Non-linear inversion of interval velocities

Daniel Carvalho Rocha Junior, CPGG/IGEO/UFBA and Milton José Porsani, CPGG/IGEO/UFBA and INCT-GP/CPGG/UFBA

This paper was prepared for presentation at the 13th International Congress of the Brazilian Geophysical Society, held in Rio de Janeiro, Brazil, August 26-29, 2013.

Abstract

The computation of interval velocities is an old and persistent problem in seismic analysis, and it is critically important to an efficient seismic imaging. The conventional method, Dix equation, is an analytical expression which permits to easily compute interval velocities from stacking velocities under certain assumptions. When stacking velocity function does not behave properly, Dix equation results in abrupt variations on interval velocity function, making the profile unrealistic. Another issue refers to model building: an accurate interval velocity field forms an estimate to a stratigraphic velocity model (suitable for seismic migration), and a possible insertion of interfaces in interval velocities would help on this estimation. This paper aims to solve this problem using an approach different from Dix's, in order to get better results. To achieve this, some inversion algorithms which take smoothness and/or simplicity of interval velocities in account will be used, making them more realistic. Inverted models containing interfaces will be obtained as well, using reflexion location information through semblance panels. A discussion of the results, comparing conventional and applied methods, will be done along this study. Stacking velocities and semblance values from synthetic and real data will be used, obtained during the seismic processing step called velocity analysis.

Introduction

Seismic reflection data and its careful analysis provide geophysicists with very important information about subsurface properties; such as depth, thickness and P-wave velocity of geological formations. All of these properties can be grouped and be called a velocity model. Seismic data itself, even when it is processed, does not yield directly to the knowledge of this velocity model.

A seismic processing step, called velocity analysis, estimates a velocity function to correct the arrival times of events in the seismogram for their varying offsets, making the stacking possible and consequently leading to the improvement of the signal-to-noise ratio of the data. These velocity values are called stacking or NMO velocities.

However, stacking velocities do not correspond to the velocity model: they just serve for normal move-out correction. The best approximation of the real formations velocities is called interval velocities, because they really correspond to the model of a subsurface divided by several layers with their own thickness and velocities. The stacking velocities, which are the available data in the end of velocity analysis, are the root mean square averages of interval velocities, considering a stratified isotropic velocity model:

$$U_j^2 = \frac{1}{t_j}\sum_{i=1}^{j} V_i^2 \Delta t_i \tag{1}$$

where $t_j = \sum_{i=1}^{j} \Delta t_i$, V_i is the interval velocity at layer i, and Δt_i is the difference between NMO times from current and previous NMO velocity, according to the summation index.

In order to find the interval velocities, the inversion of the root mean square operation is needed. Some decades ago, Dix was the first to solve analytically the problem, stating the equation which has his name today:

$$V_i^2 = \frac{U_i^2 t_i - U_{i-1}^2 t_{i-1}}{t_i - t_{i-1}} \tag{2}$$

This equation has been used very frequently in seismic processing, due to its practicality and simplicity. However, Dix formula is occasionally unsatisfactory for more complicated stratigraphy, which has more drastic variations in the stacking velocity function. There were several attempts to convey a better solution for this problem. Some of them take hold of Dix equation itself, and try to get an optimized solution, generally using least-squares type optimization.

Method

Non-linear inversion is defined by a model vector and a non-linear function resulting in predicted data:

$$\mathbf{d}^{pred} = \mathscr{F}[\mathbf{m}] \tag{3}$$

and the objective, generally, is to minimize the following objective function:

$$\phi_d = \mathbf{d}^{obs} - \mathscr{F}[\mathbf{m}] \tag{4}$$

In our problem, these terms are:

$$\mathbf{m} = [V_1, \dots, V_N] \tag{5}$$

$$\mathbf{d}^{obs} = [U_1, \dots, U_M] \tag{6}$$

$$\mathscr{F}_j[\mathbf{m}] = d_j^{pred} = \sqrt{\frac{1}{t_j}\sum_{k=1}^{j} V_k^2 \Delta t_k} \tag{7}$$

The presented method is basically a parameter estimation method for the non-linear problem so called Dix Inversion

(Harlan, 1999). Non-linear inversion is frequently achieved with Gauss-Newton type algorithms. This problem can also be treated as linear by squaring the data parameters and forward operator in Eq. (7), and then using directly LS methods (Buland et al., 2011).

Gauss-Newton

Gauss-Newton algorithm generates a least-squares iterative solution for the non-linear problem. Taylor series approximation of the non-linear forward function is implemented, yielding to a least-square system of equations:

$$\mathbf{J^T J}\delta\mathbf{m} = \mathbf{J^T}\delta\mathbf{d} \tag{8}$$

where $\mathbf{J}$ is the Jacobian matrix, containing the derivatives of the object non-linear function (in our case the RMS operation) in respect of the model parameters; $\delta\mathbf{m}$ is the increment model vector computed in each iteration, and $\delta\mathbf{d}$ is the misfit between observed and predicted data.

When possible, Jacobian matrix can be determined analytically. We can implement that for our problem, deriving analytically the forward operation for every index j with respecting to every V_i, which give us:

$$J_{ij} = \frac{\partial \mathscr{F}_j[\mathbf{m}]}{\partial m_i} = \frac{\partial U_j}{\partial V_i} \tag{9}$$

$$J_{ij} = \frac{V_i \frac{\Delta t_j}{t_j}}{\sqrt{\frac{1}{t_j}\sum_{k=1}^{j} V_k^2 \Delta t_k}} \tag{10}$$

The system of equations in (8) can be solved with the same procedure in a least-square problem, generally using gradient methods. Once $\delta\mathbf{m}$ is found, the process is repeated until results are satisfactory. In this paper the conjugate gradient method was used, which has a rapid convergence time, and a very small value of $||\delta\mathbf{m}||$ was the criterion to stop Gauss-Newton algorithm iterations.

Model objective function and Tikhonov regularization

Data misfit minimization (Eq. (4)) does not always yield to the best solution, since observed data are noisy and not trustful, making the number of possible solutions increase. In this universe of many solutions, we can eliminate unstable and unrealistic solutions, taking regard of the simplicity and smoothness of the desired model. The earth properties are simple and smooth in general, and formation velocities for ”soft” rocks are described this way (Harlan, 1999).

Besides that, due to the fact Dix equation deals with difference of squared NMO velocity, small changes in the slope of NMO velocity profile yield some large abrupt changes in interval velocities. An optimization of Dix equations surely has to take smoothness in regard.

A model objective function is inserted in our inversion, and its continuous form is (Oldenburg and Li, 2005):

$$\phi_m(m) = \alpha_s \int (m - m_{ref})^2 dt + \alpha_t \int (\frac{dm}{dt})^2 dt \tag{11}$$

When this expression is minimized, the first term aims to select the smallest (simplest) model, and the second one the flattest (smoothest) model. Smoothness is the main achievement desired for our problem, which makes us choose an α_t sufficient times higher than α_s.

Then we have a total objective function to minimize:

$$\phi(m) = \phi_d + \lambda\phi_m \tag{12}$$

and λ is called regularization parameter, which regulates the weighting between ϕ_d and ϕ_m. This insertion of model objective function modifies the Gauss-Newton system of equations:

$$(\mathbf{J^T J} + \lambda \mathbf{W_m^T W_m})\delta\mathbf{m} = \mathbf{J^T}\delta d - \lambda \mathbf{W_m^T W_m m}^{(k)} \tag{13}$$

where $\mathbf{W_m^T W_m}$ can be obtained by the discretization of the model objective function in Eq. 11, which is:

$$\mathbf{W_m^T W_m} = \alpha_s \mathbf{W_s^T W_s} + \alpha_t \mathbf{W_t^T W_t} \tag{14}$$

This damped Gauss-Newton algorithm and the problem of choosing the best regularization parameter is called Tikhonov regularization.

Models with interfaces

When a blocky model is desired, discontinuity can be implemented using an auxiliary matrix $\mathbf{B}$ in the model objective function. Therefore, the system of equations to be solved is:

$$(\mathbf{J^T J} + \lambda \mathbf{W_m^T B^T B W_m})\delta\mathbf{m} = \mathbf{J^T}\delta d - \lambda \mathbf{W_m^T B^T B W_m m}^{(k)} \tag{15}$$

where $\mathbf{B}$ is a diagonal matrix containing 0's and 1's in the diagonal, whenever there is an interface or not, respectively (Clapp et al., 1998).

Determination of these interfaces locations from seismic data itself is not trivial. Semblance panels show series of peaks, which are associated with high probability of reflections locations, and consequently, interfaces. In this paper it is suggested the computation of another coherency measure analogous to semblance, called MUSIC, which presents higher definition in low-noise data as showed in other studies (Barros et al., 2012).

MUSIC is related to signal-to-noise energy ratio, and when signal and noise are uncorrelated, it results to be (Ursin et al., 2013):

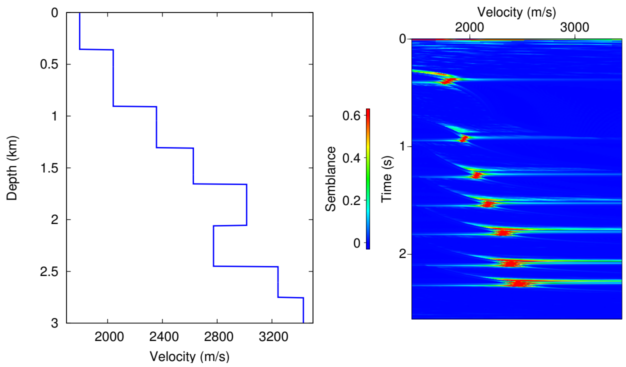
$$P = \frac{1}{1-S} \tag{16}$$

where S is semblance measure. Notice that $0 \leq S \leq 1$ and $1 \leq P \leq \infty$, which makes P (MUSIC) show more individualization of high semblance values, and therefore, it presents more precision at reflections detection.

Results

1D Case

A synthetic velocity profile was arbitrarily created in order to test the method on a single inversion (Fig. 1a). Considering



(a) (b)

Figure 1: (a) 1D synthetic model. (b) Resulting velocity analysis semblance panel, with 1% of additive random noise for better visualization.

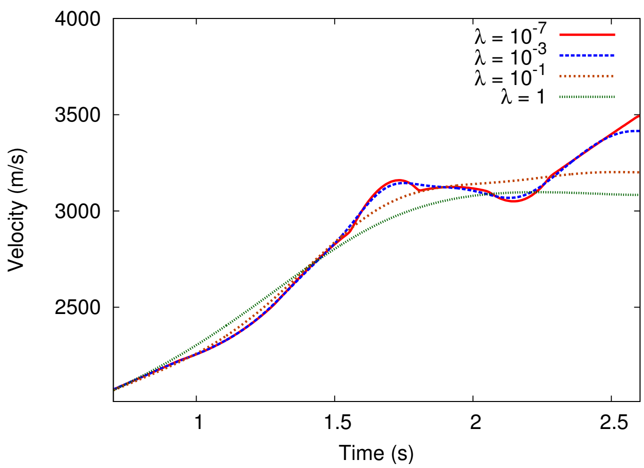


Figure 2: Inversion results with different regularization parameters.

a split-spread synthetic seismic data created using this velocity model, a semblance panel can be computed (Fig. 1b). A manual picking of NMO velocity points is perfomed, and monotonic interpolation results in a NMO velocity profile, with a velocity value at each time sample.

Along this NMO velocity profile, values of semblance at each sample are kept and form a semblance profile (Fig. 3a). In a similar way, MUSIC values form a profile as well (Fig. 3b), and their peaks will be automatically detected in order to map interfaces used in discontinuous inversion. This detection is achieved with a simple algorithm using a mobile time window, selecting peaky values within the window as it goes for all time samples.

Regularization parameters were empiracally chosen, since L-curve technique was applied but it was not sufficiently successful in this inversion. These parameters were such that the data misfit would not exceed a certain value, and closing to the supposed location of the unsuccessful L-curve corner. A plot showing results depeding on the main regularization parameter (λ) is shown in Fig. 2. Once done choosing, a continous inversion was then applied on the NMO velocity profile (Fig. 3c).

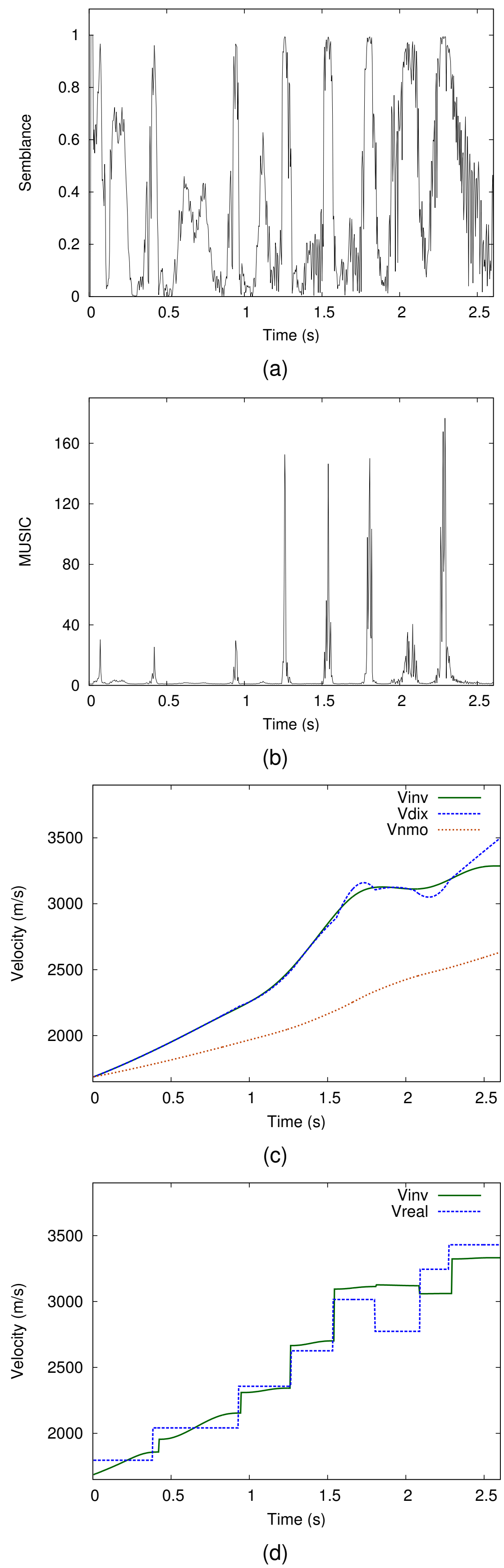


Figure 3: (a) Semblance along the NMO velocity profile, and (b) MUSIC along the same profile. Inversion results: (c) continuous and (d) discontinuous cases.

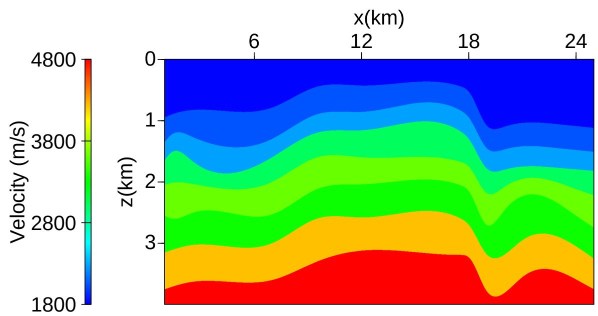


Figure 4: 2D synthetic model.

Using peaks from MUSIC measure, discontinuity locations are determined and blocky inversion can be applied (Fig. 3d).

2D Case

A synthetic model was created (Fig. 4) based on an area of Tacutu Basin, whose real data will also be inverted:

Synthetic seismograms were obtained using Gaussian Beam seismic modeling on this velocity model. These seismograms were acquired in 320 split-spread shot gathers, each one containing 100 traces, having 50m of shot and geophone spacing. Then, seismograms were sorted into CMP gathers and velocity analysis was applied jumping 35 CMP's, in a total of 871 CMP's, resulting in 25 velocity analyzed CMP's gathers. NMO velocities were interpolated for all CMP's, resulting in a NMO velocity field (Fig. 5a).

A discontinuous inversion was applied, using MUSIC measures from those 25 velocity analyzed CMP's. Peaks from MUSIC measures were obtained in a similar manner of the 1D case for each CMP. Incorporating the discontinuity points into inversion, an inverted model with interfaces was produced (Fig. 5c).

Real Case

Land real data were acquired by PETROBRAS from the region of Tacutu intracontinental rift, and they were used in order to apply the method on a real case, verifying whether continuous and discontinuous inversion approaches are possible. Processing steps such as ground-roll filtering and dip-moveout were already applied by Da Silva (2004), making seismic data for this paper ready for velocity analysis.

Once velocity analysis is done for 28 CMP's, similarly to the synthetic 2D case, a NMO velocity field is obtained (Fig. 6a). Inversion is applied using these NMO velocity values, and Dix results are computed as well, in order to compare both (Fig. 6b and 6c).

Persistent noise level in data made the automatic detection of interfaces through MUSIC measure inaccurate. Therefore, a manual picking of interfaces is applied, based on the stacked section brightest reflectors. Four reflectors were chosen to be interfaces, and inverted model is shown in Fig. 6d. NMO velocity field (Fig. 6a) was laterally smoothed prior to serve as input for discontinuous inversion.

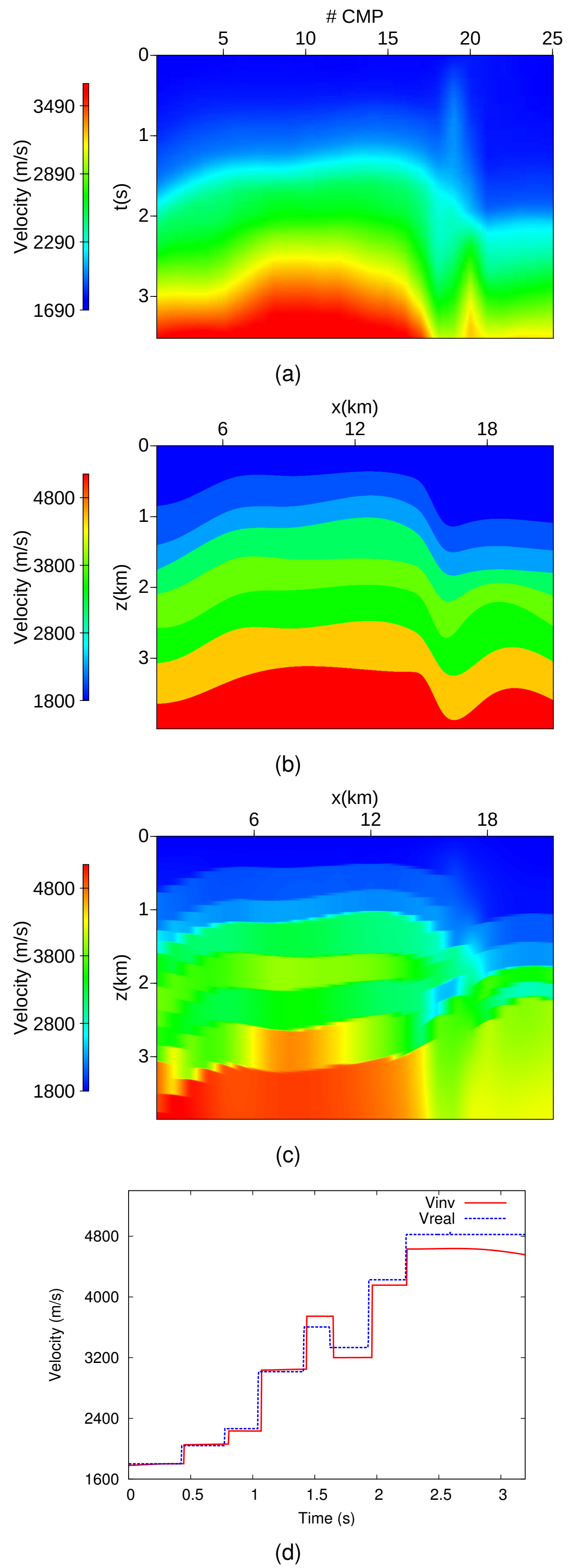


Figure 5: (a) NMO velocity field, input for inversion. (b) 2D synthetic model, windowed to the area of interest. (c) Inverted model, with interfaces, in depth. (d) Comparison between velocity profiles, taken out from 2D models (b and c), at 13.55 km in x position.

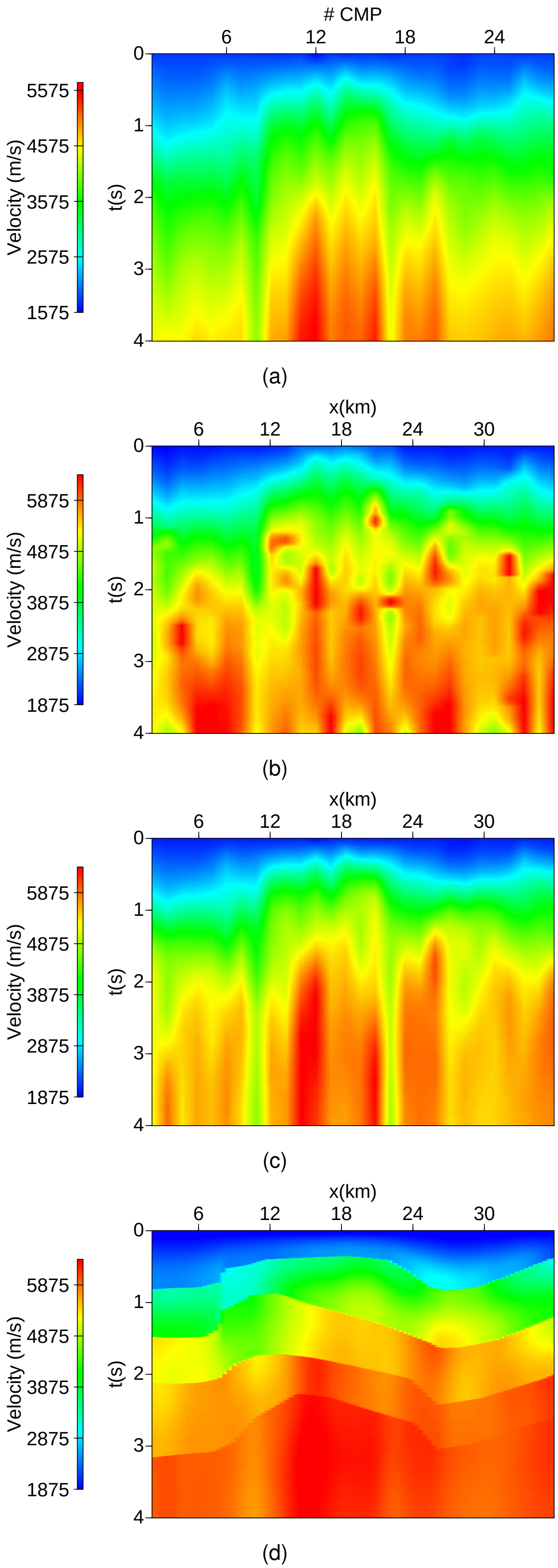


Figure 6: (a) NMO velocity field (input data for inversion). (b) Conventional result by Dix formula, and (c) Inversion result. (d) Inversion with interfaces.

Conclusions

The regularized and linearized inversion has achieved the objective of obtaining plausible velocity models, which are not obtained in some situations by Dix formula. In presented real case, especially, the continuous inversion result was way better than conventional method, whereas in synthetic case, it obtained results smoother enough to optimize Dix results. In 2D synthetic case, regions with high inclination made the results fail locally.

The insertion of information about the interfaces made a discontinuous inversion possible, and consequently, a method to estimate a stratigraphic velocity model in both synthetic and real cases. Automatic detection of interfaces by MUSIC measure failed on real case, but even though, the procedure with chosen interfaces outputs a velocity field that can be developed into a model proper to migration of these real data from Tacutu Basin.

Further studies for this method are required. An automatic regularization parameters estimation is needed, and implementation in other seismic data, maybe marine type, are encouraged.

References

Barros, T., Lopes, R., Tygel, M., and Romano, J., 2012, Implementation aspects of eigenstructure-based velocity spectra: 74th EAGE Conference - Copenhagen.

Buland, A., Kolbjornsen, O., and Carter, A., 2011, Bayesian dix inversion: Geophysics, **76**, no. 2, 15–22.

Clapp, R., Sava, P., and Claerbout, J., 1998, Interval velocity estimation with null-space: Stanford Exploration Project Report.

Da Silva, M. G., 2004, Processamento de dados sísmicos da bacia do tacutu: Master thesis, Universidade Federal da Bahia, Salvador - Brazil.

Harlan, W. S., 1999, Constrained dix inversion: Webpage: http://www.billharlan.com/papers/rmsinv.pdf.

Oldenburg, D., and Li, Y., 2005, Inversion for applied geophysics - a tutorial:, volume 13 , SEG, Tulsa, OK.

Ursin, B., Da Silva, M., and Porsani, M., 2013, Signal and travel-time parameter estimation using singular value decomposition: NTNU/UFBA.

Acknowledgments

The authors wish to express their gratitude to INCT-GP/CNPq/MCT, PETROBRAS, ANP, FINEP, FAPESB Brazil for financial support. We also thank LANDMARK for the licenses granted to CPGG-UFBA.