Generalized semblance coefficients using singular value decomposition

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Abstract

For a given type of travel time function and travel time parameters, a data matrix is generated for a time window centered around this travel time for each spatial data channel. We normally compute a coherence measure from the data matrix corresponding to these travel time parameters. There are two main classes of normalized coherence measures. Generalized semblance which is normalized between zero and one, and so-called high-resolution coherence measures which are bounded from below.

Singular value decomposition (SVD) of the data matrix results in several new types of generalized semblance. The first SVD subspace or eigenimage gives three different generalized semblance coefficients which take into account the main signal energy for a signal with the travel time corresponding to the parameters which were used to generate the data window. They can all be efficiently computed by the power method. Velocity analysis was performed on synthetic data with various noise levels and with amplitude-versus-offset (AV0) effects and on marine seismic data. The new semblance functions all gave better results than the classical one. Our recommendation for signal parameters estimation (and, in particular, for seismic velocity estimation) is to compute three coherency measures: (1) A reduced semblance coefficient, taking into account only the first eigenimage; (2) The normalized energy of the first eigenimage and (3) The logarithm of MUSIC corresponding to the generalized semblance of the first eigensubspace. These should be analysed jointly to determine the travel time parameters for the signal in the data set.

Introdução

Semblance (Taner and Koehler, 1969) is the most common coherence measure used in seismic velocity analysis. It is defined as the ratio between the energy of the estimated signal and the energy of the data in the analysis window. It is normalized to be between zero and one. Other coherence measures based on normalized covariance functions (Neidell and Taner, 1971; Sguazzero and Vesnaver, 1987) proposed the multiple signal classification (MUSIC) coherence measure. This is defined as the inverse of the data projected onto the noise subspace. It can also be expressed as the inverse of one minus the projection of the data onto the signal subspace. Barros et al (2012) define MUSIC as the inverse of one minus a normalized coherence measure which can be regarded as a generalized semblance coefficient. It is normalized to be between zero and one. We propose a new coherence measure which is defined as the ratio between the energy of the signal and the energy of the noise. For uncorrelated signal and noise this is equal to semblance divided by one minus semblance.

The relationship between semblance and MUSIC coherence measures has been analysed using the eigenstructure of the spatial covariance matrix (Biondi and Kostov, 1989; Kirlin, 1992; Key and Smithson, 1990; Sacchi, 1998).

Singular value decomposition (SVD), (Golub and Van Loan, 1996) has been used in seismic data processing for noise removal (Freire and Ulrych, 1988; Porsani et al., 2013).

Here we shall use SVD extensively to analyse and define various types of generalized semblance coefficients corresponding to different signal and noise models. The classical semblance coefficient is based on a signal model which consists of a common time signal which has the same amplitude on each trace. We assume the same signal model for the first L eigenimages of the data matrix. This gives a generalized semblance coefficient which is the sum of the first L singular values squared times the square of the average of the elements of the corresponding spatial singular vector, properly scaled. For $L = 1$ it gives the generalized semblance coefficient used by Barros et al. (2012) in defining MUSIC.

A second type of generalized semblance coefficient is obtained by assuming a signal model which has the same time signal on each trace multiplied by a spatially variant amplitude function. The optimal signal estimate is then the first eigenimage of the data, and the semblance is the first singular value squared divided by the energy of the data. This gives a signal-to-noise energy ratio which is equal to a scaled and shifted version of one coherence measure proposed by (Schmidt, 1986; Sacchi, 1998).

By multiplying the two previous generalized semblance coefficients, corresponding to the first eigenimage, we obtain a reduced semblance coefficient. It is equal to the square of the correlation between the first temporal singular vector and the sum of the traces, normalized by the number of spatial channels times the squared norm of the data matrix.

In the numerical example we compare the classical semblance coefficient with three generalized semblance coefficients derived from the first eigenimage. These are the generalized semblance coefficient for the first SVD subspace, the normalized energy of the first eigenimage and the reduced semblance coefficient.

We compute these four semblance coefficients, and the corresponding logarithm of the MUSIC measure, in synthetic data examples with various levels of noise. Two closely spaced arriving signals are modeled with moderate and strong amplitude-versus-offset (AVO) behaviour. We also apply the four semblance coefficients, and one log MUSIC coherence measure, to marine seismic data from offshore north-east Brazil. Based on analytic considerations and the numerical results, we finally propose a new procedure for travel time parameter estimation.

Normalized coherence measures

We assume that the data $d(t,x)$, function of time t and direction x, can be represented by a sum of arriving pulses, $s_k(t)$, with different arrival times $T(\theta_k,x)$ which have the same form, but different values of the travel time parameters vector θ. That is

$$d(t,x) = \sum_{k=1}^{K} s_k(t - T(\theta_k,x)) + n(t,x) \qquad (1)$$

where $n(t,x)$ is a noise term. The travel time or arrival time function may have the form:

$$T(\theta,x) = \left[T(0)^2 + \frac{x^2}{v_{NMO}^2}\right]^{1/2}, \quad \theta = [T(0), v_{NMO}], \qquad (2)$$

the hyperbolic travel time approximation used standard seismic velocity analysis (Mayne, 1962; Taner and Koehler, 1969) where $T(0)$ is zero-offset travel time and v_{NMO} is the normal moveout velocity (often approximated by stacking velocity). With a trial value for the travel time parameter vector θ, the data within a window $T(\theta,x) \pm (N_t-1)/2)\Delta t$ are aligned with $T(\theta,x)$ to give

$$d(t_j,x_n) = D_{jn} = \{j = 1, \ldots, N_t, n = 1, \ldots, N_x\} \qquad (3)$$

with $N_t \leq N_x$. This procedure is illustrated in Figure 1, and we use linear interpolation to obtain the data values between sampling grid points. The data matrix $\mathbf{D}$ is the sum of signal plus noise $\mathbf{D} = \mathbf{W} + \mathbf{N}$, where the signal matrix $\mathbf{W}$ and the noise matrix $\mathbf{N}$ are assumed to be uncorrelated.

The classical coherence measure used in seismic velocity analysis is semblance (Taner and Koehler, 1969), defined as the ratio between the signal (W) energy and the energy of the data (D), that is

$$S = \frac{||\mathbf{W}||^2}{||\mathbf{D}||^2} \qquad (4)$$

We shall study the signal-to-noise energy ratio

$$Q = \frac{||\mathbf{W}||^2}{||\mathbf{N}||^2} = \frac{||\mathbf{W}||^2}{||\mathbf{D}||^2 - ||\mathbf{W}||^2} = \frac{S}{1-S} \qquad (5)$$

This new coherence measure is unbounded and $Q \geq 0$.

The MUSIC coherence measure is defined as the inverse of the steering vector projected onto the noise subspace. With the assumption of uncorrelated signal and noise, MUSIC can also be defined as the inverse of one minus the steering vector projected onto the signal subspace (with proper normalization). In this way, a generalized MUSIC coherence measure is (Barros et al., 2012)

$$P = \frac{1}{1-S} = Q + 1 \qquad (6)$$

where S can be any generalized semblance coherence measure normalized so that $0 \leq S \leq 1$. We see that P is unbounded and $P \geq 1$.

SVD subspace coherence measures

The classical semblance coefficient (Taner and Koehler, 1969) is based on the signal model

$$\mathbf{W} = \mathbf{s}\mathbf{e}^T \qquad (7)$$

where $\mathbf{e}^T = [1, 1, \ldots, 1]$ is an $N_x \times 1$ vector of only one's (the steering vector), and the $N_t \times 1$ vector $\mathbf{s}$ is the time signal which is assumed to be the same on all channels. An optimal estimate of the signal (Ursin, 1979) is $\mathbf{s} = (1/N_x)\mathbf{De}$. Then the semblance is, according to equation 4:

$$S = \frac{N_x||\hat{\mathbf{s}}||^2}{||\mathbf{D}||^2} = \frac{||\mathbf{De}||^2}{N_x||\mathbf{D}||^2} \qquad (8)$$

where $||\hat{\mathbf{s}}||^2 = \sum_{j=1}^{N_t} \hat{s}(t_j)^2$.

In the following we shall use the reduced SVD (Golub and Van Loan, 1996) of the data matrix

$$\mathbf{D} = \mathbf{U}\Sigma\mathbf{V}^T = \sum_{k=1}^{N_t} \mathbf{u}_k \sigma_k \mathbf{v}_k^T \qquad (9)$$

where the $N_t \times 1$ vectors $\mathbf{u}_k$ are orthonormal, and the $N_x \times 1$ vectors $\mathbf{v}_k$ are also orthonormal. The singular values σ_k are sorted such that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_{N_t} \geq 0$, assuming $N_t \leq N_x$. With equation 9, semblance in equation 8 becomes

$$S = \frac{\sum_{k=1}^{N_t} \sigma_k^2 \bar{v}_k^2}{N_x \sum_{k=1}^{N_t} \sigma_k^2} \qquad (10)$$

with $\bar{v}_k = \mathbf{v}_k^T\mathbf{e}$, being the sum (stack) of all elements in the vector $\mathbf{v}_k$.

We now introduce an SVD based signal and noise model where the signal is composed of the first L eigenimages of the data, and the noise is composed of the $N_t - L$ last eigenimages. That is

$$\mathbf{D} = \mathbf{W}_L + \mathbf{N}_{N_t-L} = \sum_{k=1}^{L} \mathbf{u}_k \sigma_k \mathbf{v}_k^T + \sum_{k=L+1}^{N_t} \mathbf{u}_k \sigma_k \mathbf{v}_k^T \qquad (11)$$

By averaging the signal matrix we get the estimate

$$\hat{\mathbf{s}} = \frac{1}{N_x}\mathbf{W}_L\mathbf{e} = \frac{1}{N_x}\sum_{k=1}^{L} \mathbf{u}_k \sigma_k \bar{v}_k \qquad (12)$$

We can define an SVD subspace semblance

$$S_L = \frac{N_x||\hat{\mathbf{s}}||^2}{||\mathbf{W}_L||^2} = \frac{||\mathbf{W}_L\mathbf{e}||^2}{N_x||\mathbf{W}_L||^2} = \frac{\sum_{k=1}^{L}\sigma_k^2\bar{v}_k^2}{N_x\sum_{k=1}^{L}\sigma_k^2} \quad (13)$$

Since $\bar{v}_k^2 = |\mathbf{v}_k^T\mathbf{e}|^2 \leq N_x$ we have that $0 \leq S_L \leq 1$. For $L = N_t$ this is the classical semblance coefficient in equation 10, and for $L = 1$ it becomes

$$S_M = \frac{1}{N_x}\bar{v}_1^2 = \frac{1}{N_x}|\mathbf{v}_1^T\mathbf{e}|^2 \quad (14)$$

This is the function used by Barros et al (2012) to define the MUSIC coherence measure as in equation 6.

Eigenimage Signal Energy

The signal model in equation 7 corresponds to a common signal, with unit amplitude on each trace. An extension to this is to assume that the signal shape, $s(t)$, is the same, but that there is an amplitude variation $a(x)$. Then the signal model is

$$\mathbf{W} = \mathbf{s}\mathbf{a}^T \quad (15)$$

A least-squares estimate of the signal is the first eigenimage (Golub and van Loan, 1996):

$$\mathbf{W} = \mathbf{u}_1\sigma_1\mathbf{v}_1^T \quad (16)$$

Semblance is computed from equation 4 which gives

$$S_E = \frac{\sigma_1^2}{\sum_{k=1}^{N_t}\sigma_k^2} = \frac{\sigma_1^2}{||\mathbf{D}||^2} \quad (17)$$

The signal-to-noise energy ratio is

$$Q_E = \frac{\sigma_1^2}{\sum_{k=2}^{N_t}\sigma_k^2} \quad (18)$$

Reduced semblance coefficient

The generalized semblance coefficients S_M and S_E are both derived from the first eigenimage of the data. The classical semblance coefficient in equation (10) can be expressed by

$$S = S_M S_E + \frac{\sum_{k=2}^{N_t}\sigma_k^2\bar{v}_k^2}{N_x||\mathbf{D}||^2} \quad (19)$$

5 where we have used equation (14) and (17). That is, the product $S_M S_E$ contains all the information in the semblance coefficient related to the first eigenimage. We shall refer to it as the reduced semblance coefficient.

$$S_R = S_M S_E = \frac{\sigma_1^2|\mathbf{v}_1^T\mathbf{e}|^2}{N_x||\mathbf{D}||^2}. \quad (20)$$

Data examples

Synthetic data were generated with two reflections with the same $T(0) = 1s$, and with $v_{NMO} = 2500m/s$ and $v_{NMO} = 3000m/s$, respectively. The data set consists of 60 traces from $x = 20m$ to $x = 1200m$ with a sampling distance $\Delta x = 20m$. The data are computed from $s = 0.8s$ to $s = 1.4s$ with a sampling interval $\Delta t = 0.004s$. The signal with $v_{NMO} = 3000m/s$ has an amplitude which varies from 1.0 at zero offset to 0.8 at the far offset. The signal with $v_{NMO} = 2500m/s$ has a polarity reversal with amplitude equal to 1.0 at zero offset and amplitude equal to -0.5 at the far offset. These data are termed AVO data. Three data sets were prepared with no noise, little noise and much noise, as shown in Figure 2.

Using a data window of 11 samples, we computed four types of generalized semblance, S, S_M, S_R and S_E, and their corresponding log MUSIC functions, $\log_{10}P = -\log_{10}(1 - S)$.

For noise-free AVO data, S_E detects both signals with $\log_{10}P_E$ giving enhanced peaks as shown in Figure 3, top panels. On the AVO data with little and much noise, only the signal with moderate AVO behaviour can be detected. S_M performs better than S_R which is better than S. For AVO data with much noise, S_E has a maximum for the correct travel time parameters for the signal with moderate AVO variation, but the image in $T(0) - v_{NMO}$ - plane on Figure 4 is very noisy.

Figure 5, top right, shows a CMP gather from a marine seismic data set from north-east Brazil. It is recorded from $t = 0s$ to $t = 4s$ with a sampling interval of $\Delta t = 0.0004s$. There are 60 traces with a minimum offset equal to $150m$ and a distance between data channels $\Delta x = 25m$. Figure 5 also displays S, S_R, S_M, S_E and $\log_{10}P_M$. these coherence measures were computed with a time window of 17 samples. These semblance coefficients all gave better results than the classical one.

Conclusion

We have reviewed, renormalized and extended previous theory for normalized coherence measures. In travel time parameters estimation we may use a generalized semblance coefficient S, normalized such that $0 \leq S \leq 1$, or a corresponding high-resolution coherence measure.

The signal-to-noise energy ration and the MUSIC coherence measure are both related to a generalized semblance coefficient. They provide improved parameter estimates only for large values of semblance. For small values of semblance, they behave like semblance. They can vary over a large range of values, so they logarithm of MUSIC is easier to interpret.

Singular value decomposition (SVD) of the data matrix results in several new types of generalized semblance. The first SVD subspace or eigenimage gave three different generalized semblance coefficients which take into account the main signal energy for a signal with the travel time corresponding to the parameters which were used to generate the data window. They can all be efficiently computed by the power method. These new semblance coefficients are: (i) a reduced semblance coefficient, taking into account only the first eigenimage, (ii) the normalized energy of the first eigenimage, and (iii) a generalized semblance coefficient computed from the first eigenimage or eigensubspace.

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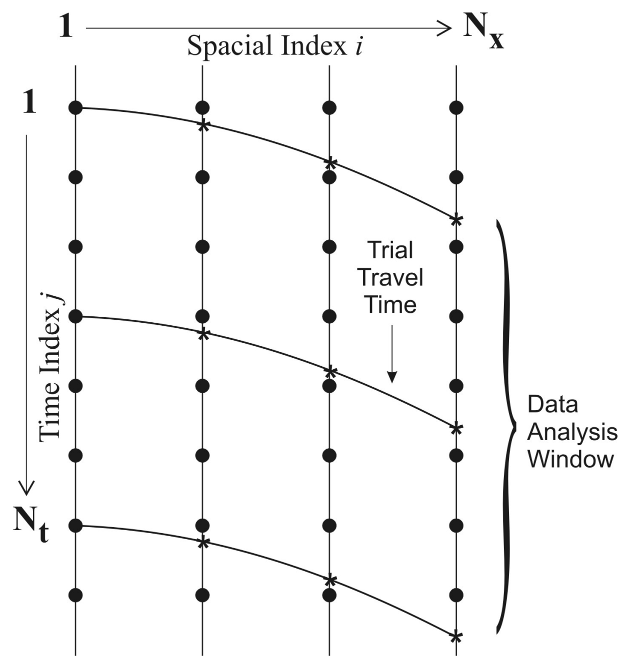

Figure 1: Data analysis window.

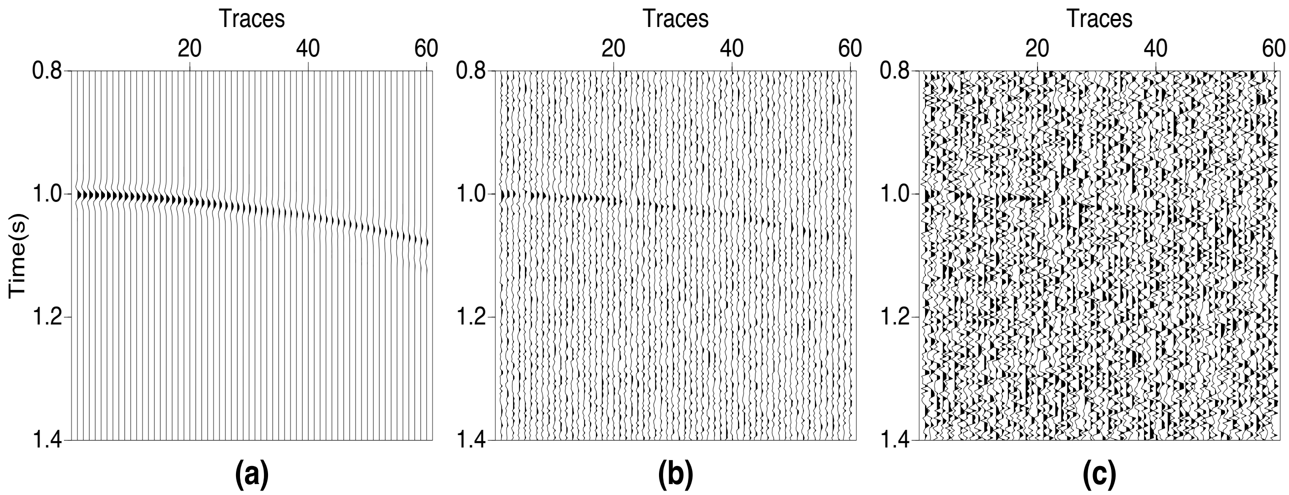

Figure 2: Synthetic AVO data: No noise (a), Little noise (b), Much noise (c).

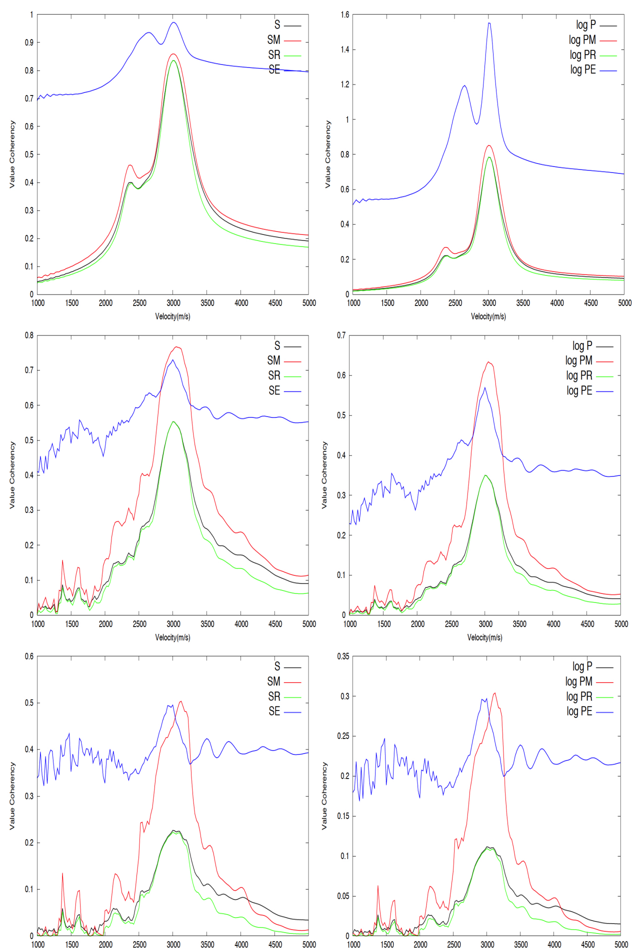

Figure 3: Semblance (left panels) and log MUSIC (right panel) for $T(0) = 1s$ and different velocities. Noise-free AVO data (top panels), little-noise AVO data (middle panels) and much-noise AVO data (bottom panels).

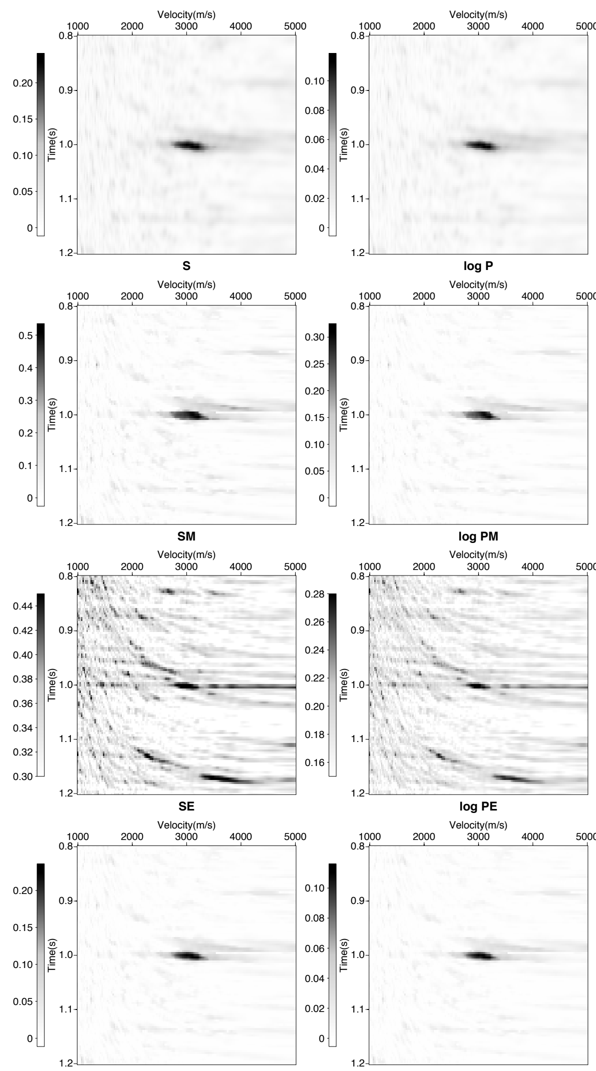

Figure 4: Much-noise AVO data. Semblance functions (left panels), and the respective log MUSIC functions (right panels).

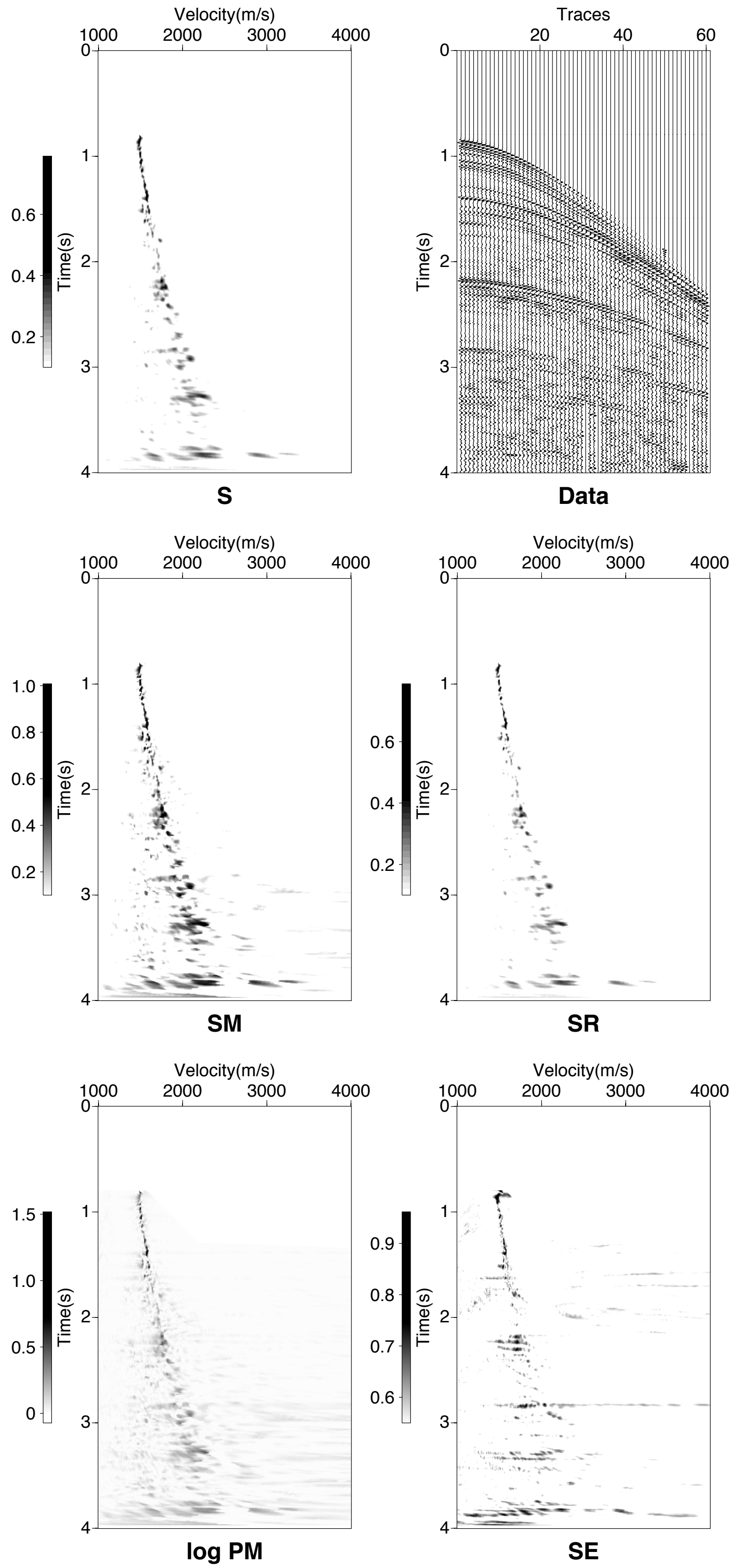

Figure 5: Different coherence measures in velocity analysis of marine seismic data.