Noise effects in the Bak-Sneppen model

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Abstract

In these work we explore the response to noise of a simple model critically auto-organized (the Bak-Sneppen SOC model) with the aim of learning the ways in which it is most effectively obtained a degradation of power-laws in distribution functions of event sizes. The basic model which essentially consists in a set of random barriers, that in this paper will be considered an abstract entity (although a pointer to earthquake is clear), systematically removes those barriers with higher absolute values, and theirs neighbors. At the stationary state avalanches of all sizes are observed. We have introduced two types of noise in simulations, a local one (just systematically and randomly changing a few barriers with a given frequency) and a global one (by slightly changing the values of all the barriers with a certain frequency). It is the aim of this work the search for methods allowing to trigger catastrophic events before the accumulated energy becomes enough to represent a challenge. We have obtained degradations of power-laws of several degrees. The possible connections of our simulations with real systems as well as the way in which an actual degradation of power-laws could be obtained are discussed at the end of the work. Future directions of research are also depicted.

Introduction

It is well-known the existence of many natural systems driven by stationary states characterized by self organized criticality (SOC). Contrary to what happens in the so-called normal systems, where exponential laws are the usual distributions for events sizes, in systems that present SOC the distributions for event sizes are described by power-laws [1-3]. This is the case, for example, for earthquakes, solar flares and magnetic storms. From the physics point of view, the presence of power-laws implies the possibility of the occurrence of large events with probability essentially different from zero. The consequences of these large events (for instance, in the earthquake case) can be catastrophic. Large efforts have been devoted to develop forecasting techniques for these catastrophic events. However, they are far of being considered successful and, at best, they predict the occurrence of large events in a probabilistic way which prevents the application of protective actions (by transferring populations at each alert, for example). Thus, it would be interesting to study the ways in which these power laws can be destroyed in such a way that catastrophic events never reach the threaten level for the human kind. On the other hand, the existence of power laws seems to be related to healthy states of the brain, for example, so, the inverse problem of finding ways to construct power laws from arbitrary distribution functions is also a challenging problem. We will concentrate in this letter in the first problem: the best way to destroy power laws or, in other words, to trigger catastrophic events harmless to human life.

This works, then, involves the proposal of triggering catastrophic events but in such a way that larger events are avoided by frequently starting small events.

The triggering of earthquakes is a long standing matter and has been studied from several points of view: by the triggering and synchronization of stick slips [4], through the cumulative Benioff strain-release [5], through the study of spatial and temporal distributions of earthquakes [6-9], through aftershocks series [10] and many others that include from meteorological to tide triggered earthquakes [11-19]. In the northern part of Brazil have been reported earthquakes supposedly triggered by wells and artificial lakes.

While all the previous works studied the possible natural ways of triggering earthquakes, the present work is the first attempt (to the best of our knowledge) to propose men voluntary actions of doing so.

The rest of the paper is organized as follows: first, we briefly present the model (it is the subject of many previous scientific publications and we will not extend to much on this point), later on we present the methods used to introduce noises and to study some dynamical characteristics of the model, followed by an exposition of our results and a discussion. Finally, we present our conclusions and some possible trends for future works.

The Model

We have simulated systems composed by connected elements that can accumulate energy coming from arbitrary sources. Once the energy in any of the elements has reached a given threshold, the element releases an arbitrary portion of its energy to the neighbors or out of the system. Some of the neighbors can eventually reach the threshold with this income and, consequently, release part of its energy to the neighbors or out of the system. In this way a single element can start a chain reaction (avalanche) that will stop only when all the elements present energies below the threshold. This is essentially a Bak-Sneppen (BS) model. The BS model is probably the simplest SOC model and was originally introduced to explain catastrophic mass extinctions during the Earth's history. It has found applications in several other research fields (evolution, the brain, and many others). Depending on the particular problem the model models, its parameters represent different physical quantities. However, even if we are essentially thinking in earthquakes, we will present it in a very general and abstract fashion and let particular applications for the future.

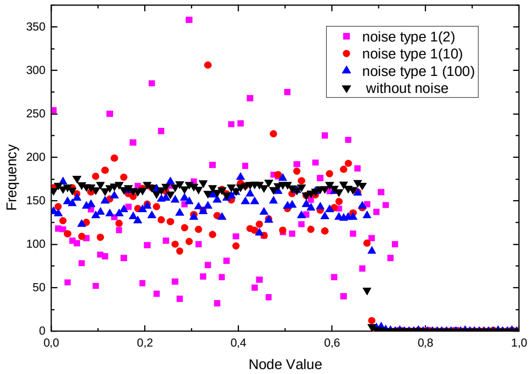

Figure 1.- Distribution of nodes values at the stationary state (black triangles) without noise. We present also the distribution for some type 1 noise levels (see inset).

The updating algorithm is composed by two relative simple steps: first, to detect the larger node of the system, and second, to substitute it by a new random value as well as the values of the neighbors. After many (10^6 to 10^8) updating steps the systems reach a stationary state characterized by a step-like distribution for the nodes values and a triangle distribution for the higher values (see Figures 1 and 2, black triangles). It is also characterized by power-laws in the distribution of avalanches (see reference [2] for a detailed description), first return times and distance between consecutive activities.

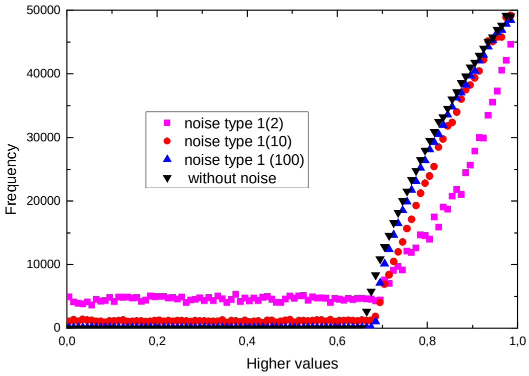

Figure 2.- Distribution of higher node values at the stationary state (black triangles) without noise. We present also the distribution for some type 1 noise levels (see inset).

The Method

To study the response of the systems to external noises we have introduced two types of them. The first, type 1, by breaking the selection rule (of selecting the highest barrier) at each n time steps (we have simulated the system for n = 1, 10 and 100) and selecting an arbitrary one to change its value and the values of the neighbors to new random numbers. The second, type 2, by changing in a small amount x all the barriers at each n time steps (we have simulated, once more, n = 1, 10 and 100, and x = 0.05) and then apply the ordinary selection/updating rule. Through this methodology we have analyzed the way in which the distribution function in the form of power-laws for avalanches, first return times and distance between consecutive activities (the ones which characterize the existence of a self-organized criticality) degrade. To illustrate, however, we have preferred to use the degradation of the stationary distributions in figures 1 and 2.

Power-laws are defined by the equation,

$$f(t) = c.t^d \tag{1}$$

where f (t) is the frequency distribution of t values, c is a proportionality constant and d is the exponent of the power-law. In the present model, as insinuated above, the distribution functions of avalanches, first return times and distance between consecutive activities follow power-laws, pointing to the existence of a critical self-organized state where activities of all sizes (avalanches) can be observed. The unique limitation for avalanche sizes is the sizes of the system.

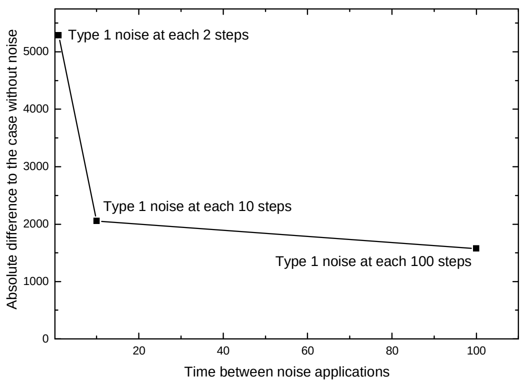

Figure 3.- Sum of the absolute value of the difference between equivalent bins for the distributions in Figure 1 (note that all the graph are histograms) taking as the common reference distribution the stationary one.

A second methodology involves the study of dynamical characteristics of the systems. In particular, we look obtaining the time elapsed to a system in the critically self-organized state evolve to a disordered one and the time elapsed to a disordered system attain the critically self-organized state. In more realistic models (and in real systems) this would serve to evaluate the time needed, under noise, for the system to abandon the critically self-organized state, i.e., when no more extremely destructive events should be expected.

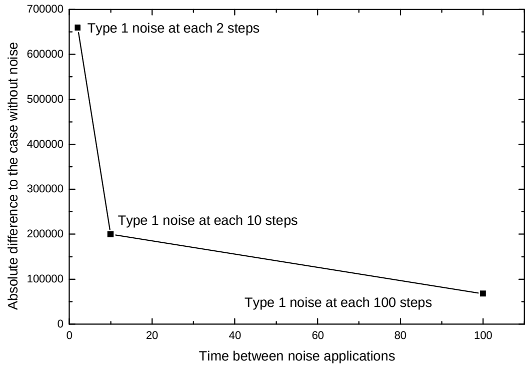

Figure 4.- Sum of the absolute value of the difference between equivalent bins for the distributions in Figure 2 (note that all the graph are histograms) taking as the common reference distribution the stationary one.

Results

Figure 3 presents the sum of the absolute value of the difference between equivalent bins for the distributions in Figure 2 taking as the common reference distribution the stationary one. The type 1 noise was applied at different rates (after one step, after 10 steps and after 100 steps each). The noise seems to more effective only at high rates.

Figure 4 presents the sum of the absolute value of the difference between equivalent bins for type 2 noise (a not shown graph, similar to Figure 2). The type 2 noise was applied at the same rates than the type 1. Here also, the noise seems to more effective only at high rates.

However, from the comparison between figures 3 and 4 it becomes apparently that the type 2 noise is more effective in destroying the SOC state than the type 1. The ratio for noise application rates 1/100 is around 2.5 for type 1 noise while is around 7 for type 2 noise.

It is worth to mention here that we have used the higher values distributions instead of the distribution of nodes because the former is much more behaved that the second one (implying in smaller errors).

Just for illustration we include in Figure 5 the distributions of distances between consecutive activities in 1D at different rates of type 1 noise applications. The power-law, one of the signatures of SOC in these systems is also gradually destroyed.

Finally, we present in Figure 6 the dependence of the higher values on time for a 300x300 system. We have done this type of graph for different size. The results for the time τ to the stationary state as a function of size are presented in Figure 7, which brings an estimative for the time the system needs to organize or disorganize itself. This could be an important factor in real cases.

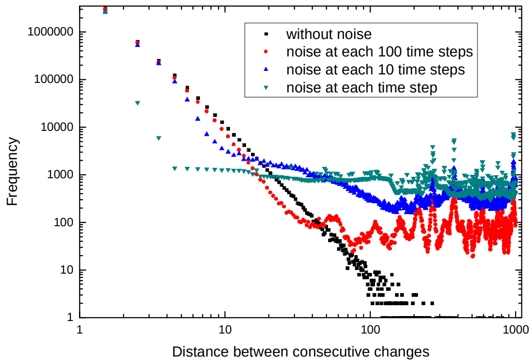

Figure 5.– Frequency distribution for Euclidean distance between consecutive changes in nodes. Note that the distribution goes to a plateau as the frequency in which the noise is applied increases, i.e., there is no longer a power law distribution, nor a critical state.

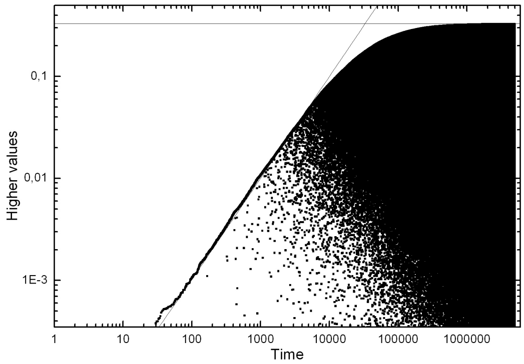

Figure 6.- Dependence of the higher values on time for a 300x300 system. From the interception of the power-law (note that is a log-log graph) with the plateau extrapolation, it is estimated the time to reach the stationary state for a given system size. The simulation begins at the completely disordered state.

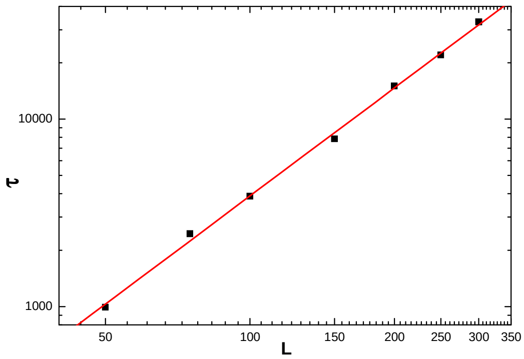

Figure 7.- Time to the plateau (see Figure 6) as a function of system size.

Conclusions

We have introduced two types of noise in a simple SOC model and observed that, upon sufficiently high rates of noise application the critical state is practically destroyed which is translated in the absence of large destructive events. We have also evaluated the time needed to effectively destroy the SOC state. It should not be expected with a simple model, like the one used by us to illustrate this facts, detailed results. However, some knowledge on general lines and methodologies to apply in more detailed models, or even in real cases, has been obtained. More realistic models should include other kinds of noises, for example, local noises simulated by locally changing the threshold values in some regions of the simulated systems, to simulate different geological characteristics along faults (in the case of earthquakes). Works along those lines are in course and their results will be published elsewhere.

Acknowledgments

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