Approximate Sensitivities with Adjoint Fields for 2D Magnetotellurics

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Abstract

We present the sensitivity matrix with approximate adjoint fields which can be used in the process of inversion of magnetotelluric (MT). The sensitivity matrix (Jacobian) is one of the main bottlenecks in the process of inversion and its calculation is done by brute force which takes most of the time in the inversion process. The calculation of the approximate sensitivity matrix is performed using the theorem of reciprocity, where the adjoint fields are calculated with appropriate sources for the method that is working. We show that the results obtained with the approximate sensitivity matrix by adjoint fields is very similar to the sensitivity matrix calculated by brute force and with a much smaller computational time.

Introduction

Traditionally, MT inversion is performed using gradient methods, like the Gauss-Newton algorithm, where it is necessary to calculate sensitivity matrices. In those problems, the large computational time required to build those matrices is a serious drawback. Even with technological advances in recent decades, like the increase in the availability of computer memory and the popularization of parallel computing, the calculation of the sensitivity matrix in inversion problems is still a limiting factor for larger problems, with large sets of data and with a very big number of parameters to invert.

The alternative explored in this work is the adjoint state method, as detailed in Mc Gillivray et. al. (1994). By making use of Green's functions and reciprocity theory, those authors showed that the sensitivity matrix can be calculated using the electric fields generated by the actual source of the problem under study and those of an auxiliary adjoint source. The method has been used successfully in numerous inversion studies (Mitsuhata et al., 2002; Li et al., 2009; Sasaki, 2004; Ramananjaona et al., 2011; Abubakar et al., 2008; Han et al., 2008).

As a first step in a research work that will include the inversion of marine MT and CSEM data, we have applied the adjoint state method to build an approximate sensitivity matrix for the magnetotelluric fields in a two-dimensional model. By making the auxiliary adjoint fields to be those of electric dipoles over one-dimensional layered earth, we can build the sensitivity matrix much faster than by calculating Frechét derivatives, and with an excellent approximation to the true sensitivities.

Method

In the case of a block inversion of magnetotelluric data, where the parameter domain is discretized in a rectangular grid, the parameters are the resistivities of the grid cells. These parameters will build the models to generate the data to match the observations, which are apparent resistivities and phases of the components of the impedance tensor or the real and the imaginary parts of those components.

The most direct way to build the sensitivity matrix for any inversion process is through the calculation of Fréchet derivatives (Mc Gillivray, P.R. and Oldenburg, D.W., 1990). In the example shown here we evaluate these derivatives by calculating the whole observations vector twice for each parameter, varying the value of the parameter between 5% above and 5% below the current value in each iteration in the inversion process, and then taking the ratio of the corresponding differences. We will refer to this process as calculating sensitivities by brute force.

If an observation value is y_i and the parameters are the conductivities of each inversion cell σ_k, the derivatives are calculated thus:

$$\Delta\sigma_k = 0.05\sigma_k \tag{1}$$

$$\frac{\partial y_i(\sigma,\omega)}{\partial\sigma_k} = \frac{y_i(\sigma_{(\sigma_k+\Delta\sigma_k)},\omega) - y_i(\sigma_{(\sigma_k-\Delta\sigma_k)},\omega)}{2\Delta\sigma_k} \tag{2}$$

Calculating derivatives in this fashion is an extremely time consuming task, because we have to build and solve an entire 2D problem a great number of times: the number of frequencies times twice the number of parameters. For a 3D problem, the total time grows exponentially.

In order to avoid the burden of calculating Frechét derivatives, we can make use of auxiliary electric and magnetic sources and work with the relation (Gillivray et al., 1994):

$$\int_\Omega \left(\tilde{\mathbf{M}}_s \cdot \frac{\partial \mathbf{H}}{\partial\sigma_k} + \tilde{\mathbf{J}}_s \cdot \frac{\partial \mathbf{E}}{\partial\sigma_k} \right) dv = \int_\Omega \tilde{\mathbf{E}} \cdot \mathbf{E}\, \psi_k \, dv \tag{3}$$

Where, $\tilde{\mathbf{M}}_s$ and $\tilde{\mathbf{J}}_s$ are the magnetic and electric adjoint sources, respectively, and ψ_k are the basis functions used to define the sub-domains where the integrals are taken.

By defining the appropriate adjoint sources, we are able to calculate the sensitivities of the magnetic $\mathbf{H}$ and electric $\mathbf{E}$ fields. For example, to obtain the sensitivity for H_y at an observation location x_0, let $\tilde{\mathbf{M}}_s = \delta(x-x_0)\hat{y}$ and $\tilde{\mathbf{J}}_s = 0$. Then equation 3 becomes

$$\frac{\partial \mathbf{H}_y(x_0)}{\partial\sigma_k} = \int_\Omega \tilde{\mathbf{E}} \cdot \mathbf{E}\, \psi_k \, dv \tag{4}$$

$\tilde{\mathbf{E}}$ is defined as the electric field in the block due to a y-directed magnetic dipole with unit moment at the receiving location, and $\mathbf{E}$ is the electric field due to the plane-wave MT source. In the typical situation in which the model is composed of rectangular cells of uniform conductivity, in a block or cell inversion, we define ψ_k as 1 inside block k and 0 elsewhere.

For the two-dimensional magnetotelluric problem, the integral in equation 4 is changed to an area integral over each inversion cell in the model, by transforming the function $\tilde{\mathbf{E}}(x,y,z)$ into a suitable function of two variables $\tilde{\mathbf{E}}'(x,z)$ (Farquharson, 1996):

$$\frac{\partial \mathbf{H}_y(x_0)}{\partial \sigma_k} = \int_A \tilde{\mathbf{E}}' \cdot \mathbf{E}\, ds \qquad (5)$$

In the case of the electric field, we make $\tilde{\mathbf{J}}_s = \delta(x - x_0)\hat{x}$ and $\tilde{\mathbf{M}}_s = 0$. Then, the sensitivity for the component E_x at an observation location x_0 becomes:

$$\frac{\partial \mathbf{E}_x(x_0)}{\partial \sigma_k} = \int_A \tilde{\mathbf{E}}' \cdot \mathbf{E}\, ds \qquad (6)$$

Now $\tilde{\mathbf{E}}$ is the electric field in the block due to an x-directed unit electric dipole at the receiving location, and $\mathbf{E}$ is the electric field due to the plane-wave MT source.

The observations in the inversion of magnetotelluric data are usually the apparent resistivity and phase of the components of the impedance tensor. In the transverse magnetic mode (TM) in relation to the x direction, those are:

$$\rho_a = \frac{1}{\omega\mu}\left|\frac{E_x}{H_y}\right|^2 \qquad \phi = arctan\left(\frac{E_x}{H_y}\right) \qquad (7)$$

According to Farquharson (1996) the sensitivities of the apparent resistivity and phase can be expressed in terms of electric and magnetic fields as follows:

$$\frac{\partial \rho_a}{\partial \sigma_k} = 2\rho_a \left\{ \Re e\left(\frac{1}{E}\frac{\partial E}{\partial \sigma_k}\right) - \Re e\left(\frac{1}{H}\frac{\partial H}{\partial \sigma_k}\right) \right\} \qquad (8)$$

$$\frac{\partial \phi}{\partial \sigma_k} = \Im m\left(\frac{1}{E}\frac{\partial E}{\partial \sigma_k}\right) - \Im m\left(\frac{1}{H}\frac{\partial H}{\partial \sigma_k}\right) \qquad (9)$$

In this formulation, our adjoint fields are those generated by the dipole sources over the two-dimensional earth. However, the level of approximation required in the calculation of the sensitivities to be used in the inversion can be achieved by making the adjoint field sources over a 1D model, or even over a conductive half-space, which is the case in our example.

The 2D modeling program calculates the fields in every position required in one run for each frequency. If we call NF the number of frequencies, NX the number of positions in which the observations are measured and NP the number of parameters that composes the model, then in the calculation of the sensitivity matrix by brute force we have to run the complete 2D direct model program a number of times equal to 2 x NF x NP. On the other hand, if we use the adjoint fields, as described, in the calculation of the sensitivities we have to run a 1D code for the adjoint fields only NF x NO x NP and the 2D program only once for each frequency. Therefore, the adjoint state method takes a considerably shorter times to calculate the sensitivities.

Results

We consider the 2D model shown in figure 1. In an inversion process, we discretize the model with rectangular cells of uniform conductivity. In this example we will have only one observation: the value of the apparent resistivity, taken in the position marked with a triangle in figure 1, at a frequency of 0.02Hz. To illustrate the use of the method we calculate the sensitivities of this apparent resistivity with respect to the conductivity of each one of the 840 cells used in the discretization of the model.

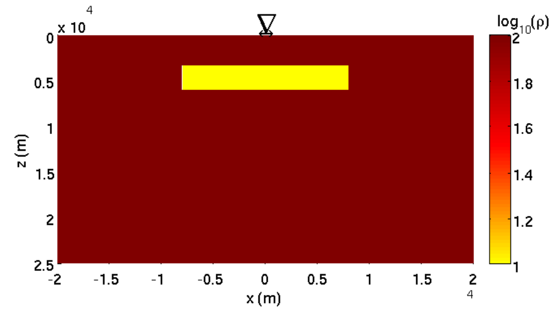

Figure 1: The two-dimensional resistivity model used here to illustrate the approximate sensitivities with adjoint fields.

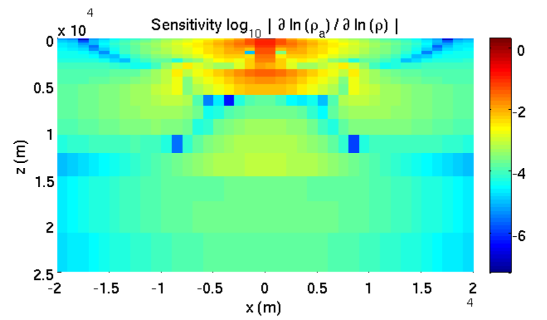

Figure 2: Brute-force sensitivities for the TM-mode apparent resistivity for the resistivity model shown in Fig. 1 with one observation made at z = 0 and x = 0.

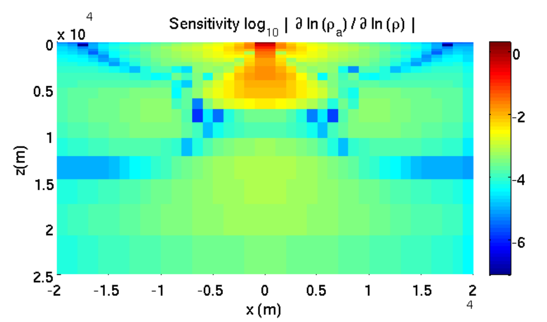

Figure 3: Approximate sensitivities for the TM-mode apparent resistivity for the resistivity model shown in Fig. 1 with one observation made at z = 0 and x = 0.

When we calculate the sensitivities using Frechét derivatives in our "brute force" approach, the results are those shown in figure 2. The sensitivities calculated with the adjoint fields are shown in figure 3.

While the computation of the sensitivities for the 840 parameters takes approximately 12.5 minutes by brute force on a laptop computer, the computation time with the adjoint fields is done in less than 1 second.

Summary and Conclusions

Our results show that in using the adjoint fields we can achieve an acceptable level of approximation to the sensitivities of the magnetotelluric observations, in a much shorter time than when we calculate Fréchet derivatives. These results are the first step in a research with the goal of performing the inversion of electromagnetic data. In our next step, we will use the adjoint sources over the same 2D model on which the actual fields are calculated and we will investigate what level of approximation we can achieve in that case.

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