# Determination of the framework of homogeneous magnetic sources through Genetic Algorithm with elitism

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## Abstract

We present a method for inverting total-field anomaly for determining the framework of 3D magnetic sources such as: batholiths, dikes, sills, geological contacts, kimberlite and lamproite pipes. We use genetic algorithm with elitism to obtain magnetic sources' frameworks and their magnetic features simultaneously. Specifically, we estimate the magnetization direction (inclination and declination), the total dipole moment intensity, and the horizontal and vertical positions, in Cartesian coordinates, of a finite set of elementary magnetic dipoles. The spatial distribution of these magnetic dipoles makes up the skeletal outlines of 3D geologic sources.

## Introduction

Aeromagnetic and terrain surveys play an important role in the exploration of natural resources of economic interest, as well as in regional geologic mapping. The goal of magnetic prospecting is to infer, mainly through inversion, both the geometry and magnetization of the geologic structure that causes the observed magnetic anomalies. However, the solution of these inverse problems are non unique and some efforts are necessary to stabilize them.

In the last decades, several inversion algorithms were developed to interpret magnetic anomalies, assuming that the basic source body is an inductively magnetized prism with no remanent magnetization. Bhattacharyya (1980) proposed a least-squares semiautomatic method of inversion to find the magnetization distribution in a three-layer model constituted by contiguous prisms. To reduce ambiguity in the solution, this method allows fixing the depth of a particular prism.

Different methods to constrain solutions also exist. For example, Last and Kubik (1983) used the concept of minimum volume to find a compact and structurally simple solution. Guillen and Menichetti (1984) adopted the minimum moment of inertia with respect to the center of mass. Zeyen and Pous (1991) used Bayes's theorem to develop an algorithm that allows the inclusion of a priori information on model parameters. Their model consists of several right-rectangular prisms in which the unknown parameters are the depth to top and base, susceptibility, inclination, declination, and intensity of remanent magnetization. In the method of Li and Oldenburg (1996), the 3D region was discretized into a set of rectangular cells, each having constant susceptibility, and then the 3D distribution of susceptibility was inferred from inverting the magnetic data. Prior information can be incorporated into their objective function via a reference model and by 3D weighting functions that counteract the natural decay of the magnetic field with distance. Beiki and Pedersen (2011) describe a non-linear constrained inversion technique for 2D interpretation of high resolution magnetic field data along flight lines using a simple dike model. They first estimate the strike direction of a quasi 2D structure based on the eigenvector corresponding to the minimum eigenvalue of the pseudogravity gradient tensor derived from gridded, low-pass filtered magnetic field anomalies, assuming that the magnetization direction is known. Then the measured magnetic field can be transformed into the strike coordinate system and all magnetic dike parameters – horizontal position, depth to the top, dip angle, width and susceptibility contrast – can be estimated by non-linear least squares inversion of the high resolution magnetic field data along the flight lines.

Our method estimates the 3D Cartesian coordinates of a set of elementary magnetic dipoles whose spatial distribution forms the skeleton of a homogeneous magnetic source. The method also provides a single direction of magnetization (declination and inclination) and a rough estimate of the magnetic dipole moment intensity of the magnetic source. To this end, we set the problem of minimizing an objective function that favors fitting the observed and the predicted data. In addition, we impose the condition of proximity between nearest dipoles by the minimum spanning tree problem. We set a graph and obtain the minimum spanning tree that connects the whole set of magnetic dipoles. Synthetic tests are done to validate the minimum spanning tree regularization approach. This hard non-linear inverse problem is solved by the genetic algorithm with elitism (Goldberg 1989, Chakraborty and Chaudhuri, 2003).

### Inverse Problem

Let $\boldsymbol{b^o} = [b_1^0, b_2^0, ..., b_N^0]$ be a set of *N* total-field anomaly observations at $(x_i, y_i, z_i)$, *i=1,…,N*, produced by a 3D magnetic source with arbitrary geometry and uniform magnetization (Figure 1).

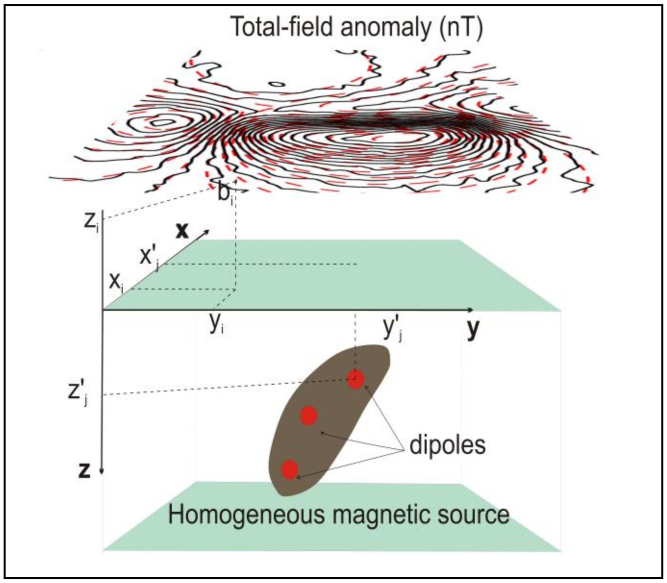


Figure 1: Methodology. The red circles are magnetic dipoles. $x_i$,$y_i$,$z_i$ are measurements points. $b_i$ is the ith total-field anomaly $x'_j$,$y'_j$,$z'_j$ are the *j*th dipole Cartesian coordinates. The upper plot represents the observed total-field anomaly (black contour lines) superposed by the predicted total-field anomaly (red contour lines).

To estimate the skeleton of this 3D source, we assume a set of *M* elementary magnetic dipoles, all of them with the same magnetization direction and the same magnetic dipole moment intensity. The *j*th dipole is located at$(x'_j, y'_j, z'_j)$ , j =1,...,*M*, as we can see in Figure 1. Given a set of predicted magnetic data $\boldsymbol{b^p}(\boldsymbol{q}) = [b_1^p, b_2^p, ..., b_N^p]$ , the problem of estimating the vertical and horizontal positions, the magnetic inclination and declination and the magnetic dipole moment intensity can be formulated as the minimization of the following objective function:

$$\varphi(\boldsymbol{q}) = \sum_{i=1}^{N} \left| b_i^o - b_i^p(\boldsymbol{q}) \right|^2 + \mu f_e(\boldsymbol{p}), \qquad (1)$$

where vector $\boldsymbol{q}$ can be expressed in partitioned form as: $\boldsymbol{q} = \binom{\boldsymbol{p}}{\boldsymbol{u}}$, $\boldsymbol{p} = (x'_1, y'_1, z'_1, \dots, x'_M, y'_M, z'_M)^T$ and $\boldsymbol{u} = (i, d, m_0)^T$. The superscript *T* stands for transposition. The first part of Equation 1 is the sum of squared differences between the observed data and the predicted data (Euclidean norm). As we already know, the majority of inverse problems are ill-posed, which means that the information contained in the data is not sufficient to estimate the parameters in a stable way. That's why we need *a priori* information to stabilize the inverse problem.

The second part of Equation 1, the equidistance function $f_e$(**p**), measures how the magnetic dipoles are not equidistant from each other. If all dipoles are at the same distance from its nearest neighbor, then $f_e$(**p**) is equal to zero. Otherwise, $f_e$(**p**) will be equal to the sum of squared differences between these distances. The function $f_e$(**p**) penalizes the fit between predicted data and observed data through the imposition of compactness to the *M* elementary dipoles. This imposition avoids that some dipoles deviate from the set. With this, we estimate a compact and homogeneous distribution of dipoles that better represent a compact and homogeneous 3D magnetic source.

In order to control the compromise between fitting the data (first part of the objective function) and the equidistance function (second part of the objective function), we introduce a real positive number, called the regularizing parameter *(μ).*

### *A priori* information: Minimum spanning tree regularization

Instead of using traditional Tikhonov regularization (Tikhonov and Arsenin, 1977) and Last and Kubic compactness regularization (Last and Kubic 1983), we calculate the equidistance function by the minimum spanning tree. Let *G(V,E)* be a non-oriented, acyclic and connected graph, where *V* is the set of dipoles (or vertices of the graph) and *E* is the set of possible interconnections (the edges) between the pairs of dipoles. Each interconnection has a weigh represented by the distance between nearest dipoles. So we want an acyclic subset of edges that connects all dipoles and whose weight is minimized. To achieve this goal, we implement the Kruskal's algorithm (Cormenet al., 2002). The Kruskal's algorithm is a greedy algorithm in graph theory that finds a minimum spanning tree for a connected weighted graph.

### Inversion tool: Genetic Algorithm with Elitism (EGA)

Genetic algorithms consist of a random search algorithm based on the mechanics of natural selection and natural genetics (Goldberg, 1989). Genetic algorithms are widely used in linear and nonlinear optimization problems due to its capacity of finding the global minima of multimodal functions (Smith and Ferguson, 2000).

Genetic algorithms require a set of initial estimates (i.e., an initial population). In general, the initial population can be randomly selected between a range values in the parameter space. In this methodology, we raffle both the vectors ***u*** and ***p*** with regular distribution inside boundaries values. The boundaries values are referred to as search limits. The search limits play a crucial role in the convergence of the genetic algorithms. If the search limits are too large, genetic algorithms will need an equally large number of iterations (i.e., generations) to converge and the computational cost can make the problem intractable. Otherwise, if the search limits are wrongly set up, genetic algorithms can converge to unwished minima.

The relevant stages in the implementation of the genetic algorithm, such as selection of parents, mutation and crossover are widely discussed in Goldberg (1989). We consider an extra stage in our implementation called elitism. At this stage, the fittest individuals of the current generation are replicated to the next generation. The elitism can be considered as a convergence accelerator, because it allows the appearance of a "super-man" at the last generations. Genetic algorithms with this strategy are referred as genetic algorithms with elitism or EGA (Chakraborty and Chaudhuri, 2003).

### Synthetic example

As primary results, we apply our methodology to a noise-corrupted synthetic magnetic data set produced by a 2 x 2 x 6 $km^3$ vertical prism, with magnetic inclination of 20° and magnetic declination of 45°. We simulate a geomagnetic field with magnetic inclination of -17° and magnetic declination of -20°. In this test, we use 10 magnetic dipoles. The EGA runs for 150 generations with a population of 30 individuals. Six fittest individuals are replicated at elitism. The search limits for the dipoles Cartesian coordinates are defined by total-field anomaly amplitudes. The search limits for the magnetic inclination are defined between 10º and 40º, while the search limits for magnetic declination are defined between 20° and 60°. The search limits for the dipole moment intensity are one magnitude order up and down from an average value, set to be 24 A.$m^2$. Figure 2A presents the spatial estimates of the dipoles using the EGA without the minimum spanning tree regularization (μ = 0).

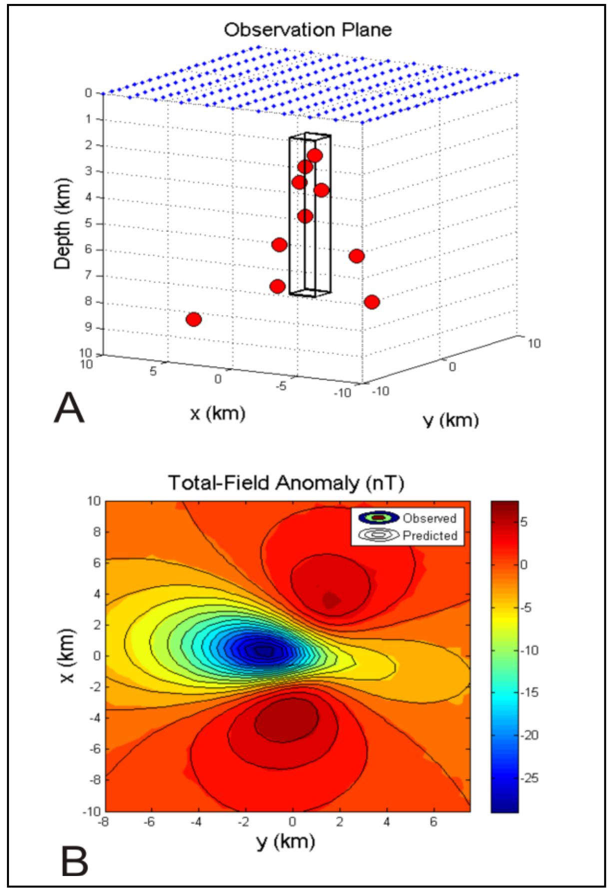


Figure 2: (A) Spatial estimates of dipoles (red circles) without minimum spanning tree regularization. The prism is the true homogeneous magnetic source. (B) Observed data (color map) and predicted data (black lines)

We can see in Figure 2A that the deeper dipoles deviated from the set of dipoles that fit the data. In Figure 2B, we validate the result by plotting the observed data (color map) and the predicted data (black lines on the color map) superposed. To better represent compact sources and rescue the deviated dipoles of Figure 2A, we repeat the same test applying the minimum spanning tree regularization. We set μ = 1000. In Figure 3A, we observe that the dipoles became more compact, what is expected to better represent the homogenous magnetized prism. In Figure 3B, we observe a good agreement between observed and predicted Total-field anomaly.

The Table 1 shows the estimates of the magnetic features for both tests. Despite small discrepancies, our EGA offers good estimates of the magnetic inclination and declination in both tests.

| Test | *i* | *i'* | *d* | *d'* | *mo* | *mo'* |
|---|---|---|---|---|---|---|
| Test1 (figure2A) | 20.0 | 21.2 | 45.0 | 43.7 | 24 | 27 |
| Test2 (figure3A) | 20.0 | 18.0 | 45.0 | 45.1 | 24 | 27 |

Table 1: (**i,*d***) are the true magnetic inclination and declination. (***i',d'***) are the predicted magnetic inclination and declination for both tests. ***mo*** is the true dipole moment intensity and ***mo'*** is the estimated dipole moment intensity for both tests.

Later on, we will test this promising methodology to other synthetic cases, validated by a good correlation between observed data and predicted data. After that, we will be able to apply our methodology to real data sets.

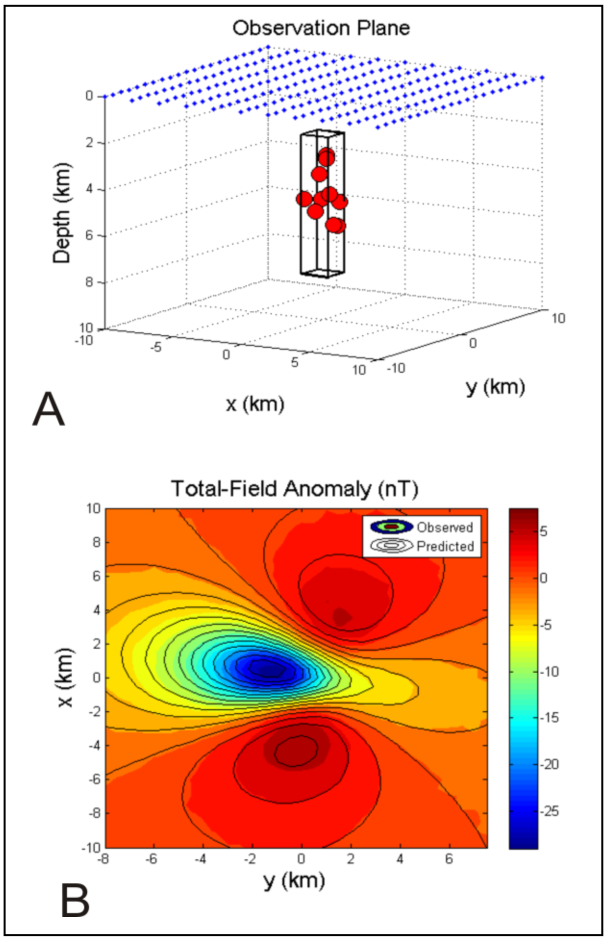


Figure 3: (A) Spatial estimates of dipoles (red circles) with some minimum spanning tree regularization. The prism is the true homogeneous magnetic source. (B) Observed data (color map) and predicted data (black lines)

**Conclusions and future work**

We have developed a method for estimating the Cartesian coordinates of a finite number of elementary magnetic dipoles that represent 3D homogeneous magnetic sources. In addition, we estimate a magnetization direction (inclination and declination) and a dipole moment intensity for all dipoles. Our inversion incorporates the minimum spanning tree regularization to impose compactness to the set of dipoles. Tests on synthetic data show good performance of the EGA in recovering the direction of the magnetization vector and the skeleton of the simulated magnetic source**.**

Later on, we will test this promising methodology to other synthetic cases, validated by a good correlation between observed data and predicted data. After that, we will be able to apply our methodology to real data sets.

Other future work will be the multi-objective optimization. With this, we will create a set of solutions without dealing with the regularizing parameter.

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