Proposal for a test site for the teaching of gravimetry.

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Abstract

This paper proposes a tool for the teaching of the Gravity Method in the form of a controlled test site to perform gravity measurements over a cylindrical well dug in the soil. We have calculated the gravimetric field due to a cylinder and we have used this calculation to model a gravimetric survey over an empty well. The site can be used for classes in which the students would perform the surveys, the gravimetric corrections and the interpretations.

Introduction

The teaching of the Gravity Method (Chang, 1987; Telford *et al.*, 1990 ; Silva and Luiz, 1995), in the general context of the geophysical methods, encompasses two fundamental aspects of the graduation of the professional geophysicist: the theory of the method, based on the study of potential theory, and the practice of acquisition, processing and interpretation of gravity data. In the Federal University of Pará, Brazil, the field practice of gravimetry is done by undergraduate students during field trips, which are difficult and expensive. Generally, this means an extensive and hard field work, with the necessity of transporting equipment and the groups of students to a site with previously known gravity field distribution and geological sources.

While such trips are important for the students to have a taste of the actual work on a geophysical survey, it would be useful for them to have a controlled laboratory environment for gravity measurements for a first contact with the equipment and methods, prior to the field work itself. A survey in such controlled condition would allow the teacher to explore several aspects of data acquisition, processing and interpretation with the purpose of identifying a target which is known in detail.

Such controlled test site for gravity measurements must include structures that generate strong enough variations on the local gravity field to be detected by the gravity meters. This is a hard to meet requirement, especially for those who have to work with older equipment, not in the micro-gravimetry level.

In this work, we propose the design of a controlled test site for the teaching of gravimetry that fulfills all requirements for a first contact of the students with the practice of the method.

The most basic feature of the test site is a cylindrical well, dug in the ground, which will generate a negative anomaly, as illustrated in figure 1.

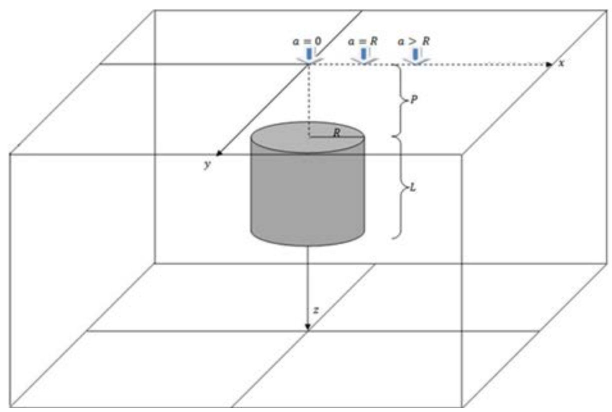


Fig. 1- Scheme illustrating the model where the gravity survey will be performed.

If the cylinder is just an empty cavity in the ground, the total anomalous mass will be given by the volume of the cylinder times the density of the soil around it. In this case, a strong enough negative anomaly can be measured if the volume is large enough and it is close to the surface.

In order to be detectable by the Lacoste & Romberg gravity meter that belongs to the UFPA, a gravity field anomaly must be at least 0.01mGal. This is the theoretical minimum necessary anomaly to be generated by our cylindrical cavity.

We need, then, to determine what is the required size of the cylinder dug in a soil with an average density. We have performed this calculation as follows.

Gravity field of cylinder

To calculate the vertical component of the gravimetric field of the cylinder, we define the coordinate system shown in the figure 2. The gravity field was calculated with the cylinder's top at depth $\boldsymbol{p}$. The cylinder's length is $\boldsymbol{L}$ and its radius is $\boldsymbol{R}$. We calculate the vertical component of gravity's acceleration over the points on the $\boldsymbol{x}$ axis indicated by the variable $\boldsymbol{a}$. The soil density is $\boldsymbol{\rho_s}$, considered uniform. We considered the cylinder's density equal to zero. The vertical component of gravity's acceleration in a distant point is $\boldsymbol{g_0}$. We take this value as reference to the calculation of the surface field variation $\boldsymbol{\Delta g}$ .The vertical component at the surface will be:

$$\boldsymbol{\Delta g = g - g_0 = - g_c}$$

$$\Delta \boldsymbol{g} = -\boldsymbol{G\rho_s} \int_{-R}^{R} \int_{-(R^2-y^2)^{\frac{1}{2}}}^{(R^2-y^2)^{\frac{1}{2}}} \int_{p}^{p+L} \frac{\boldsymbol{z\, dzdxdy}}{[(\boldsymbol{a}-\boldsymbol{x})^2+\boldsymbol{y}^2+\boldsymbol{z}^2]^{\frac{3}{2}}}$$

We solved this triple integral in three steps:

- First the integral in dz

$$I_z = \int_{p}^{p+L} \frac{zdz}{[(a-x)^2+y^2+z^2]^{\frac{3}{2}}}$$

$$= \left[\frac{-1}{[(a-x)^2+y^2+z^2]^{\frac{1}{2}}}\right]_p^{p+L}$$

$$I_z = \left[\frac{1}{[(a-x)^2+y^2+p^2]^{\frac{1}{2}}} - \frac{1}{[(a-x)^2+y^2+(p+L)^2]^{\frac{1}{2}}}\right]$$

- Second integral in dx

$$I_x = \int_{-(R^2-y^2)^{\frac{1}{2}}}^{(R^2-y^2)^{\frac{1}{2}}} I_z(x,y)\, dx$$

$$I_x = \ln\left\{\frac{a+[R^2-y^2]^{\frac{1}{2}}+\left[a^2+p^2+R^2+2a(R^2-y^2)^{\frac{1}{2}}\right]^{\frac{1}{2}}}{a-[R^2-y^2]^{\frac{1}{2}}+\left[a^2+p^2+R^2-2a(R^2-y^2)^{\frac{1}{2}}\right]^{\frac{1}{2}}}\right\} + \ln\left\{\frac{a-[R^2-y^2]^{\frac{1}{2}}+[a^2+L^2+2Lp+p^2+R^2+2a(R^2-y^2)^{\frac{1}{2}}]^{\frac{1}{2}}}{a+[R^2-y^2]^{\frac{1}{2}}+[a^2+L^2+2Lp+p^2+R^2+2a(R^2-y^2)^{\frac{1}{2}}]^{\frac{1}{2}}}\right\}$$

- Finally, the integral in dy.

$$I_y = \int_{-R}^{R} I_x(y)\, dy$$

Which was calculated numerically. To express the Δg values in Gal, we use p, L and R in meter, ρ_s in g/cm^3 and G in Nm^2/kg^2, multiplying the result by 10^5. Finally, to express the values in mGal we multiply by 10^3.

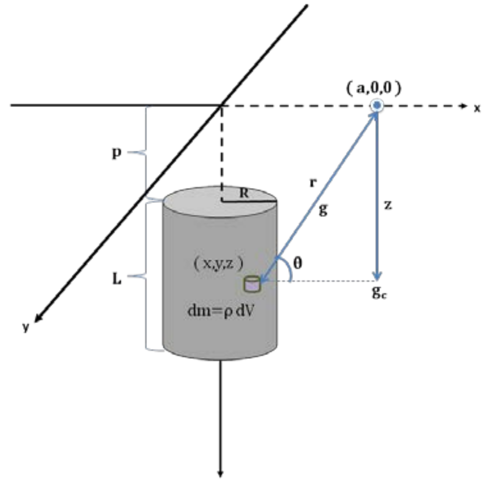


Fig. 2- Scheme illustrating the variables used in the calculation.

Results for the proposed model.

In our models, we assume that the top of the empty cylinder is at a depth of 5 cm from the surface. Thus in the calculation $p = 0{,}05\ m$. Soil density is $\rho_s = 1.5\ g/cm^3$ (Sengik, 2010).

We began the calculation with $R = 1m$ e $L = 2m$. The result for this model is shown in figure 3.

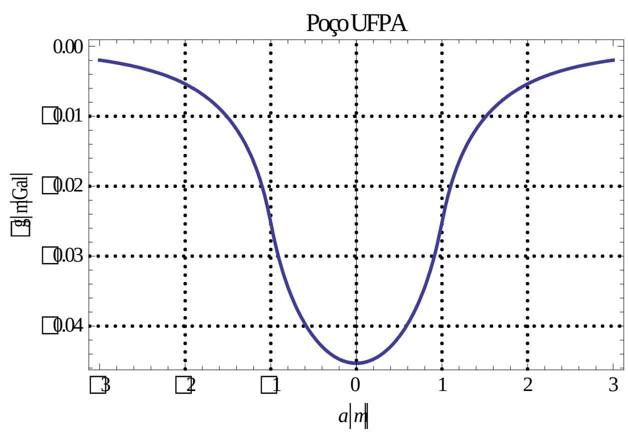


Fig. 3- Negative anomaly due to the well with $R = 1m$, $L = 2m$ and density's contrast $\Delta\rho = -1{,}5\ g.cm^{-3}$.

For this model, the peak of the anomaly reached the value of -0.045 mGal. The limit of 0.01 mGal was reached at a distance of about 1.5 m of the cylinder's center.

Now we assume a cylinder again with $R = 1m$, but with $L = 3m$, which represents a volume of 1.5 times the length of the first model. The result is illustrated in figure 4.

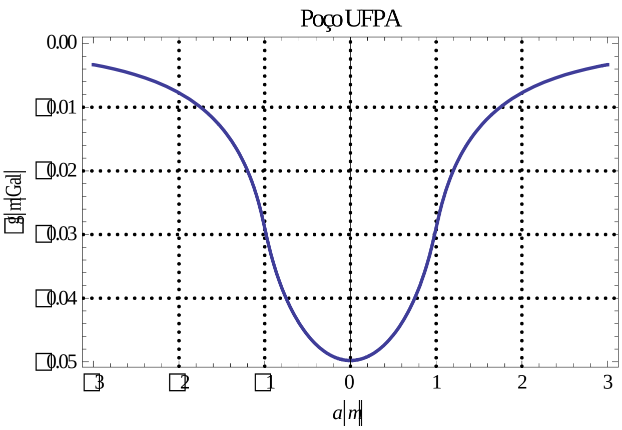


Fig 4- *Negative anomaly due the well with* $R = 1m$, $L = 3m$ *and density's contrast* $\Delta\rho = -1.5\ g.cm^{-3}$.

Now the anomaly's peak reached the value of -0,05 mGal

For the third model, we increase the cylinder's volume, assuming now $R = 1.5m$ and keeping $L = 2m$, like in the first model. Now the cylinder's volume is 2.25 times the volume of the first model. The result for this model is shown in the figure 5.

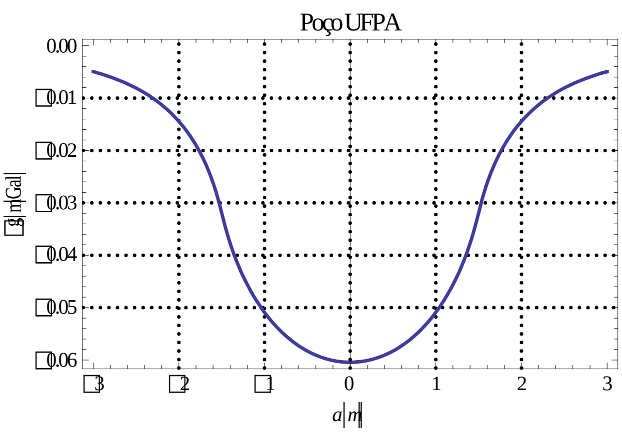


Fig 5- *Negative anomaly due the well with* $R = 1{,}5m$, $L = 2m$ *and density's contrast* $\Delta\rho = -1{,}5\ g.cm^{-3}$.

For this model, the peak value of was -0,06 mGal.

A gravimetric survey should detect the cylinder through the measurement of its anomaly. The cylinder's volume varies linearly with depth and with the square of the radius. The final dimensions of a well like this will be determined by factors like the cost of building it and the difficulty imposed by the soil geology.

The data must be accurate to allow the processing, interpretation and a possible inversion to determine the well geometry. The survey will be planned to reach a high detail level in an area relatively small around the well. The work may be planned to take enough time to become necessary to perform tide and drift corrections prior to interpretation.

The test site may include variations in the land elevation, so that free-air and Bouguer corrections may be necessary.

While we seek the necessary funding to build this test site, we needed real measurements over a similar structure, to assess the real accuracy of our gravity meter and to verify our calculations. We found a cylindrical cavity like the one we needed in the campus of the UFPA, in a location shown in figure 6.

Fig 6- Image illustrating the site of the well in the UFPA where the measurements were taken.

The well has diameter of 3m, length of 1.6m, and is covered by a cement slab 5cm thick. Figure 7 illustrates how the survey was performed over the well, on a line with three measurements. In figure 8 we plot the measured data and the result of our modeling with the same geometrical parameters as the real well.

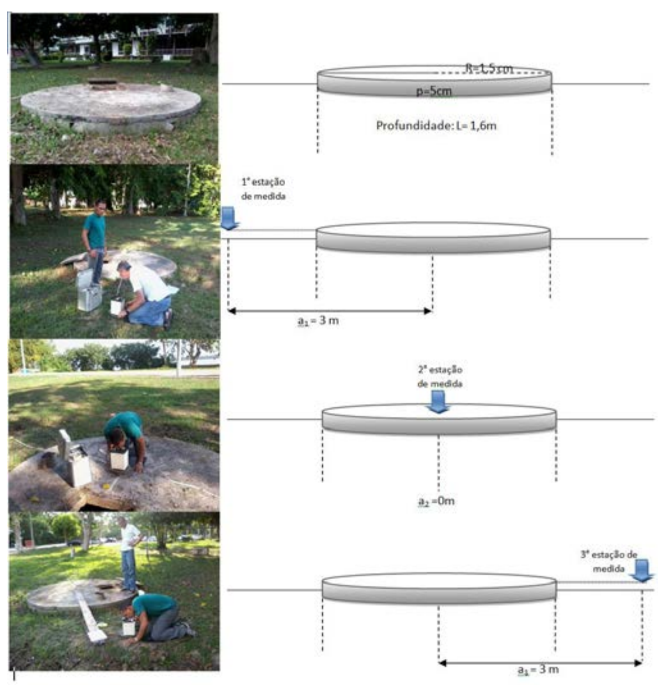
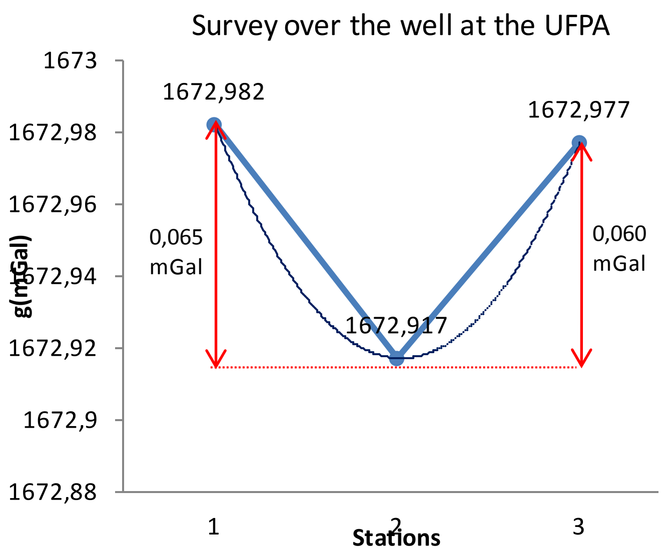


Fig. 7- Scheme illustrating the gravimetric survey over the empty well at the UFPA and photos of the survey.



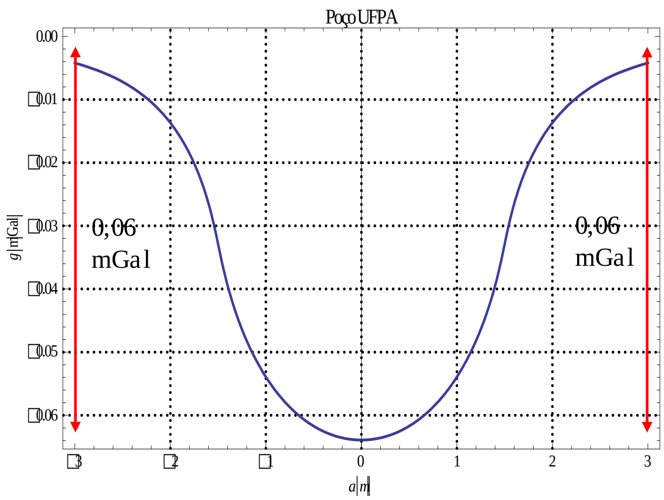


Fig. 8- Comparison between the anomaly due to the real well and the anomaly calculated in the computational model with the parameters of the real well (R=1,5m; p=0,05m; L=1.6m; ρ=1.75g.cm^{-3}).

Other possible features of the controlled test site

Figure 9 illustrates a possible design for a more complete test site. This consists in a combination of sources of different anomalies. At the area there are two wells: the first (P_1) is filled with a material with higher density than the soil around it, which will produce a positive anomaly; the second well (P_2) is empty. The material that will fill the P_1 well can be composed by scrap metal, with an average density that will be a function of the shape and size of the pieces employed. With an adequate material, the final mean density will be significantly higher than the soil.

Other possibility, making use of the stronger signal of the filled well, is the building of an elevated part of the land over it. The elevation of the land would initiate some meters before the filled well's edge and would be done in horizontal steps, to allow easy measurements in its side faces. A height variation of just one meter is enough to generate a change in the signal in the same order of the signal generated by the target of the gravity survey. The students, therefore, have to include free-air and Bouguer corrections to achieve the correct interpretation of the data.

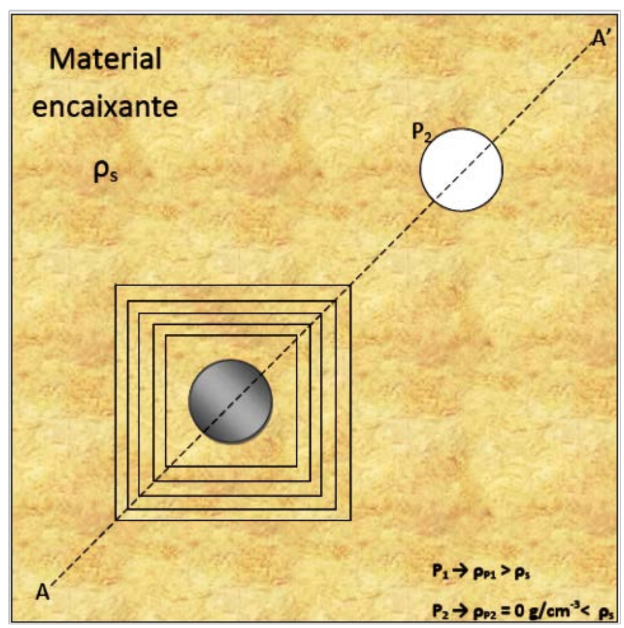
This model of a test site to the teaching of gravimetry will need an area of 300 m^2 to 1000 m^2, depending on how the features in the area will be distributed. The survey can be done on a single line, which will pass over the top of the anomaly's sources, or in an area that will include both the sources, to produce a map of isovalues of the gravimetric field.

Conclusions

The results of the gravimetric survey over the empty well in the UFPA indicate that to achieve and surpass the signal level that we can measure with a gravimeter with resolution in the order of 0.01mGal, like the gravimeter that belong to UFPA, we need to build a well of modest size, well within the resources normally available to the University.

Obviously, the use of micro-gravimeters, with greater resolution, would improve the gravimetric survey and the applicability in the gravimetry classes.

The building of this test site will be expensive and it will occupy a large area. However, an effort to build it will be rewarded with a unique and extremely useful tool for the teaching of the gravity method.



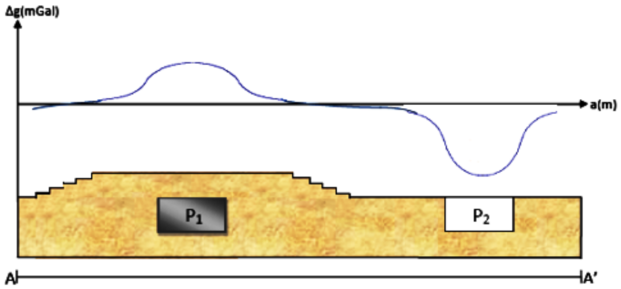


Fig.9- Profile AA'of the Gravimetric Circuit showing two wells. The well P_1 with a density's contrast positive generating, therefore a positive anomaly, and the well P_2 with density's contrast negative generate, therefore a negative anomaly.

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