Application of regularization to FWI in noisy data

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Abstract

The objective of this experiment is to evaluate the robustness of the FWI method to the presence of noise, and whether there is an increase to such robustness when regularization is inserted into the process. Three types of multiplicative regularization terms were applied to the objective functional (total variation, L_2 and weighted L_2) for data with and without noise.

Introduction

The ability to generate subsurface images from seismic data has been a cornerstone of oil and gas exploration for many years. The robustness of the seismic processing techniques involved in the creation of these images has improved greatly through these years, as the industry pushes for more geologically complex objectives.

One such technique that has gained momentum in recent years is the so called Full Waveform Inversion (FWI). Initially proposed by Tarantola (1984), FWI describes the seismic imaging method as a nonlinear inverse problem, where the goal is to create a property model that correctly fits its synthetic seismogram to real data, through the minimization of the difference of the two datasets.

However, this nonlinear approach to the seismic problem is ill-posed and frequently non-converging. Also, due to the computational costs involved in the problem, local minimization approaches must be employed, which add the chance of the problem converging to local minima. The non-uniqueness of the solution caused by the presence of local minima has led many authors to propose different implementations to FWI, the most common being the introduction of a regularizing term to the functional which is being minimized (Sun and Schuster, 1992; Zhou et al., 2002; Zhdanov, 2003; Abubakar et al., 2009).

Multiplicative regularization was proposed originally by van den Berg *et al.* (2003), as a way to introduce a priori information into the inversion scheme. From a geophysical point of view, such information could include layer continuity, dips and/or properties´ smoothness, amongst others.

In this work, we present three types of multiplicative regularization terms, as previously described by van den Berg et al. (2003) and Abubakar et al. (2009). The objective is to show how each of these terms affects the minimization of the objective function when noise is added to the data.

Methods

The variant of the FWI method used in this work is based on the solution of the acoustic wave equation on the frequency domain. Such formulation allows us to explore the multiscale approach to the inversion process in a more natural fashion.

To benchmark our results, we applied the process to a subset of the Marmousi data set. This version consists of a downsample of the original to a resolution of 122x382 samples, depth first. The result is then expanded by a frame 21 samples wide through constant extrapolation, to accommodate for the PML (Berenger, 1994) energy absorption method. Although this frame is part of the medium in the propagation processes, it is made exempt from model updating, i.e., the parameters it encloses will only update via this extrapolation.

The observed data was obtained by the very modeling process ingrained to the inversion, thus we incur in the original inversion sin (the first traditional one). Although this is not necessary, it helps to sort some problems out. Noisy datasets were obtained by addition of some noise from a normally distributed source to each frequency component (Figure 1). This addition, though, was modulated in frequency by the same source wavelet, so noise was less destructive to the higher frequencies. Each noisy dataset is associated to a different signal to noise ratio.

Each version of the velocity model is associated with an implicit kernel matrix, which encompasses the Helmholtz equation [This kernel matrix is the main computational concern of the algorithm, both in terms of storage and processing], and subsequently with a forward solution (modeling). Such association is enforced every time the parameter model updates. The functional used is the measure, in an L_2 sense, of the discrepancy between this modeled data and the original data.

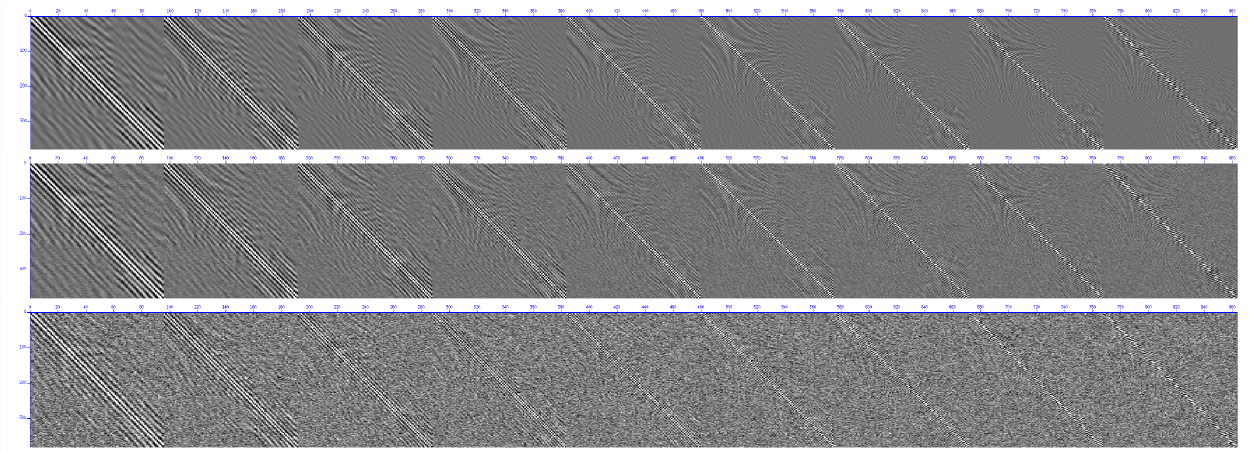

Figure 1: Input data set, source vs receiver vs frequency vs noise map. Each of the three bands denote a noise level (noiseless, 40dB and 10dB, from top to bottom. In each band are nine squares, one for each frequency (7,9,11,13,15,17,19,21,23 Hz, from left to right). Each square has all 96 sources (vertical axis) and 382 receivers (horizontal axis), and have been scaled by the inverse of the wavelet amplitude for that given frequency, for visual comparison purposes.

The smoothed model (Figure 3) was declared as the smoothing out of the original model (Figure 2) by a 15x15 2D moving average filter. The resulting input model for the inversion fairly resembles the output from a pre-processing step such as tomography.

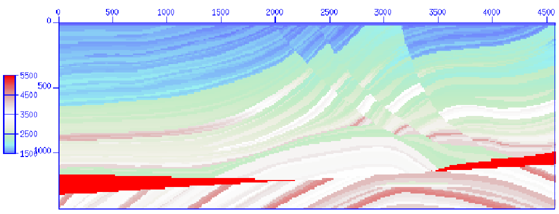

Figure 2: True velocity model (Marmousi).

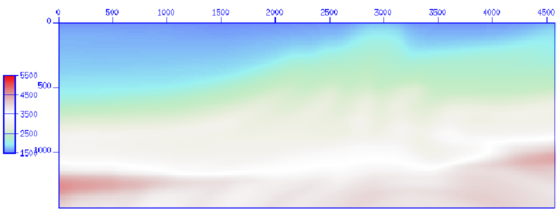

Figure 3: Smoothed velocity model.

Discussion

The effects of regularization can be seen in figure 4, which shows the inversion result for the various noise levels and regularization terms. These effects appear as a general softening of the contrast between the layers. The beneficial aspects become most apparent as noise increases on the data, as continuity is better preserved. From this we infer that regularization terms are unnecessary at low noise data. For a more detailed analysis of these results, we extract a trace at 3240m, which can be seen in figure 5.

Without regularization, the noiseless data enables an almost perfect matching result down to 220m, some wiggling around the true model down to 500m, followed by an increasing mismatch. The presence of a low level of noise does little to improve that, but as the SNR degrades, so does the result. The highest noise level provides a mismatch from top to bottom.

Grouping the results by noise levels (Figure 5), the noiseless data set gets mixed results, with better fit without regularization up to 1000m, but failing on the great velocity inversions far below. Regularization lessens this undesirable trend on that portion at the cost of smoothing results throughout.

In the presence of light noise, all regularizations keep the deep results bounded in the same range as the true profile (Figure 5). Without the regularization term, this is true at almost every velocity peak – though interestingly not at the troughs.

Heavier noise benefits even more from the regularization term, but none seems very efficient at the poorly illuminated depths.

Conclusion

Our FWI scheme was able to recover the model satisfactorily in all cases. The addition of noise, however, caused this fit to worsen as the depth increased.

When no noise was added, the inversion without any regularization terms achieved the same results as those where a multiplicative regularizer was applied. Considering the increased computational cost of these extra terms, we conclude that regularization was not necessary for our example in the absence of noise. However, the addition of noise rapidly degrades the

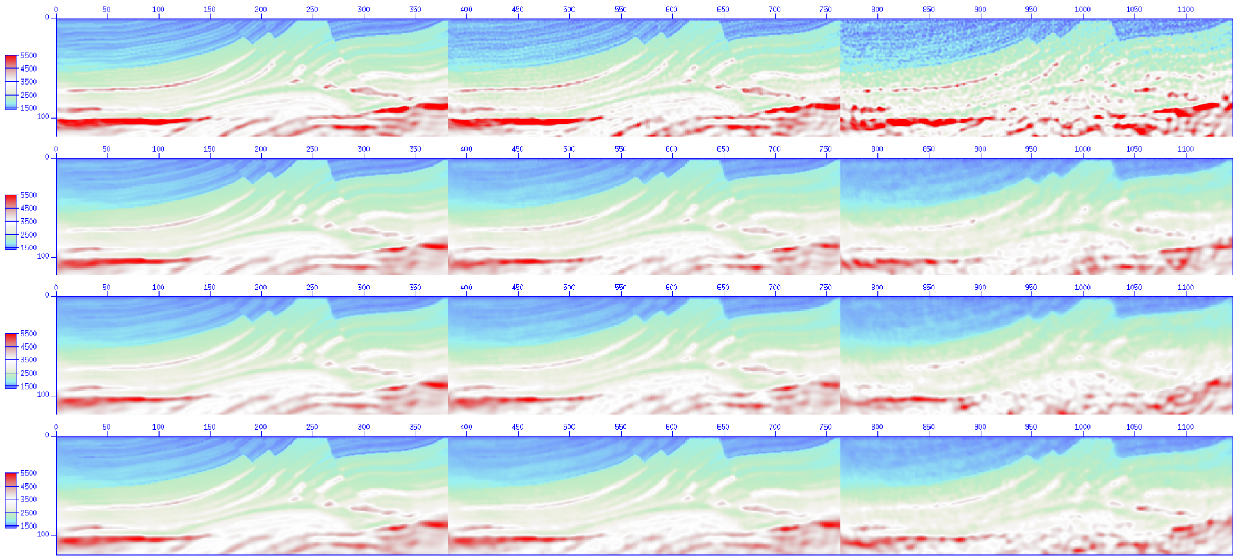

Figure 4: Final results after a single pass through all frequencies in a multiscale approach. Each of the four bands denote a different regularization formula (from top to bottom, no regularization, L_1 norm, L_2 norm and weighted L_2 norm). In each band, from left to right, the noiseless result followed by 2 decreasing signal-to-noise ratio, namely 80dB and 10dB, the same as the data shown on Figure 1. No inference is to be taken from such small pictures apart from the general notion of an earlier degradation on the topmost band. Coordinates are shown in grid points, where each point corresponds to 12 m.

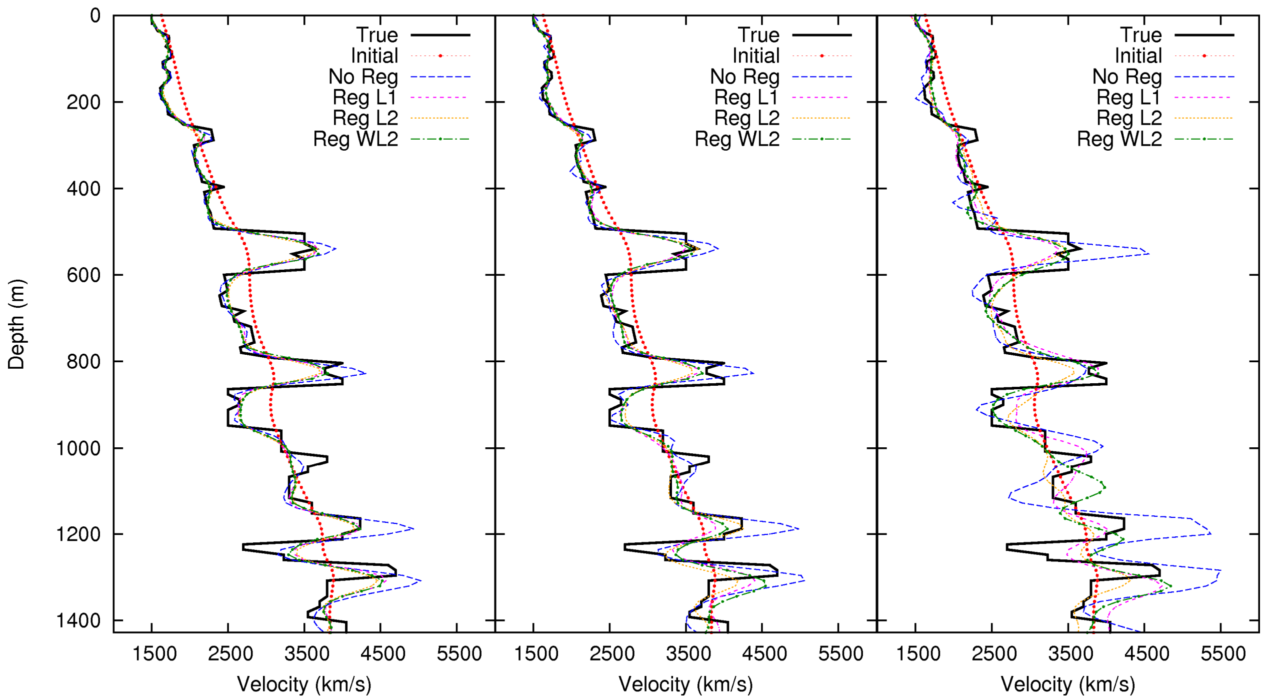

Figure 5: 1D traces extracted from the inverted images at x=3240m. Noiseless (left), low noise (center) and heavy noise (right) for all regularization implementations. Results are surimposed to the true model (black) and the initial smooth model (red).

solutions of the scheme without regularization. The errors related to this scheme are mostly due to amplitude mismatches of the velocity, while the kinematic part of the solution seems to fit the true model.

For the total variation (L_1) term, the kinematic part seems to fit the true model for noiseless and noisy data for all depths. However, the increase in noise affects the amplitude estimation of the L_1 term more strongly than in the other regularizers.

The L_2 regularization term seems to give the worse results amongst the regularizers when noise is added to the data. Both a depth misfit and an amplitude error are present for deeper layers, suggesting that the continuity imposed by the regularization is too dominant to properly handle noise.

The weighted L_2 gives the best results for the tested operators, especially in the case where strong noise is present. One possible explanation can be inferred by the governing equations (see Abubakar *et al.*, 2009). While the gradient for the L_2 regularization term tries to minimize the average error energy throughout the model, the weighted L_2 tries to minimize the error locally before averaging the error energy throughout the model. This behavior of the weighted L_2 norm resembles the one observed for the total variation scheme.

Acknowledgments

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