Anti-dispersive acoustic seismic modeling

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Abstract

This work presents comparative results of two alternative methods to the conventional finite difference which is widely used for seismic modeling in exploration geophysics. The objective is to minimize the numerical dispersion problem while keeping the goal of not significantly increasing the computational cost.

The first method uses a nearly-analytic discrete operator to approximate the partial differential operators. The second one proposes to minimize the dispersion energy error in the wavenumber domain.

Based on these schemes, many strategies have been developed, and shown their superiority in suppressing numerical dispersion over conventional algorithms according to their authors.

Introduction

Important and promising steps for seismic imaging such as optimizing acquisition parameters, migration and full waveform inversion require the ability to simulate the wave field and its effects in the most accurate and computationally efficient manner. Having become a very popular method in exploration geophysics to perform seismic modeling, the classical finite difference method has simple implementation, flexibility and efficiency. On the other hand, this method often suffers from problems such as computational cost and numerical dispersion caused by the discretization of the wave equations, especially when models have strong velocity contrasts between adjacent layers or too few samples per wavelength are used. Usually, there are two ways to suppress the numerical dispersions: use high-order schemes or finer spatial grids. Unfortunately, these solutions may not be satisfactory in many cases as both approaches increase the computation cost.

This paper presents the results obtained with the reproduction of the methodology exposed in Li et al. (2011) - Optimal Nearly Analytic Discrete Method (ONADM), which describes an optimized method for the calculation of spatial derivatives present in the wave equation, with additional information from the field gradient. In addition the methodology seen in Bogey and Bailly (2004) was adapted to minimize the dispersion error energy in the wavenumber domain. The latter one showed the best results as can be seen in details as follows.

Methods

The following sets out the main ideas of the method ONADM for an acoustic 2D. Starting with the wave equation for the case cited:

$$\frac{\partial^2 u}{\partial t^2} = c_0^2(x,z)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right). \qquad 1$$

Where u = $u\ (x, z, t)$ represents the pressure field and $c_0\ (x, z)$ the medium velocity. We continue with the following notation which facilitates setting the gradient field and its representation with the discretization.

$$U = \left(u \quad \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial z}\right)^t \quad U(x_i, z_j, n\Delta t) = U_{i,j}^n, \qquad 2$$

Where Δt is the time increment. Expanding in Taylor series the vector U for time $(n+1)\Delta t$ and $(n-1)\Delta t$ and summing these equations leads to:

$$U_{i,j}^{n+1} = 2U_{i,j}^n - U_{i,j}^{n-1} + \frac{\Delta t^2}{2}\left(\frac{\partial^2 U}{\partial t^2}\right)_{i,j}^n + \frac{\Delta t^4}{12}\left(\frac{\partial^4 U}{\partial t^4}\right)_{i,j}^n, \qquad 3$$

where terms of order Δt^6 were not considered. At this point it would be possible to calculate the time derivatives in the expression (3) by an approximation by central finite differences, for example. However, this would be rather inefficient in terms of computational cost, as it requires a large storage capacity memory raising the cost of the process and to request more information from initial conditions. To overcome this difficulty, the time derivatives appearing in equation (3) are replaced by spatial derivatives using equation (1). This leads to the following equation.

$$U_{i,j}^{n+1} = 2U_{i,j}^{n} - U_{i,j}^{n-1} + (c_0\Delta t^2)\left[\left(\frac{\partial^2 U}{\partial x^2}\right)_{i,j}^{n} + \left(\frac{\partial^2 U}{\partial z^2}\right)_{i,j}^{n}\right] + \frac{(c_0\Delta t^2)}{12}\left[\left(\frac{\partial^4 U}{\partial x^4}\right)_{i,j}^{n} + 2\left(\frac{\partial^4 U}{\partial x^2 \partial z^2}\right)_{i,j}^{n} + \left(\frac{\partial^4 U}{\partial z^4}\right)_{i,j}^{n}\right] \quad 4$$

As it is not possible to use an infinite Taylor series expansion in search for computational precision, as an alternative to the truncation exposed in equation (3), Kondoh (1994) proposed the introduction of an interpolation function and the use of associative links between neighbors for each point in an attempt to recover some of the lost information on truncation. This interpolation function and its gradient, also allow obtaining expressions for the discretized high-order spatial derivatives present in equation (4). In this study we used the interpolation function,

$$G(\Delta x, \Delta z) = \sum_{r=0}^{M} \frac{1}{r!}\left(\Delta x \frac{\partial}{\partial x} + \Delta z \frac{\partial}{\partial z}\right)^r u \,. \quad 5$$

in which the sum was calculated until $r=5$, as in equation (4) derivatives are present up to the fifth order. The connection relations are obtained for each grid point with each of its eight neighbors as shown in Figure 1.

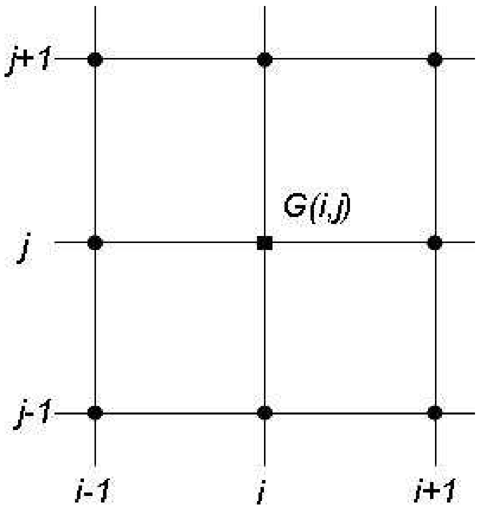

Figure 1: Neighbors of the point *i, j* used to generate the relations connecting

The other method presented here was proposed by Bogey and Bailly. Originally it describes an approximation to first order derivatives. However, the same idea can be used for higher order derivatives.

The spatial derivative can be approximated by a central, *2N+1* point stencil, finite-difference scheme as

$$\frac{\partial u}{\partial x}(x_0) = \frac{1}{\Delta x}\sum_{j=-N}^{N} a_j u(x_0 + j\Delta x)\,. \quad 6$$

Where Δx is the spacing of a uniform mesh, and the coefficients a_j are such as $a_j = -a_{-j}$.

Following Tam and Webb (1993), we apply spatial Fourier transform to equation 6

$$k^*\Delta x = 2\sum_{j=1}^{N} a_j \sin(jk\Delta x)\,. \quad 7$$

Where k* is label as the effective wavenumber. The dispersion error is the difference between the effective and the exact wavenumbers k^* and k. Finite difference schemes using 9, 11, 13 and higher points can be developed so that the dispersion error is small for a large range of wavenumbers up to $k\Delta x=\pi/2$. The coefficients a_j are defined to minimize the integral error

$$\int_{(k\Delta x)_l}^{(k\Delta x)_h} \left|k^*\Delta x - k\Delta x\right|^2 d(k\Delta x)\,. \quad 8$$

Results

In order to demonstrate the numerical dispersion present in the two alternative methods previously described and the conventional finite differences algorithms, we simulate the seismic wave field in a 2D homogeneous medium. An explosive Ricker source is located at the center of the computational domain. The modeling parameters are given in Table 1.

Velocity	1500 m/s
Cutoff frequency	30 Hz
Time step	0.001 ms
Grid point interval	20 m
Model dimensions X and Z	1024 x 1024 grid points

Table 1: Modeling parameters

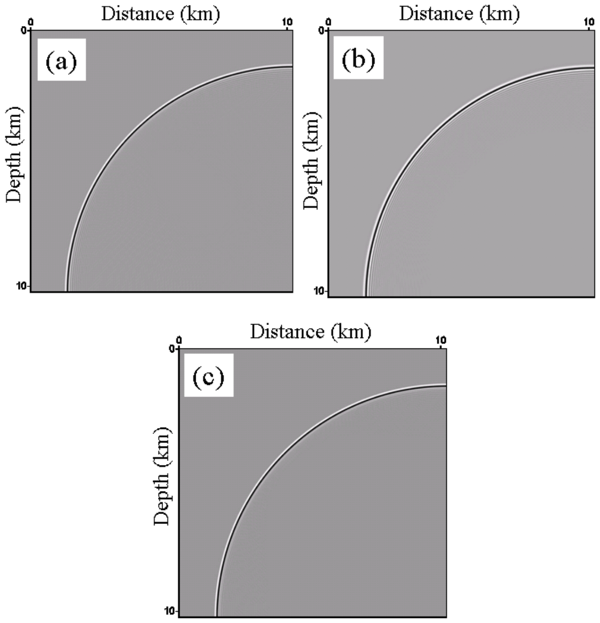

Figure 2: Snapshots obtained with the traditional finite differences method (a), the method described in Yang (2011) (b) and the method based on Bogey and Bailly (2004) (c) at t = 6 s.

Figure 2 (a), (b) and (c) show the wavefield snapshots at t=6 sec. For clarity, only one quarter of each snapshot is shown.

With the purpose of facilitating the comparison between the three methods, figure 3 depicts a horizontal profile in the same constant depth for all the snapshots present in figure 2. Each of these profiles is superimposed to a profile acquired through the method based on Bogey and Bailly (2004) with a grid point interval four times smaller then the one displayed in Table 1. In other words, the reference solution uses h=5 m.

Conclusions

A comparison between the conventional finite differences operator, to approximate the spatial derivatives existing in the wave equation, and two alternative methods are performed. The first alternative method uses a nearly-analytic discrete operator to approximate the partial differential operators and thus reduce the numerical dispersion issue. Although the results presented in the work of Yang (2011) show that this can be achieved to their conditions, when we use a model with lower velocity, which is more critical for the numerical dispersion, this method does not provide a good fit to the reference solution as it can be seen in figures 2 and 3.

The second method, on the other hand, attempts to minimize the dispersion error in the wavenumber domain, finding the best coefficients that approximate a partial derivative by a minimization process (equations 6 to 8).

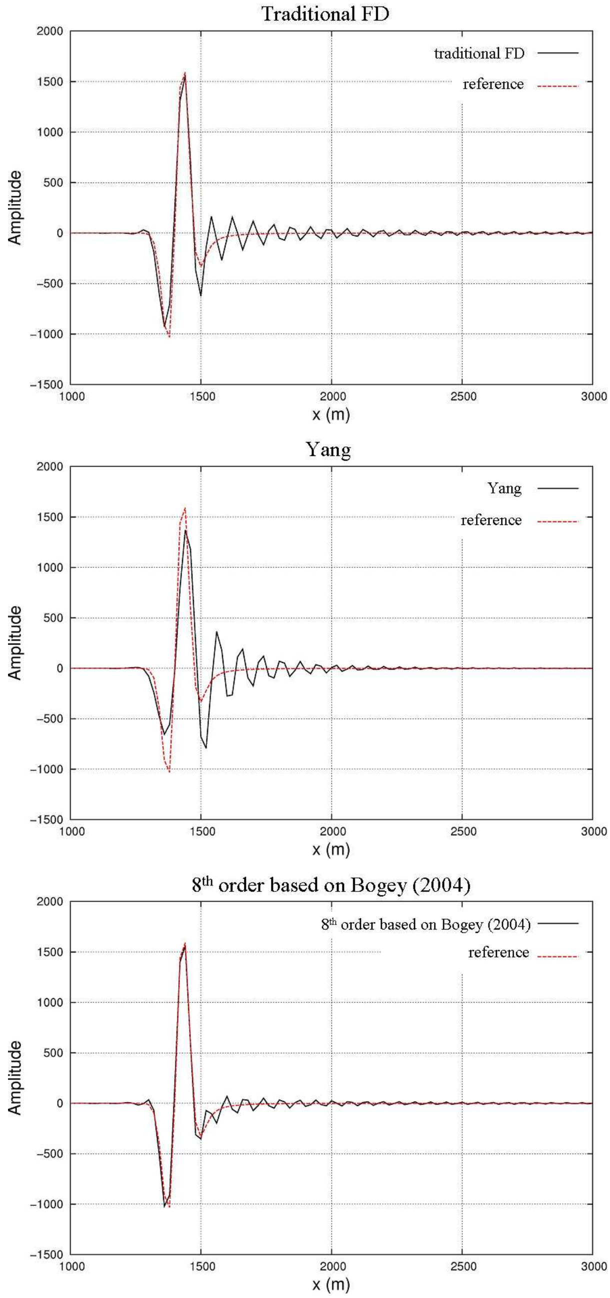

Figure 3: Horizontal profile of the snapshots exposed in figure 2 with the reference solution. From top to bottom, the traditional FD method, the method proposed by Yang (2011) and the method based on Bogey and Bailly (2004).

When compare to the reference solution, this approach shows the best fit as figures 2 and 3 demonstrate. Another advantage is the same computational cost as the traditional finite differences method because, in practice, their main difference lies in the optimized coefficients, obtained only once by the minimization described in equation 8.

Acknowledgments

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