# Comparing Kirchhoff Prestack Depth Migration Using Paraxial Traveltime Approximation and Eikonal Equation

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## Abstract

**For obtaining the seismic image of the subsurface in complex geological media, it is necessary an accurate and efficient way to evaluate the seismic wave field. In nthe geophysical literature we can find several methods based on the numerical solution of the seismic wave equation. Because the flexibility to be applied to any seismic configuration and also on the solution of target oriented image problems, the Kirchhoff pre-stack depth migration continue to be investigated in order to obtain more efficient algorithms. As a critical point is the efficient calculation of traveltimes in complex media, that is necessary for stacking the amplitudes to be downward to the image points in the depth. In this paper we calculate the traveltimes by using the numerical solution of the Eikonal equation and by the paraxial traveltime approximation. The last alternative presents many advantages, e.g. less computational artifacts, better imaging performance and easily extended for including multiarrivals, anisotropy and Gaussian beam migration.**

## Introduction

For migrating seismic data using Kirchhoff depth migration based method, we need to know an accurate velocity model of the subsurface and have an efficient way to calculate traveltime tables. Keho and Beydoun (1988) showed that the Kirchhoff depth migration using paraxial traveltime approach is computationally efficient and accurate. Vidale (1990) and Van Trier (1991) presented a method to efficiently calculate first arrival traveltime based on the numerical solution of the Eikonal equation. Some critical problems arise when we have multiarrival situations, because the most part of the migration methods only take into account for the first arrival. Liu and Palacharla (2011) presented a new approach to include multiarrivals in the Kirchhoff migration based on the Gaussian beam migration algorithm (Hill, 2001). In both cases, i.e. the multiarrival Kirchhoff beam migration and the Gaussian beam migration, the traveltimes result of the second order paraxial approximations around the central rays.

In this paper, we compare two Kirchhoff migration results by using the Marmousi dataset in a common-offset configuration. The first using the Eikonal equation solver and the last using the paraxial traveltime approximation.

## Method

Kirchhoff Pre-stack Depth Migration

In the true-amplitude Kirchhoff pre-stack depth migration (TA K-PSDM) method, for a fixed image point $M$ and a surface Cartesian coordinate $\xi$, the traveltime $\tau_D(\xi, M)$ provides us with the Huygens surface, along which the seismic wave field $\hat{U}(\xi,\omega)$ is to be weighted summed, being expressed in frequency domain by the weighted diffraction stack integral (Schleicher et al., 2003 and Bleistein, 1987):

$$\hat{I}(M,\omega) = -\frac{i\omega}{2\pi}\iint_A d\xi w(M,\xi)\hat{U}(\xi,\omega)\exp[i\omega\tau_D(\xi,M)] \quad (1)$$

By using the standard zero-order ray theory approximation to represent the principal component seismic wave field, Schleicher et al. (1993) obtained the appropriate weighting function $w(M,\xi)$ by applying the stationary phase method (Bleistein, 1984) to the diffraction stack integral (1). Within the migration aperture all parameter vectors, $\xi \in A$, specify a source-receiver positions.

In this paper, our main interest is about the kinematic aspects and improvement of the seismic imaging. The weight function $w(M,\xi)$ is considered equals unity and the determination of a more efficient and accurate way for calculating the traveltime function $\tau_D(\xi, M)$ is here the target of our research.

We concentrate in the two most popular possibilities, i.e. the numerical solution of the Eikonal equation and the paraxial traveltime approach.

Eikonal equation solution

The Eikonal equation is a nonlinear first order partial differential equation for the traveltime of the ray field, which for isotropic medium is expressed by:

$$(\nabla\tau)^2 = \left(\frac{1}{v}\right)^2 \quad (2)$$

where $\tau = \tau(x,y,z)$ is the traveltime of the seismic ray field in the velocity model $v = v(x,y,z)$. For the application on the Kirchhoff migration the traveltime is to be calculated from points on the earth surface through the grid of image points in the depth. Vidale (1988) and Van Trier and Symes (1991) provide us with elegant and efficient methods to calculate the first arrival traveltime with potential to migration and tomography. In this paper we apply this solution for determining the traveltimes of the Huygens surface $\tau_D(\xi, M)$, along which the amplitudes of the seismic data is summed and the result is accumulated at the image point $M$.

Paraxial Traveltime Approximation

Alternatively, we can calculate the traveltime from a point on the earth surface to any image point in the depth, by using an extrapolation technique so-called paraxial traveltime approximation (Cerveny, 2001). It is given by an Taylor series expansion from a point of a central ray where the ray field is well determined by the dynamic ray tracing method. Thus, the traveltime at a point O with coodinates $x_o^i, i = 1,2,3$ of a central ray curve is projected to a point $M$ with coodinates $x^i, i = 1,2,3$ of the migration grid using a paraxial distance around the central ray. It can be mathematically expressed by (Cunha, 2005):

$$\tau(x^i) = \tau(x_o^i) + \frac{\partial \tau}{\partial x_o^i}(x^i - x_o^i) + \frac{1}{2}\frac{\partial^2 \tau}{\partial x_o^i \partial x_o^j}(x^i - x_o^i)(x^j - x_o^j) \quad (3)$$

Using the traveltime formula (3), we can extrapolate the traveltime to any desired point in subsurface, and by combining these traveltimes adequately to seismic configuration we can build the appropriate Huygens surface necessary to stack the amplitudes for the Kirchhoff depth migration. The traveltime extrapolation methodology can be found in Cunha (2009),

**Results**

For comparing the application of these two methods of traveltime calculation in seismic imaging, we use the marmousi data set (Versteeg, 1991). In Figures 1 and 2, respectively, we have the results of the application of the Kirchhoff pre-stack depth migration by using the numerical solution of the Eikonal equation (1) and of the paraxial approximation for calculating the traveltime tables. In Figures 3 and 4, we have the impulse response of the Kirchhoff migration operator applied to a seismic located at the central part of velocity model. In the whole migrated sections we can see the better performance of the Kirchhoff migration using the paraxial traveltime approximation. It is to emphasize the more concentrate energy of the impulse response of the paraxial traveltime than the Eikonal solution Kirchhoff migration, what yields for strongest reflection horizon in the migrated section in Figure (2).

**Conclusions**

By using the Marmousi dataset, we compared the Kirchhoff pre-stack depth migration using the numerical solution of the Eikonal equation and the paraxial ray approximation for calculating the traveltimes. The performance of the Kirchhoff migration using the paraxial traveltime approximation is better than the Eikonal solution in the whole migrated seismic section. The extension of the paraxial traveltime approach for including multiarrivals, anisotropy and Gaussian beam migration is straightforward.

**Acknowledgments**

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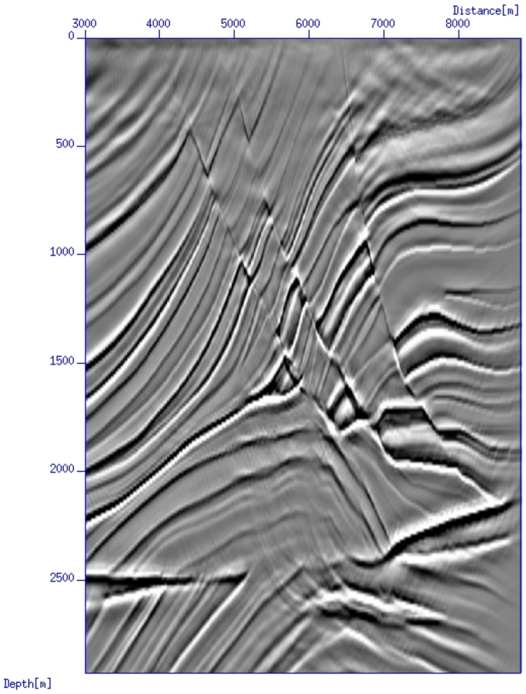


Figure 1: Kirchhoff pre-stack depth migration of Marmousi dataset using the Eikonal equation for traveltime calculation.

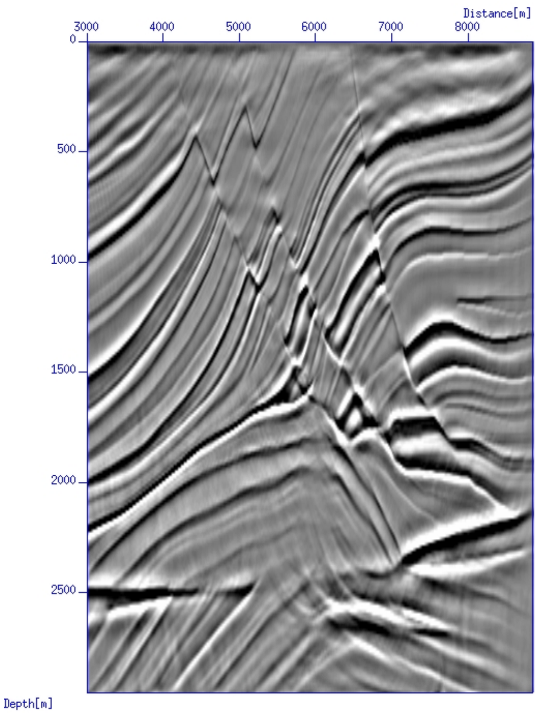


Figure 2: Kirchhoff pre-stack depth migration of Marmousi dataset using the paraxial traveltime approximation.

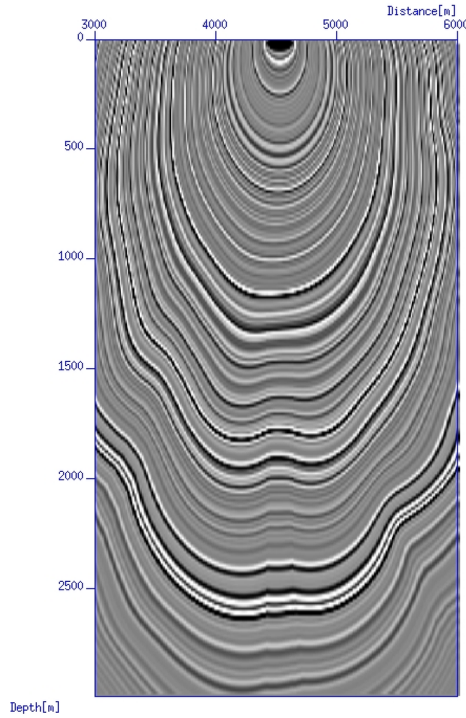


Figure 3: Impulse response of the Kirchhoff pre-stack depth migration using the Eikonal equation for traveltime calculation.

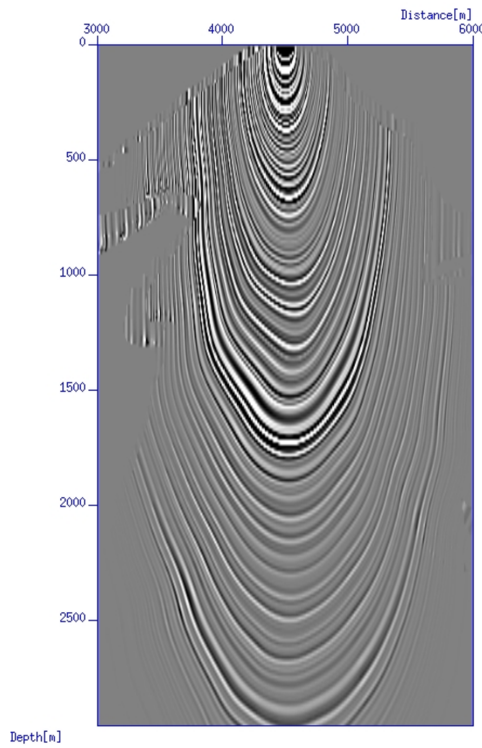


Figure 4: Impulse response of the Kirchhoff pre-stack depth migration using the paraxial traveltime approximation.