Efficient anisotropic wavefield extrapolation using effective isotropic models

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Abstract

Anisotropic wavefield extrapolation is more expensive than the isotropic case, and this is true especially of the anisotropy exhibits a tilt in the axis of the symmetry. Isotropic wavefield extrapolation is more efficient than the anisotropic extrapolation, and this is especially true when the anisotropy of the medium is tilted (from the vertical). We use the kinematics of the wavefield, appropriately represented in the high-frequency asymptotic approximation by the eikonal equation, to develop effective isotropic models, which are used to efficiently and approximately extrapolate anisotropic, poroelastic, and even elastic wavefields using the isotropic, relatively cheaper, operators. These effective velocity models are source dependent and tend to embed the anisotropy in the inhomogeneity. Though this isotropically generated wavefield theoretically shares the same kinematic behavior as that of the first arrival anisotropic wavefield, it also has the ability to include all the arrivals resulting from a complex wavefield propagation. In fact, the effective models reduce to the original, potentially complex, isotropic model in the limit of isotropy, and thus, the difference between the effective model and, for example, the vertical velocity depends on the strength of anisotropy. For reverse time migration (RTM), effective models are developed for the source and receiver fields by computing the traveltime for a plane wave source stretching along our source and receiver lines in a delayed shot migration implementation. Applications to the BP TTI model demonstrate the effectiveness of the approach.

Introduction

Despite all the complexities involved in extrapolating wavefields in anisotropic media, including shear wave artifacts and stability constraints, the improved imaging achieved by including anisotropy in areas like the Gulf of Mexico (Zhou et al., 2004; Huang et al., 2008) justify the effort. In fact, the cost of wavefield extrapolation in anisotropic media, especially if the anisotropy exhibits tilt, can be high (much higher than isotropic extrapolation). However, the cost of solving for the geometrical behavior of the wavefield, given by the traveltime, is orders lower than solving for the wavefield. Figure 1(b) shows a snap shot of the wave_eld for part of the BP TTI velocity model, given in Figure 1(a), overlain by the traveltime counter for the same snapshot time. For this source the traveltime field was computed using a finite-difference solution of the anisotropic eikonal equation, which admits the fastest arrival only. On the other hand, the wavefield was computed using the second-order in time fourth-order in space finite difference approximation of the acoustic anisotropic wave equation with a 20 Hz peak frequency source wavelet and a 0.8ms time sampling. For a 25m grid spacing, the computation of the wavefield solution took a 100 times more than the traveltime solution, which implies that traveltime computation is negligible compared to the cost of the wavefield extrapolation.

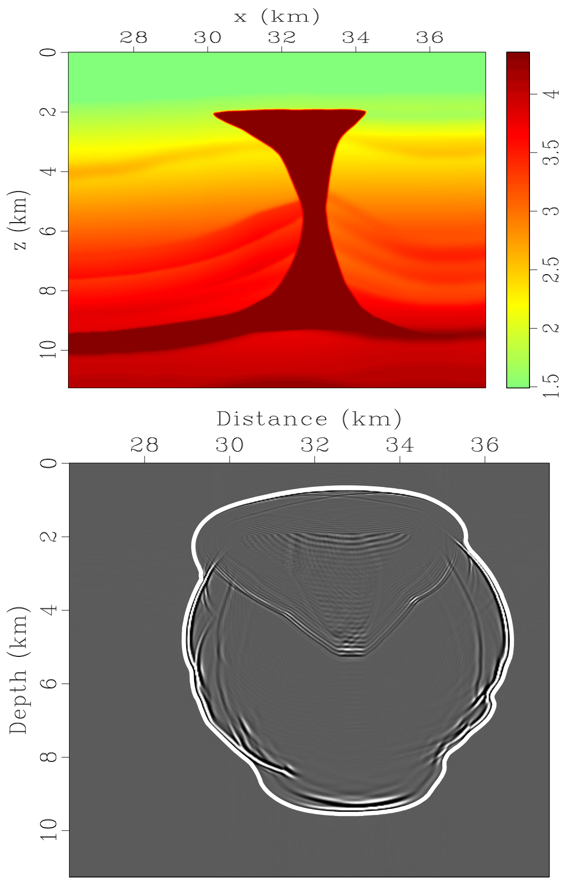

Figure 1: a) The BP TI tilt direction velocity model. b) A snap shot of the anisotropic wavefield at 1.28 seconds for a source located at position 32.75 km and depth 4 km. The anisotropic traveltime solution for the same model and source location is given by the white curve.

da Silva and Sava (2009) used the eikonal equation to compute an effective velocity model capable of approximating the kinematics of prestack wavefield. They used that effective velocity to build prestack migration isochrons using zero-offset operators. Likewise, in this abstract, we develop effective isotropic velocity models that approximate the anisotropic one by matching the kinematics of the two wavefields. The effective isotropic models are then used to extrapolate approximate anisotropic wavefields at a reduced cost. For RTM, we match the plane wave components of the phases, and thus use delayed shot migration with the plane-wave based isotropic models, which is now a function of the initial plane wave angle, to extrapolate both the source and recorded wavefields.

Method

Considering an anisotropic (elastic or acoustic) inhomogeneous three-dimensional model, the traveltime map can be extracted by solving the corresponding eikonal equation. Such an equation is given by a form of the Hamilton-Jacobi equation given by the following formula:

$$F(\mathbf{x}, \tau, \frac{\partial \tau}{\partial \mathbf{x}}) = 0\,, \qquad (1)$$

where **x** describes a location in the domain of investigation, **D**, □ is the traveltime, and $p_i = \frac{\partial \tau}{\partial x_i}$ are the components of the slowness vector. Setting Hamiltonian F to zero describes the geometry of surfaces or level sets representing the propagation of singularities (wavefronts) in solving the wave equation as an evolution in time. In its general form and in this method it also may represent the wavefront (stationary time) surfaces corresponding to an anisotropic/poroelastic or viscoelastic medium in any coordinate system with any initial/boundary surface or source. Solving Equation 1 for such level sets (traveltime surfaces) is accomplished using many of the available finite-difference methods or by extrapolating the solution along it's characteristics using ray-tracing methods. Using the isotropic eikonal equation, the effective velocity for a particular traveltime solution corresponding is computed as follows:

$$v_{eff}(\mathbf{x}) = 1/\left|\frac{\partial \tau}{\partial \mathbf{x}}\right|, \qquad (2)$$

where |.| describes to the Euclidean norm.

A similar approach can be used to obtain an effective shear wave velocity by matching the kinematics of shear waves of the isotropic model with the anisotropic model. The effective velocity can include the influence of anisotropy, density, poroelastic behavior, or/and the viscoelastic phenomena. In all cases, the effective P-wave or Shear wave velocity or both with or without density can be used in an isotropic acoustic/elastic wave equation to solve for an approximate wavefield as follows:

$$\Im[v_{eff}(\mathbf{x})]\mathbf{u}(\mathbf{x},t) = \mathbf{f}(\mathbf{x},t)\,,$$

where $\Im[v_{eff}]$ is the isotropic acoustic or elastic wave equation operator corresponding to a velocity or velocities given by the effective velocity/velocities (in the case of having shear waves). For acoustic media such an operator is given by

$$\Im[v_{eff}(\mathbf{x})] = \nabla^2 + \frac{1}{v_{eff}(\mathbf{x})}\frac{\partial^2}{\partial t^2}\,. \qquad (3)$$

Despite the kinematic only matching between both wavefields, the wavefield extrapolation will include most of the critical wavefield components, including the frequency dependency of wavefields and any caustics. For an isotropic acoustic media, the effective velocity reduces to the true velocity and the wavefield solution is that of the isotropic case.

Accuracy for the point source case

We test the approach by comparing the approximate solution to the exact one obtained using the more expensive anisotropic extrapolation. For the TTI BP model in Figure 1(a), along with the v, η, and θ fields, we solve for the traveltime for a source located at depth 4 km and lateral position 32.75 km using a fast marching method applied to the finite difference approximation of the acoustic eikonal equation for TI media, given by equation 1.

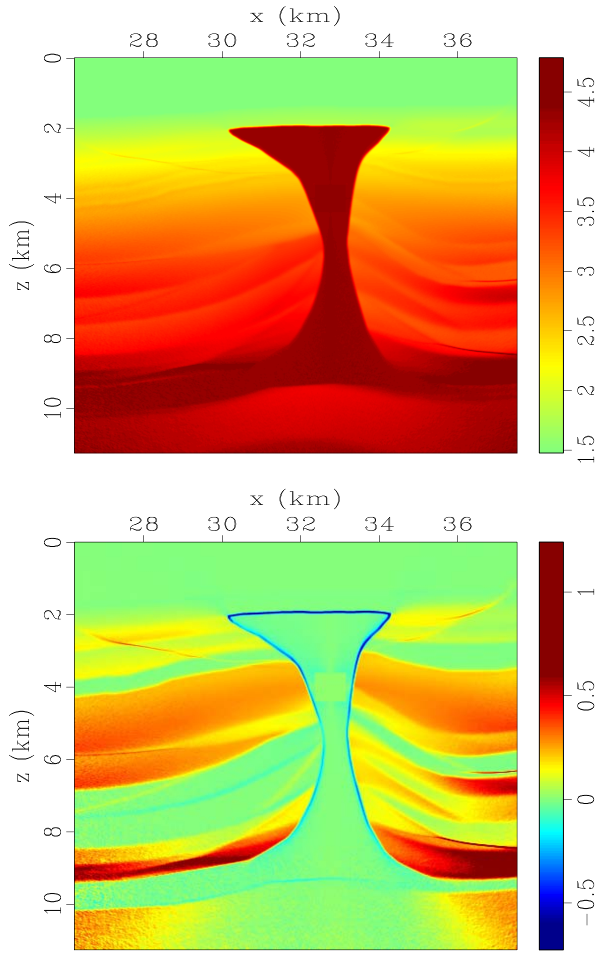

Figure 2: a) The effective isotropic velocity obtained from the TI traveltime using equation 2. b) The difference between the original tilt direction velocity model, shown in Figure 1(a), and the effective velocity model.

The resulting traveltime is used to compute an effective velocity based on equation 2, and we display that effective velocity in Figure 2(a). The difference between the tilt direction velocity for the BP model (Figure 1(a)) and the computed effective velocity is shown in Figure 2(b). This difference is related to the anisotropy and it is source location dependent. Using the effective velocity we solve the acoustic isotropic wave, equation 3, using a finite difference approach for the same source location and obtain the snap shot of the wavefield shown in Figure 3(a). The difference between the effective based wavefield and the anisotropic one (Figure 1(b)) at 1.28s, plotted at the same scale, is shown in Figure 3(b).

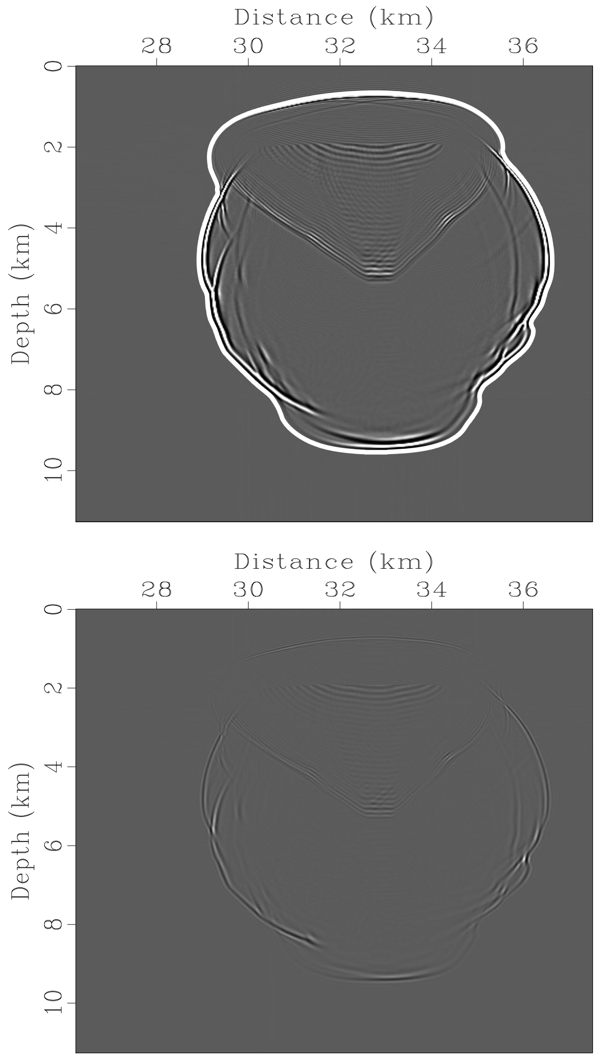

Figure 3: a) A snap shot of the approximated anisotropic wavefield at 1.28 seconds using the isotropic wave equation 3 and the effective velocity model shown in Figure 2(a). b) The difference between the anisotropic computed wavefield, shown in Figure 1(b), and the isotropically calculated one in (a), plotted at the same scale.

The difference considering we are using the cheaper isotropic acoustic wave equation is relatively small. The cost of computing the wavefield shown in Figure 3(a) is half of that in Figure 1(b) and for 3D the difference should be at least four fold.

Reverse time migration

In reverse time migration, the receiver wavefield is extrapolated by reversely propagating the recorded data at the surface. To use an effective isotropic model, the only point source representation of the recorded data at the surface is given by the mirror image of the corresponding source with respect to the image point. In this case, we will need an effective velocity model for each image point, which will require also a unique extrapolation operation per image point, which will be prohibitive. To alleviate this problem, we use a delayed shot migration implementation (Zhang et al., 2005; Vigh and Starr, 2008). Following Zhang et al. (2005), delayed shot migration is based on modeling all the sources together with a delayed time given by the gradient $\partial \tau / \partial x$, with no delay at the origin x = 0. The same delay $\partial \tau / \partial x$is used to sum the receive wavefields for all the shots prior to reversely extrapolating them. Finally, we apply the conventional zero-lag crosscorrelation imaging condition at each subsurface point to extract the image. For example, $\partial \tau / \partial x$= 0 implies starting the sources together and summing the receiver wavefields without delays. We finally sum the images from the different $\partial \tau / \partial x$RTMs. The number of $\partial \tau / \partial x$needed to obtain an accurate image is usually less than the number of sources (Etgen, 2005), which provides for some speed up. In this case, for delayed-shot migration, the effective model is calculated in the same way described earlier, but for a plane wave source initial condition. Thus, we will have an effective velocity model for each plane wave slope ($\partial \tau / \partial x$). Since the source and receiver wavefields conventionally share the same recording surface, the effective models per plane wave angle applies to both sources and receivers. Thus, initial time is given at the surface has a lateral gradient, $\partial \tau / \partial x$capable of producing the desired plane wave. The traveltime is then computed by solving the TI eikonal equation 1 for each plane wave slope. Luckily, plane waves admit less first-order traveltime errors than spherical waves due to the reduced curvature (Alkhalifah and Fomel, 2001). The gradients of the traveltime solution for each plane wave are then used to compute the effective velocity corresponding to that plane wave, v_{eff} $(x; y; z; p)$, where p corresponds to the initial slope $\partial \tau / \partial x$at the surface used in computing the anisotropic traveltime. These effective velocities are then used to reversely extrapolate the recorded data in a delayed shot migration implementation.

Conclusions

Representing the anisotropic model by effective isotropic models by fitting the geometrical features of the first traveltime arrivals corresponding to the two models for a given source allows us to use isotropic (cheaper) wavefield extrapolation methods to compute approximate anisotropic wavefields. Using isotropic extrapolation operators, the cost of wavefield extrapolation is reduced by as much as five fold for the case of 3-D tilted symmetry axis anisotropy. For reverse time migration, we reformulate the effective models by matching the kinematics of plane waves spanning the receiver surface in a delayed shot migration implementation. Application to the BP TTI model and data set demonstrates the accuracy of the approach.

Acknowledgments

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