Equivalent source: A natural choice for gridding scatter gravity data.

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Abstract

Measurements of the gravity field, specially those taken at ground level are acquired on a scatter distribution due to the difficulties imposed by terrain conditions. Most of the times, the irregular sampling interval leaves large empty areas that need to be interpolated during the gridding. For such situations, mathematical interpolation like minimum curvature technique, for instance, can lead to erroneous results. In this paper, I propose to address the gridding of scatter gravity data using the equivalent source technique and show that it has advantages over the minimum curvature. In addition, I also propose a simple method to efficiently estimate the true field. The method was successfully tested with a set of ground gravity data from Recôncavo Basin, Brazil.

Introduction

Since long ago, it is well known in the potential fields community that the ambiguity in determining the causative source of the field from values outside the source region is a disadvantage of the potential fields interpretation. As suggested by Dampney (1969), such drawback can be turned into advantage through the use of the equivalent source technique.

The logic behind the equivalent source technique states that if it is possible to find a specific source distribution that generates the observed potential field at the measurement points, such distribution can be used to calculate the field anywhere above the source distribution. The calculated source does not need to resemble the true unknown source distribution because Green's third identity assures that the calculated source distribution causes the same potential field in a restricted region.

As gravity field, more specifically, ground gravity measurements usually are acquired on scatter locations, equivalent source technique can be used to find a alternative source distribution that fits the gravity field at the measurement points and then to project this field onto a regularly gridded horizontal plane. Having the observations laying in a regular horizontal plane is a necessary requirement for machine contouring and also for many potential fields applications, like all filtering techniques that make use of Fourier transforms, for instance.

Methods for interpolating scatter gravity measurements in a regular grids are available in the majority of commercial software that deals with potential fields. However, the methods applied to perform the interpolation are usually mathematical or statistical techniques that do not take into account the particular characteristics of the potential fields. Minimum curvature (Briggs, 1974) and kriging (Journel and Huijbregts, 1978) are examples of these techniques usually found in potential field softwares.

The applicability of the equivalent source technique is not restricted to gridding. As Dampney (1969) suggested, once the equivalent source is computed it can be used for upward continuation, for instance. It can also be used for interpolation between data points as well as to extrapolate data beyond the range of observation. Field extrapolation can be very useful when working with Fourier transforms as suggested by Cordell and Grauch (1982).

In this paper, I show that equivalent source technique is a more appropriate choice for gridding scatter gravity data by comparing the results of both minimum curvature and equivalent source gridding with a synthetic field computed from a known source distribution. In addition, I show that the results of equivalent source are strongly related to their depths and propose a simple method for depth choosing that is able to produce a quasi-optimal field.

The proposed technique is illustrated by a real case application using a ground gravity survey acquired in Recôncavo Basin, Brazil.

Method

The equivalent source gridding is a two-step procedure composed by the inverse problem of finding a source distribution that fits the observed gravity field and a forward calculation that provides the field at the desired locations once the alternative source is found.

The approach chosen here to compute the equivalent source is based on approximating the source distribution by a series of discrete thin prismatic bodies. Assuming that the observed gravity field is known at *M* locations, the equivalent source can be represented by *N* prisms placed at a suitable depth. This inverse problem can be written in the traditional matrix form

$$g = Ad \quad , \tag{1}$$

where g is the M gravity observations, A is MxN matrix that represents the geometric relations between the observations and sources positions, and d is the vector of the N unknown density values of each prisms. Finding a solution for d in the system of equation described by Equation (1) is a underdetermined linear inverse problem since the number of equations M is smaller than the number of unknowns N. Underdetermined inverse problems do not have unique solutions, but it does not represent a problem for equivalent source technique. As Green's equivalent layer states, the potential caused by a three-dimensional density distribution is indistinguishable from that of a thin layer of sources spread over any of its equipotential surfaces. Therefore, the same potential can be caused by an infinite variety of sources, which do not need to have any relation at all with the true source distribution. Any vector solution d of Equation 1 is valid as long as it fits the observations.

Once a hypothetical source distribution is found, the next step is to use it to compute its gravity response at the grid nodes. This step may be done using any available three-dimensional forward gravity modeling tool found in the most popular commercial software. It is important to mention that different from other gridding techniques that suffer from spatial aliasing, there is no restrictions on the grid resolution. This characteristic alone is already a strong advantage equivalent source shows over other gridding techniques. In addition, because the interpolated values are computed from a gravity equivalent source, they must show the correct gravity potential field behavior, which does not hold for other gridding techniques.

The comparison between the minimum curvature and equivalent source techniques was carried out by constructing a three-dimensional density distribution and computing the field at sets of both regularly spaced and random locations. The first represents the true field used as the basis for comparison while the second simulates a survey that will be the starting point for gridding.

For minimum curvature gridding I choose a popular commercial software. Although it was possible to play with several parameters, I decided to keep it as simple as possible and left all parameters as default, except by the cell size. To allow comparison, the cell size was set to be the same as that used to compute the also regularly spaced true field.

Besides the cell size, which was set to fit the same size as used in the true field, another parameter for equivalent source gridding is the layer depth. As evaluated by Dampney (1969) the layer depth plays important role in the equivalent source result and there will be a specific range of depths in which the equivalent source should be positioned. Such range is problem dependent and Dampney (1969) have empirically determined the best values for his case history in terms of the cell size. In this paper I decide to keep this idea and define the depths in terms of cell size units. So, equivalent sources were calculated for different depths and the field computed from each one compared to both the minimum curvature result and the true field.

Quantitative comparison were performed in terms of the root mean square errors (RMS) between the results of both gridding techniques and the true field. For qualitative comparison I have analyzed the absolute difference maps between the resulting grids and the true field.

Results

In order to demonstrate the advantages of the equivalent source over minimum curvature gridding I have selected a synthetic 3D model of 15 x 15 x 2.5 km and represented its density distribution by a set of 400 rectangular prisms having constant density contrast of 300 kg/m^3 to the surroundings (Figure 1). First, the synthetic gravity field was calculated along 2601 nodes of a grid having cell size of 300 m in both x- and y- directions. This will be considerate as the true field for comparisons with both minimum curvature and equivalent source gridding techniques. I have also simulated a survey by computing the field at 100 random locations spread over the area. Figure 2 shows the true synthetic gravity field and the locations of the simulated measurement stations.

The set of simulated stations was then gridded using minimum curvature technique with a 300 m cell size. The absolute difference between the minimum curvature grid and the true field was calculated and shown in Figure 3. Although the majority of the difference values (66 %) may be considered as low values (< 0.05 mGal), it is easy to realize that the greater differences (> 0.05 mGal) locate in between the simulated measurement points where high frequency anomalies were expected to occurs due to the shallow depths of the density contrast in this region. The large sampling interval, reaching 2 km in some regions, does not allow the capture of the high frequency anomalies and consequently, the minimum curvature technique is not able to interpolate such high frequencies causing differences up to 8.24 mGal.

As the equivalent source technique is strongly dependent on the source depth, I compute several grids representing the equivalent source at different depths ranging from 0.5 to 10 units of grid cell size (150 to 3000 m). The RMS error between each of these grids and the true field were calculated and plotted in Figure 4. The result in Figure 4 shows a reduction in the error as the depth increases. At a depth of 7.5 grid cells, the minimum error is reached and after that, the error starts to increase again. Therefore, the curve in Figure 4 proves there is a quasi-optimal depth for the equivalent source that produces the best approximation to the true field. For comparison, the RMS error of the minimum curvature (21.03 mGal) is included in the chart. The equivalent grid fits the true field better than minimum curvature grid for grids at depths below 3 grid cells where the smallest RMS error is 8.42 mGal.

The absolute differences between the best equivalent grid and the true field is shown in Figure 5. The majority of the difference values (89 %) stays below 0.05 mGal and the maximum difference is only 1.33 mGal. Similarly to what happens to minimum curvature grids, the larger differences (> 0.05 mGal) are located between the simulated measurement points at locations where high frequencies are expected to occur. Although the sample interval problem also affect the equivalent source grid, the differences here are much smaller because the interpolation is made respecting the behavior of a gravity potential field.

A careful analysis of the chart in Figure 4 shows that RMS errors between two consecutive depths are large at low depths and reduces as the equivalent source moves deeper. This becomes clear when comparing the differences between the errors at depths 0.5 and 1.0 grid cells with those at depths 3.5 and 4 grid cells, for instance. From depths between 5 and 8 grid cells the RMS error flats and only subtle changes occur in the differences. The differences start to increase again for the deepest depths. So, there is a set of depths, close to the best one, where the RMS error becomes almost flat. Any depth picked out from this set would produce a close estimate to the true field. However, it is necessary to know the true field in order to compute the RMS error, which is very unlikely in real life applications.

Based on the behavior observed in the RMS-depth curve, I propose an approach to estimate the best depth to place the equivalent source. The approach takes into account that it is not necessary to know the true field to compute the difference square between two equivalent sources at consecutive depth. Although the behavior of the difference square curve is not exactly the same as the RMS error differences it is, in general, similar enough to allow a good estimate of the quasi-optimal depth without knowing anything about the true field, as shown in Figure 6. The minimum of the square difference curve is located at depth 6.5 grid cells while the RMS error curve would point out the correct value of 7.5 grid cells for the equivalent source depth. It is possible to see from Figure 4 that the RMS error at depth 6.5 and 7.5 grid cells are very close, which gives confidence for the estimate. The absolute differences between the estimated equivalent grid (depth 6.5 grid cells) and the true field is shown in Figure 7. The majority of the difference values (88 %) stays below 0.05 mGal and the maximum difference is only 1.38 mGal. The similarity between this statistics and that of the quasi-optimal equivalent source depth, reinforce the importance of the method.

The equivalent source technique was applied to a set of 472 ground gravity stations acquired in a region of the Recôncavo Basin. Figure 8 shows the behavior of the square difference curve between equivalent source grids calculated at consecutive depths. As expected, the curve shows a minimum, which according to the method, indicates the best depth (4 grid cells) to place the equivalent source for this data. Considering that the grid cell size was chosen to be 100 m in this example, the best depth is 400 m below the surface. The resulting grid computed with the equivalent source at 400 m deep is shown in Figure 9.

Conclusions

The advantages of using equivalent source compared to minimum curvature technique to grid scatter gravity data were discussed based on comparisons with a synthetic field. Because the equivalent source technique respects the behavior of gravity potential field, it is able to better recover the high frequencies even in regions with reduced sampling. It was also possible to show that there is a set of depths in which the equivalent source results become better estimates of the true field than minimum curvature. In fact , there is a quasi-optimal depth that can be approximated by the minimum of a square difference curve between the equivalent grids resulting from consecutive depths. The importance of this estimator resides in the fact that it ensures a good representation of true field without knowing it. This is of fundamental importance for real case applications where knowing the field is the main objective. The method was successfully tested with real data from Recôncavo Basin producing a reasonable gravity field estimate.

Acknowledgments

The author would like to thanks PETROBRAS for the permission to publish this work.

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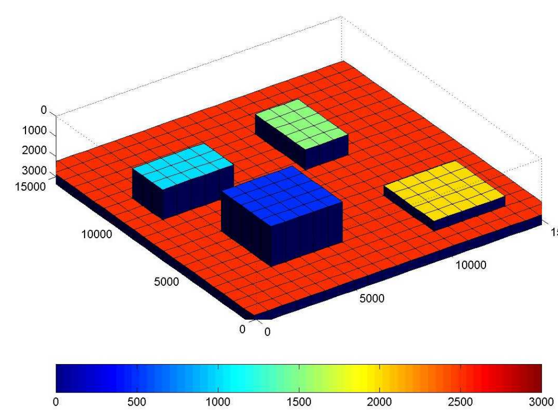

Figure 1 – Synthetic 3D model representing the density distribution in a 15x15x2.5 km region. The model is composed by a set of 400 rectangular prisms having constant density contrast of 300 kg/m^3 to the surroundings.

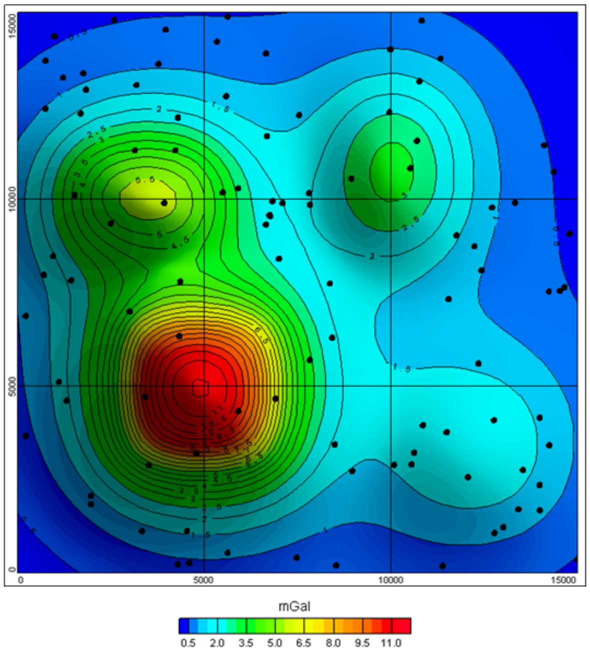

Figure 2 – True synthetic gravity field and the position (dots) of the 100 randomly located measurement stations that simulates a survey.

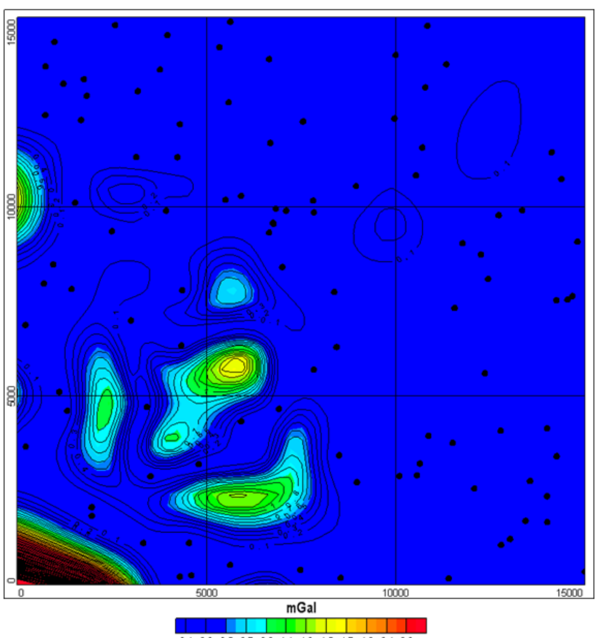

Figure 3 – The absolute difference between the minimum curvature grid and the true field. About 66% of the differences are below 0.05 mGal. The maximum difference is 8.24 mGal.

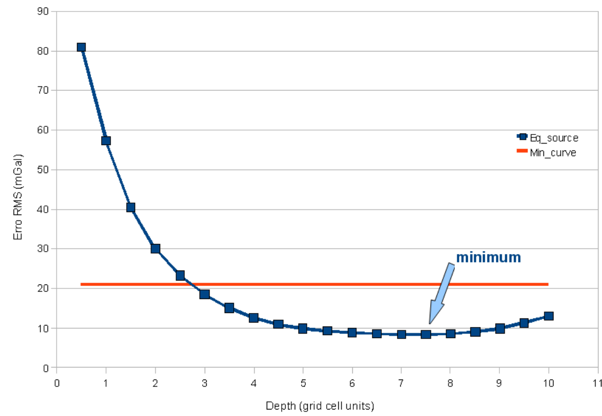

Figure 4 – The RMS error between the equivalent source grids at different depths (blue line) and the true field. The RMS error of the minimum curvature grid (red line) is also presented for comparison.

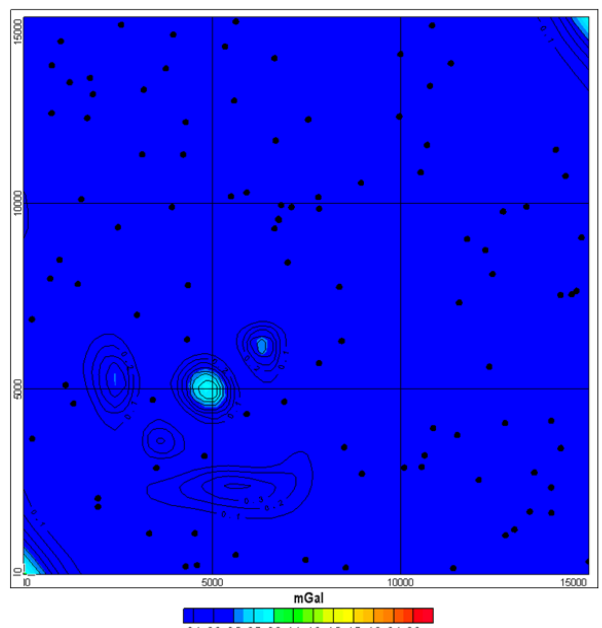

Figure 5 – The absolute difference between the best equivalent source grid (depth 7.5 grid cells) and the true field. About 89% of the differences are below 0.05 mGal. The maximum difference is 1.33 mGal.

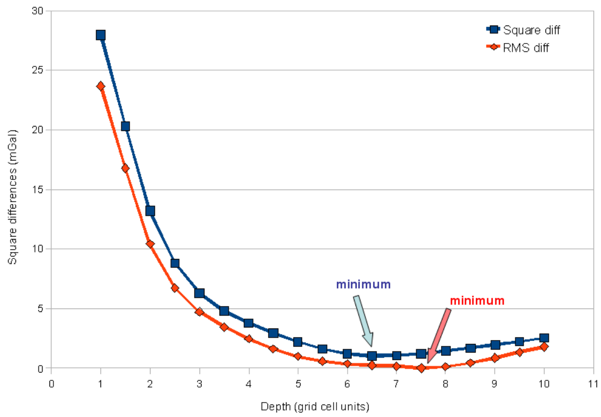

Figure 6 – The behavior of the square difference curve (blue) and the RMS error differences (red) showing great similarity. The minimum in the first is a good estimate of the quasi-optimal depth represented by the minimum at the later.

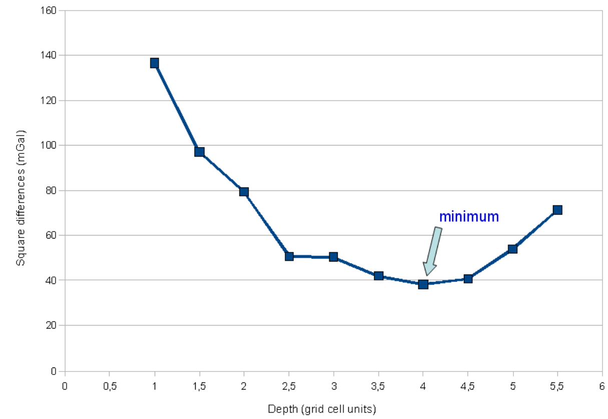

Figure 8 – The behavior of the square difference curve for the Reconcavo basin data set. According to the proposed method, the minimum in the curve (4 grid cells) represents the best estimate for the equivalent source depth.

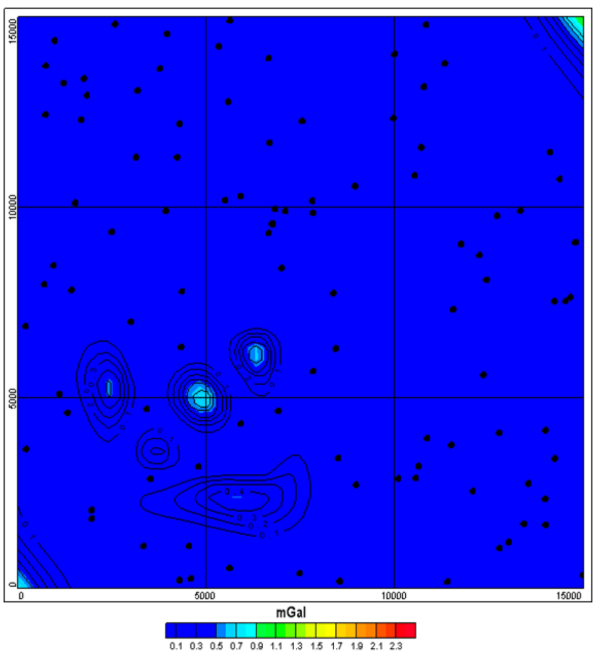

Figure 7 – The absolute difference between the estimated equivalent source grid (depth 6.5 grid cells) and the true field. About 88% of the differences are below 0.05 mGal. The maximum difference is 1.38 mGal.

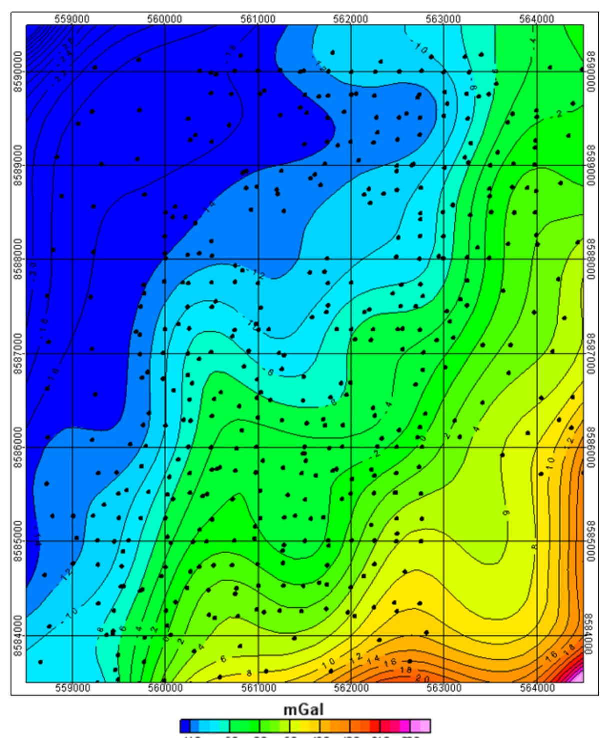

Figure 9 – The gravity field of the Recôncavo Basin area calculated with the equivalent source gridding technique. The black dots represent the position of the 472 stations that compose the original survey.