Estimation of the total magnetization direction using Helbig's method: A comparative study using the Reduction to the Pole operator

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Abstract

This paper presents the first steps of the study about the use of Helbig's Integrals to estimate the total magnetization direction and a comparative study using the Reduction to the Pole operator.

The Reduction to the Pole (RTP) operator in the wavenumber domain was implemented as a Matlab function, allowing the user to specify both the reference field and the total magnetization directions.

After that a Matlab function was implemented for the evaluation of the cartesian components of the anomalous field from Total Intensity Magnetic Anomalies by the use of Fourier Filtering. This function is a necessary preliminary step to calculate the Helbig's Integrals.

Our implementation was validated by comparison with results obtained using the software Oasis Montaj v.7.0.1.

The RTP computed by our implementation is similar to the one computed by Oasis Montaj, except that Oasis Montaj shows a short wavelength ripple which seems to be a processing artifact.

A similar short wavelength ripple was observed in the cartesian components of the anomalous field computed by Oasis Montaj. Apart from this, our implementation gives almost identical results.

Therefore, the algorithms showed reliable stability. In this way, we are ready to continue the project and implement the Helbig's Integrals to calculate the total magnetization direction.

Introduction

The total magnetization plays a very important role in the geophysical interpretation. In this paper the Reduction to the Pole (RTP) operator was used as an example of its application.

The successful application of RTP requires the knowledge of the total magnetization direction.

Brazil is located under the South Atlantic Anomaly (SAA), which implies in low intensity values of the induced magnetization. For this reason, the knowledge of the remanent magnetization is fundamental for a reasonable geophysical interpretation, as shown at Shukowsky and Cordani (2009).

As is well known, the total magnetization is a vector sum of the remanent magnetization and the induced magnetization, but the induced contribution can be removed using the method proposed by Cordani and Shukowsky (2009), *VPMA – Virtual Pole from Magnetic Anomaly.*

In other words, if one wants to obtain reliable information about the magnetic anomalies sources such as the depth to the top and the lateral extent (Louro, 2013) or even indirectly estimate the age of cristalization (Cordani and Shukowsky, 2009), it is important to know with good precision the total magnetization direction.

Thus, this paper shows the first-steps in order to implement the Helbig's method to compare with the total magnetization direction provided by the MaxiMin technique (Fedi et al. 1994) and its consequence to the RTP operator.

Therefore, aiming a better analysis of the results achieved by our algorithms, we performed tests using synthetic anomalies in each stage of the project.

Reduction to the Pole

A magnetic source with positive susceptibility contrast located close to the magnetic equator produces, in general, an anomaly formed by a minimum flanked by two positive lobes (Telford et al. 1990).

However, the anomaly morphology depends not only on the body's geographic location, but also on the presence or lack of remanent magnetization.

It is known that as higher the magnetic inclination module of the reference field (International Geomagnetic Reference Field - IGRF) and the magnetization of the body, more pronounced should be the positive lobe and more attenuated should be the negative lobe.

Then, if both magnetization directions (induced and remanent) were +90° or -90°, the positive lobe must be centralized above the source and the negative lobes should be strongly attenuated.

The Reduction to the Pole operator will transform a generic anomaly into the one as should be seen from the magnetic pole with an inclination of +90° or -90° for the total magnetic field (the vector sum of the induced and remanent magnetization).

The equation below represents the operator calculated in the wavenumber domain, (Blakely, 1995).

$$P_F(u,v) = \frac{(u^2+v^2)}{(iu\alpha_1 + iu\alpha_2 + \sqrt{u^2+v^2}\,\alpha_3)\,(iu\beta_1 + iu\beta_2 + \sqrt{u^2+v^2}\,\beta_3)} \quad (1)$$

And,

$$\begin{aligned} \alpha_1 &= \cos I_M \cos D_M & \beta_1 &= \cos I_R \cos D_R \\ \alpha_2 &= \cos I_M \operatorname{sen} D_M & \beta_2 &= \cos I_R \operatorname{sen} D_R \\ \alpha_3 &= \operatorname{sen} I_M & \beta_3 &= \operatorname{sen} I_R \end{aligned} \quad (2)$$

The parameters, I_R and D_R are relative to reference field direction and the I_M and D_M are relative to total magnetization direction.

Helbig`s Theory

Klaus Helbig (1963) developed integral relations to estimate the direction of total magnetization of an anomaly magnetic source.

According to Phillips (2005), given the magnetic field components (ΔX, ΔY, ΔZ) produced by magnetic source, the following integrals are used to calculate the total magnetization of the body:

$$
\begin{aligned}
I_6 &= -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,\Delta X\,dx\,dy = m_z\\
I_7 &= -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\,\Delta Y\,dx\,dy = m_z\\
I_8 &= -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,\Delta Z\,dx\,dy = m_x\\
I_9 &= -\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y\,\Delta Z\,dx\,dy = m_y
\end{aligned}
\quad (3)
$$

And the module |m|, the inclination i_m and the declination d_m of the total magnetization are given by:

$$|m| = \sqrt{m_x^2 + m_y^2 + m_z^2} = \sqrt{I_8^2 + I_9^2 + (I_6 + I_7)^2/4} \quad (4)$$

$$i_m = \arcsin\left[\frac{m_z}{|m|}\right] = \arcsin\left[\frac{(I_6 + I_7)}{2|m|}\right] \quad (5)$$

$$
d_m = \begin{cases}
\arctan\left[\frac{m_y}{m_x}\right] = \arctan\left[\frac{I_9}{I_8}\right] & ; m_x \geq 0\\
\arctan\left[\frac{m_y}{m_x}\right] + 180^\circ & ; m_x < 0, m_y \geq 0\\
\arctan\left[\frac{m_y}{m_x}\right] - 180^\circ & ; m_x < 0, m_y < 0
\end{cases}
\quad (6)
$$

Fedi et al. 1994 - MaxiMin

The MaxiMin technique is completely based on the Reduction to the Pole (RTP) operator described by Baranov (1957). As shown in the equations (1) and (2), it is necessary to provide the total magnetization direction and the reference field direction to the RTP.

On the other hand, this algorithm allows to estimate the total magnetization direction using only the reference field direction information.

It is known how must be a well reduced to the pole anomaly. That "ideal" anomaly reduced has the maximum attenuation of the negative lobe. So, the algorithm provides many pairs of values of the total magnetization direction and evaluates the RTP operator for each one.

After that, it assigns the value of the negative lobe to a g(x) function and then this function is maximized. The pair of values of total magnetization direction that maximizes the g(x) function, it is the best estimation obtained by the MaxiMin technique.

Implementation and Results

- **Reduction to the Pole**

The RTP operator available by the Oasis Montaj and the MaxiMin technique do not allow the user to input the total magnetization direction. For this reason, the first implementation done was the calculation of the Reduction to the Pole operator based on the equation (1).

We choose to use the GXF extension for the grid from Oasis Montaj because it is accepted in others softwares like Surfer and Model Vision, better explained by Grid Exchange File (1999).

Nevertheless, we still have the same problems with the instability of the RTP filter when applied near the magnetic equator. This instability is largely discussed in the literature such as Mendonça (1993) who proposes a truncated stable operator to attenuate this problem.

Therefore, were used synthetic anomalies to study the behavior of the implemented algorithm, and the software Oasis Montaj v.7.0.1.

The first synthetic anomaly used, figures 1 and 2, was a Pipe with only induced magnetization, -29.71° of inclination and -20.58° of declination. For that reason, it is expected that either Oasis Montaj or our algorithm must be able to compute the correct Reduction to the Pole.

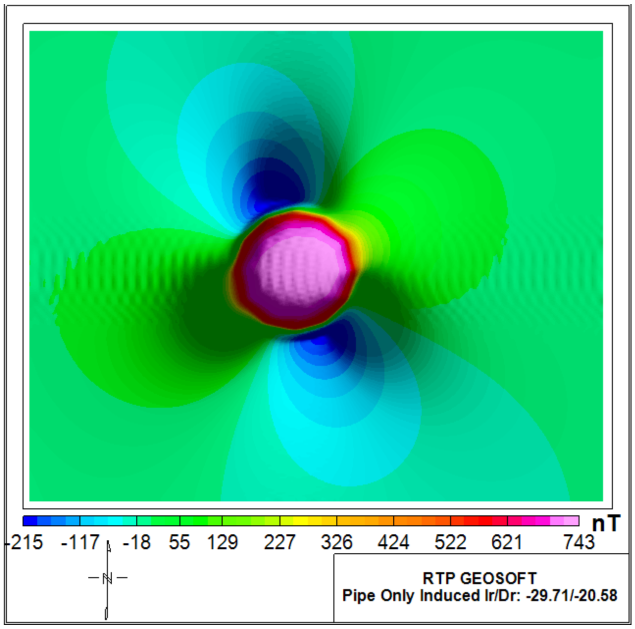

Fig. 1 - Synthetic anomaly reduced to the pole by Oasis Montaj.

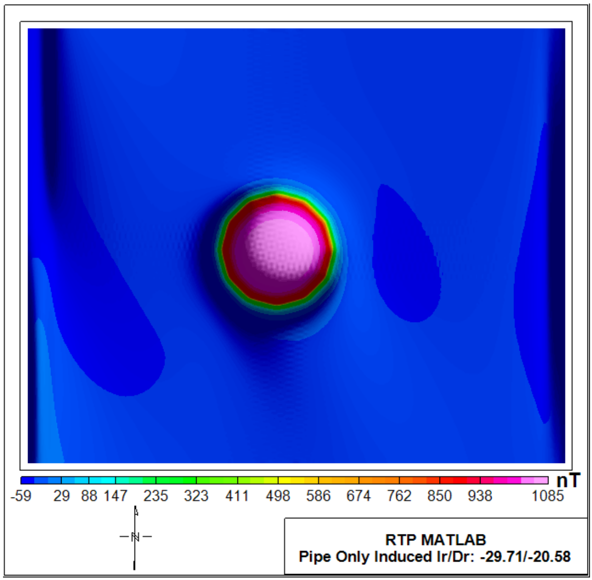

Fig. 2 – Synthetic anomaly reduced to the pole by our MATLAB function.

The two attenuated lobes shown in figure 1 can be correlated with a parameter used by Oasis Montaj (1972) to correct the values of the amplitude. And it is also observed a short wavelength ripple in the W-E direction.

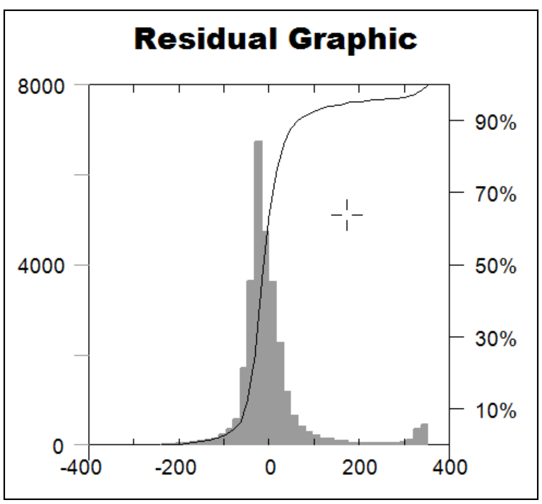

Fig. 3 – Residual graphic from the subtraction of figures 1 and 2 above.

As shown by the Residual Graphic, the results given by the figures 1 and 2 present a very good correlation. There is a major concentration of data near from zero. Therefore, was obtained the same result but without the short wavelength ripple in the W-E direction.

The second synthetic anomaly used, figures 4 and 5, was a Pipe with induced magnetization and remanent magnetization. The magnetic anomaly was created with -29.71° for the induced inclination, -20.58° for the induced declination and 30.0° for the total inclination and 39.4° for the total declination.

The Köenigsberg ratio, given by the equation 7 bellow, is used to measure the contribution of the remanent magnetization in relation to the induced magnetization.

$$K = \frac{M_r}{M_i} \tag{7}$$

Where Mr is the remanent magnetization and Mi means the induced magnetization. The synthetic anomaly has K=2.

It's expected that the results provided by Oasis Montaj using this synthetic anomaly differs from our result, because the Oasis cannot handle with the total magnetization direction.

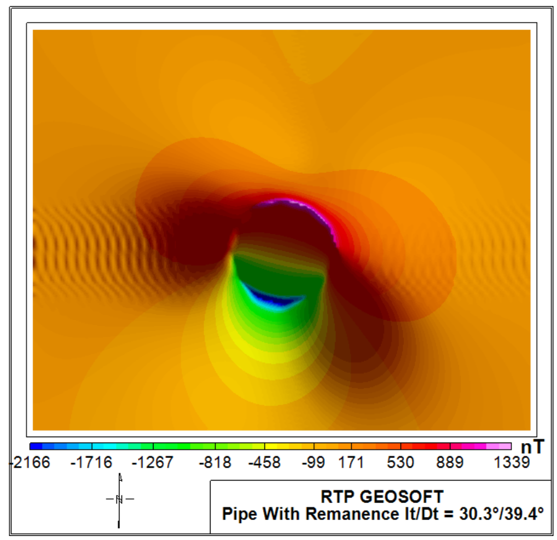

Fig. 4 – Synthetic anomaly with remanent component, reduced by Oasis Montaj. The angle with the Induced Magnetization vector is 60°.

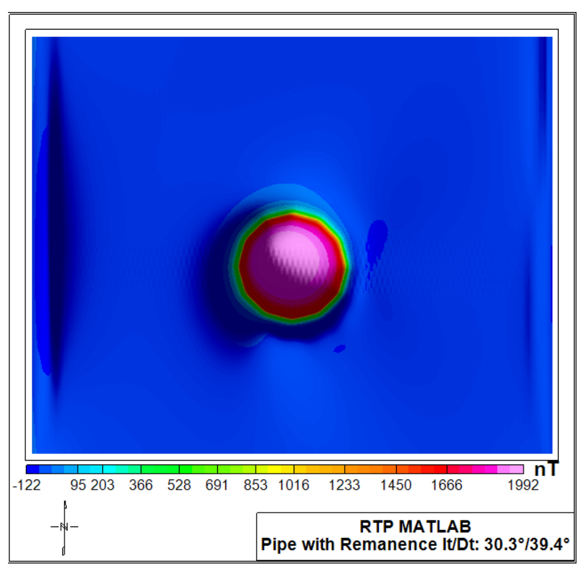

Fig. 5 – Synthetic Anomaly with remanent component, reduced by our Matlab function. The angle with the Induced Magnetization vector is 60º.

We observe that due to the contribution of the remanent magnetization the result given by Oasis Montaj tries to recover the shape of the anomaly. However as shown in figure 4, the anomaly have not the characteristic of a reduced anomaly. We also can identify the same short wavelength ripple in the W-E direction.

On the other hand, our algorithm recovers precisely the shape of the anomaly, as seen by comparing the figures 5 and 2. Proceeding with the residual graphic analyses, the width of the data distribution shown in figure 6 is nearly 5 times wider than the residual graphic shown in figure 3.

This implies that it is not presenting a good correlation.

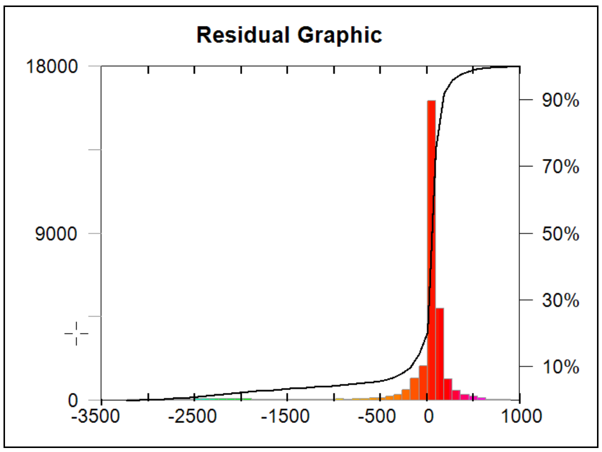

Fig. 6 – Residual graphic of the subtraction from Matlab and Geosoft Anomaly Reducted. The median of this histogram was 68.5.

Others tests were done using synthetic anomalies with different inclinations of the remanent magnetization vector into the Matlab and the Oasis Montaj.

The results acquired were similar, our algorithm recovers accurately the shape of the anomalies and the Oasis could not handle well when the anomaly had an expressive contribution of the remanent magnetization.

The MaxiMin technique also could be used to compare the results, but using synthetic anomalies, the technique present the same results of our algorithm. And it was expected because both RTP techniques use the equation (1) to evaluate the operator.

The MaxiMin technique will be better used to compare the estimation between the total magnetization direction provided by Helbig's Integrals and by itself.

- Conversion between Fields Components

Now, we can move forward to the main point that is implement the Helbig's technique aiming to obtain the total magnetization direction.

In order to apply Helbig's equations to total field aeromagnetic data, is necessary to convert the gridded magnetic data to vector component grids using Fourier filtering techniques (Blakely, 1995).

However, it is important to look closer to this operation because all our subsequent work inside the Helbig's integrals will depend entirely of the results acquired by the conversion operator.

The software Oasis Montaj already have a similar filter inside the MAGMAP called *Conversion Between Fields Components,* however was found more questions than answers looking to the results given by this operator.

Therefore, this was the second algorithm that needed to be implemented. Looking for the theory behind this operator into Blakely (1995), we derive the following equations to each component transformation:

B_z from H total:

$$\pounds(\psi_i) = \frac{|k|}{|k|\widehat{f}_z + i(k_x\widehat{f}_x + k_y\widehat{f}_y)} \tag{8}$$

B_x from H total:

$$\pounds(\psi_i) = \frac{ik_x}{|k|\widehat{f}_z \; + \; i(\widehat{f}_x k_x + \widehat{f}_y k_y)} \tag{9}$$

B_y from H total:

$$\pounds(\psi_i) = \frac{ik_y}{|k|\widehat{f}_z \; + \; i(\widehat{f}_x k_x + \widehat{f}_y k_y)} \tag{10}$$

Where:

$$\begin{aligned} fz &= sin(I_r) \\ fx &= cos(I_r)sin(D_r) \\ fy &= cos(I_r)cos(D_r) \end{aligned} \tag{11}$$

$$|k| = \sqrt{k_x^2 \; + \; k_y^2 \; + \; k_z^2} \tag{12}$$

And k_x, k_y, k_z are the wavenumbers referent to x,y,z.

The implementation of the algorithm was similar to the RTP operator, also into the wavenumber domain. The information requested by the function is the *grid_name*, the inclination and declination directions of the reference field (IGRF).

Below, it is shown the comparison between the conversion of the field due to a synthetic anomaly with -29.71° inclination and -20.58° declination to the reference field.

It is important to highlight the equations (8), (9) and (10) because to calculate these functions it is only needed to specify the values of the reference field. In this way, the remanent magnetization is not an important parameter for these functions. The concept behind the conversion between field components shows that the directions of an anomaly can be dissociated into two components, remanent direction and induced direction.

The equations (8), (9) and (10) are showing that the remanence direction of the anomaly remains unchanged and it is changing only the direction of the reference field.

In other words, the equation (8) for example, is showing one operation that fixes the remanent direction and calculates Bz the vertical component of the magnetic field from the Total Intensity Field anomaly.

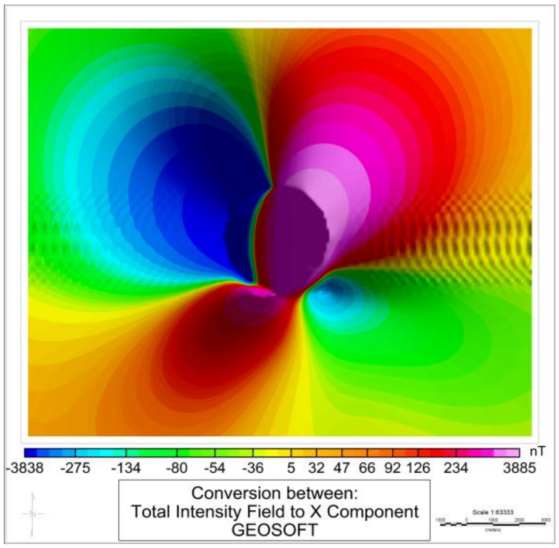

Fig. 7 – Synthetic anomaly used to evaluate the Conversion between Component Field operator.

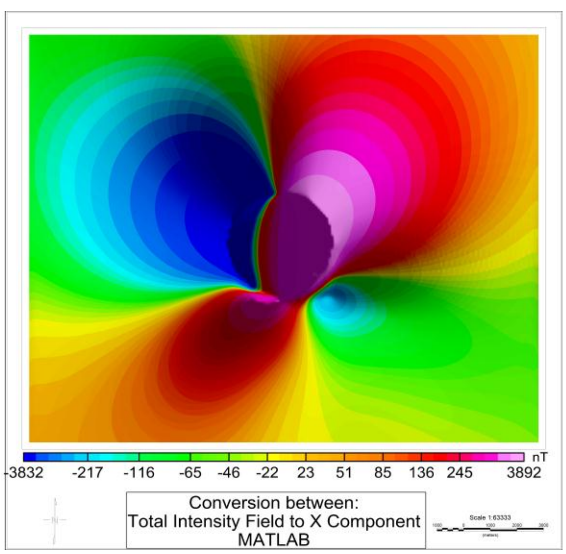

Fig. 8 – The same synthetic anomaly shown in the figure 7, but calculated from the developed MATLAB function.

As observed in figure 7, the conversion to the Bx component of the magnetic field from the Total Intensity Field anomaly provided by Oasis, has an undesirable effect due to the high frequencies wavelength.

The figure 8, shows the result acquired by our algorithm. There is not the presence of the high frequency interference and the result is nearly the same.

The figure 10 shows the Bz component of the magnetic field. Was chosen not use the By component in this paper because weren't observed differences between the techniques.

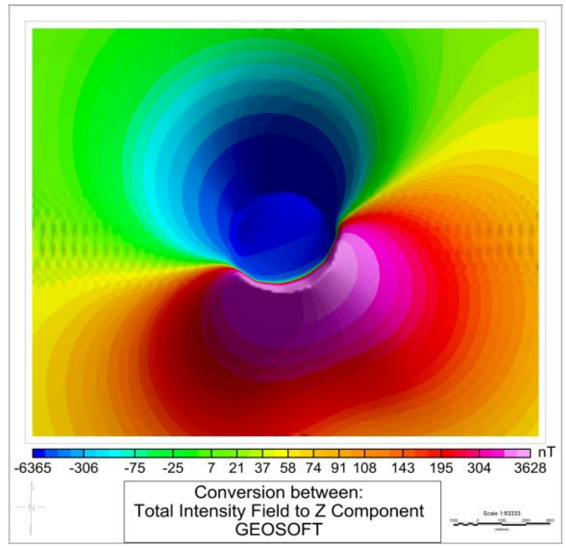

Fig. 9 – The Component Z of the field obtained from the Total Intensity Field by the Oasis Montaj.

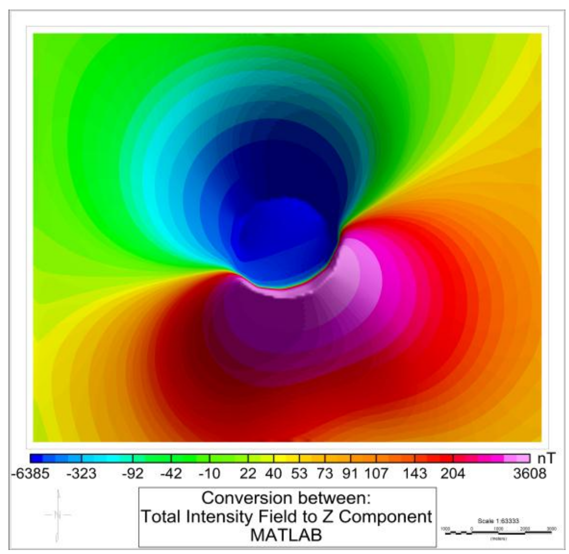

Fig. 9 – The Component Z of the field obtained from the Total Intensity Field using the MATLAB algorithm implemented.

The interference effects observed in figures 7 and 10 could be associated with some manipulation done by the software Oasis Montaj when the grid was prepared to calculate the FFT or to avoid high amplitudes.

As the results show, our effort to understand the operation described by the equations 7 to 11, was important to the development of the project. We reached better results than the provided by Oasis Montaj and we had the complete understanding about what the algorithm was doing when the operation was in process.

Conclusions

The importance of estimate the total magnetization is clearly abundant in the processing and interpretation of magnetic data. The method proposed by Helbig (1963) theoretically did not depends of an unstable operator to estimate the total magnetization direction.

For example, using a 3-axial instrument measuring the components of the field during the acquisition, it is possible to use this data directly inside the integrals.

However, if we had to generate the vector components of the field from the Total Intensity Field, it is necessary to pay attention because the operator is not stable and interference can be detected as saw in the results.

At this point, we already have a RTP operator that allows the user to use the information of total magnetization direction, and the first and important step done to develop the Helbig's technique that is the cartesian components of the anomalous field.

The next steps of the study are implement the Helbig's Integrals as a MATLAB function in order to apply the results into a comparative study. For this end, will be used synthetic and real anomalies to compare the MaxiMin technique and the Helbig's technique.

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