to compare the separation of two systems (an original and a slightly altered copy of the former) as a function of time is one of the aims of future works to be published elsewhere.

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